$\vec{u} \to \langle u_1, u_2 \rangle$ then $\vec{u}\,\vec{v} = u_1 v_1 + u_2 v_2$
$\vec{v} \to \langle v_1, v_2 \rangle$

Precalculus Name _____

Dot Products and Unit Vector Notes

Date _____ Period ____

Find the dot product of the given vectors.

1) $\mathbf{u} = \langle -7, -9 \rangle$
$\mathbf{v} = \langle 3, -2 \rangle$

$(-7)(3) + (-9)(-2) =$
$-21 + 18 = \boxed{-3}$

2) $\mathbf{u} = \langle 4, -2 \rangle$
$\mathbf{v} = \langle 4, -6 \rangle$

$(4)(4) + (-2)(-6)$
$16 + 12 = \boxed{28}$

$\vec{u}\,\vec{v} = |\vec{u}| \cdot |\vec{v}| \cdot \cos\theta$ $\theta \to$ the angle between the 2 vectors

Find the measure of the angle between the two vectors.

3) $\mathbf{u} = -8\mathbf{i} - 3\mathbf{j}$
$\mathbf{v} = -2\mathbf{i} - 4\mathbf{j}$

$\vec{u}\,\vec{v} = (-8)(-2) + (-3)(-4)$
$= 16 + 12$
$= 28$

$|\vec{u}| = \sqrt{8^2 + 3^2} = \sqrt{73}$

$|\vec{v}| = \sqrt{2^2 + 4^2} = \sqrt{20}$

$28 = \sqrt{73}\sqrt{20}\cos\theta \to \cos^{-1}\left(\frac{28}{\sqrt{73}\sqrt{20}}\right) = \theta$

$\theta = 42.88°$

4) $\mathbf{u} = 9\mathbf{i} \to \langle 9, 0 \rangle$
$\mathbf{v} = -6\mathbf{j} \to \langle 0, -6 \rangle$

$\vec{u}\,\vec{v} = 90 + 0 - 6 = 0$

$|\vec{u}| = 9$

$|\vec{v}| = 6$

$0 = 96\cos\theta$

$\cos^{-1}\left(\frac{0}{96}\right) = \theta \to \boxed{\theta = 90°}$

5) $\mathbf{u} = \langle 2, -9 \rangle$
$\mathbf{v} = \langle -4, 18 \rangle$

$\vec{u}\,\vec{v} = (2)(-4) + (-9)(18)$
$= -8 - 162$
$= -170$

$|\vec{u}| = \sqrt{2^2 + 9^2} = \sqrt{4 + 81} = \sqrt{85}$

$|\vec{v}| = \sqrt{4^2 + 18^2} = \sqrt{16 + 324} = \sqrt{340}$

$-170 = \sqrt{85}\sqrt{340}\cos\theta$

$\theta = \cos^{-1}\left(\frac{-170}{\sqrt{85}\cdot\sqrt{340}}\right) \to \boxed{\theta = 180°}$

→ perpendicular

State if the two vectors are parallel, orthogonal, or neither.

6) $\mathbf{u} = 5\mathbf{i} - 35\mathbf{j}$
$\mathbf{v} = -\mathbf{i} + 7\mathbf{j}$

parallel since we see the same ratio

7) $\mathbf{u} = \frac{3}{2}\mathbf{i} - 3\mathbf{j}$
$\mathbf{v} = 3\mathbf{i} - 6\mathbf{j}$

parallel (we see same ratio)

8) $\mathbf{u} = 18\mathbf{i} - 18\mathbf{j}$
$\mathbf{v} = 6\mathbf{i} + \mathbf{j}$

Neither

→ $(18)(6)+(-18)(1) \neq 0$ → not ⊥

→ Not same ratio → not ∥

9) $\mathbf{u} = 2\mathbf{i} + 3\mathbf{j}$
$\mathbf{v} = -\frac{3}{2}\mathbf{i} + \mathbf{j}$

$(2)\left(-\frac{3}{2}\right)+3(1)$

$-3+3=0$

orthogonal

Find the component form of the resultant vector.

10) Given: $T = (4, 4)$ $X = (2, 6)$
Unit vector in the direction of $\overrightarrow{TX}$

mag $\sqrt{(-2)^2+2^2} = \sqrt{8}$

$\overrightarrow{TX} = \langle 2-4, 6-4 \rangle$

$\overrightarrow{TX} = \langle -2, 2 \rangle$

unit vector means mag(length) = 1

$\hat{TX} = \left\langle \frac{-2}{\sqrt{8}}, \frac{2}{\sqrt{8}} \right\rangle$

11) Given: $P = (-8, -5)$ $Q = (-6, -6)$
Unit vector in the opposite direction of $\overrightarrow{PQ}$

$\overrightarrow{PQ} = \langle 2, -1 \rangle$

$-\overrightarrow{PQ} = \langle -2, 1 \rangle$

Mag or $|\overrightarrow{PQ}| = \sqrt{2^2+1^2} = \sqrt{5}$

$\hat{-\overrightarrow{PQ}} = \left\langle \frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right\rangle$

Express the resultant vector as a linear combination of unit vectors i and j.

12) $|\mathbf{f}| = 4, 198°$ → magnitude

Unit vector in the direction of **f**

$x \to \frac{4\cos 198°}{4} \to \frac{-3.804}{4}$

(i)

$y \to \frac{4\sin 198°}{4} \to \frac{-1.224}{4}$

(j)

$\hat{f} = -.95\,i - .309\,j$

13) $|\mathbf{u}| = 22, 115°$

Unit vector in the direction of **u**

$\hat{u} = \langle \cos 115°, \sin 115° \rangle$

$\hat{u} = -.423\,i + .906\,j$

Find the magnitude and direction angle of the resultant vector.

14) Given: $T = (5, 7)$ $X = (-5, 6)$
$Y = (6, -5)$ $Z = (-7, 6)$
Find: $\overrightarrow{TX} - \overrightarrow{YZ}$

$\overrightarrow{TX} = \langle -10, -1 \rangle$

$\overrightarrow{YZ} = \langle -13, 11 \rangle$

$\overrightarrow{TX} - \overrightarrow{YZ} = \langle -10+13, -1-11 \rangle$

$= \langle 3, -12 \rangle$

Mag $\sqrt{3^2+(-12)^2}$

$\sqrt{9+144}$

$\sqrt{153}$ or 12.37

$\tan\theta = \frac{-12}{3}$

$\theta = \tan^{-1}(-4)$

$\theta = -75.964°$

15) Given: $P = (0, 7)$ $Q = (4, 2)$

Unit vector in the opposite direction of $\overrightarrow{PQ}$

$\overrightarrow{PQ} = \langle 4, -5 \rangle$

$\overrightarrow{QP} = \langle -4, 5 \rangle$

$\theta = 128.66°$

mag 1