 Name________________________

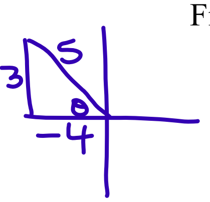
Double Angle and Half Angle Notes

Date________________ Period____

Use a double-angle identity to find the exact value of each expression.

1) $\sin\theta = \frac{3}{5}$ and $90° < \theta < 180°$

Find $\cos 2\theta$



$\cos(2\theta) = 1 - 2\sin^2\theta$

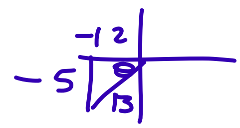
$= 1 - 2\left(\frac{3}{5}\right)^2$

$= \frac{25}{25} - 2\left(\frac{9}{25}\right)$

$\cos(2\theta) = \frac{7}{25}$

2) $\sec\theta = -\frac{13}{12}$ and $180° < \theta < 270°$

Find $\sin 2\theta$



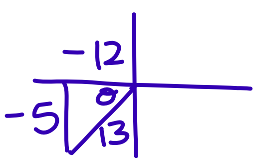
$\sin(2\theta) = 2\sin\theta\cos\theta$

$= 2\left(\frac{-5}{13}\right)\left(\frac{-12}{13}\right)$

$\sin(2\theta) = \frac{120}{169}$

3) $\cot\theta = \frac{12}{5}$ and $180° < \theta < 270°$

Find $\tan 2\theta$



$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$

$= \frac{2\left(\frac{5}{12}\right)}{1 - \left(\frac{5}{12}\right)^2}$

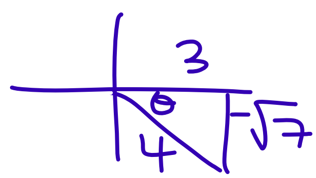
$= \frac{\frac{10}{12} \quad \frac{12}{12}}{\frac{144}{144} - \frac{25}{144}}$

$\tan(2\theta) = \frac{120}{119}$

4) $\cot\theta = -\frac{3\sqrt{7}}{7}$ and $270° < \theta < 360°$

Find $\cos 2\theta$

$-\frac{3}{\sqrt{7}}$



$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$= \left(\frac{3}{4}\right)^2 - \left(\frac{-\sqrt{7}}{4}\right)^2$

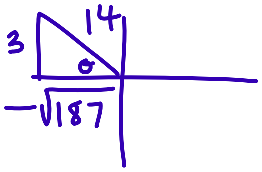
$= \frac{9}{16} - \frac{7}{16}$

$= \frac{2}{16} = \frac{1}{8}$

5) $\tan\theta = -\frac{3\sqrt{187}}{187}$ and $90° < \theta < 180°$

Find $\cos 2\theta$

$$\frac{-3}{\sqrt{187}}$$



$$\left(-\sqrt{187}\right)^2 + 3^2 = 187 + 9 = 196$$

$$\sqrt{196} = 14$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

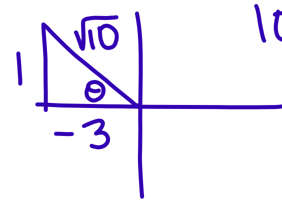
$$= \left(\frac{-\sqrt{187}}{14}\right)^2 - \left(\frac{3}{14}\right)^2$$

$$= \frac{187-9}{196} = \frac{178}{196} = \frac{89}{98}$$

6) $\sec\theta = -\frac{\sqrt{10}}{3}$ and $90° < \theta < 180°$

Find $\tan 2\theta$

$10 - 9 = 1$



$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

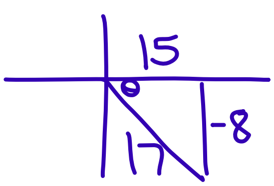
$$= \frac{2\left(-\frac{1}{3}\right)}{1-\left(-\frac{1}{3}\right)^2}$$

$$= \frac{-2/3}{9/9 - 1/9}$$

$$= -\frac{2}{3}\ \frac{9}{8} = \frac{-18}{24} = \frac{-3}{4}$$

7) $\sin\theta = -\frac{8}{17}$ and $\frac{3\pi}{2} < \theta < 2\pi$

Find $\tan 2\theta$



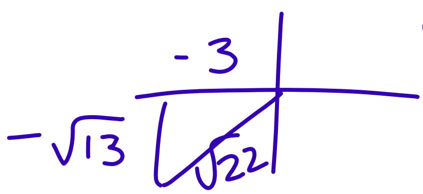
$$\tan(2\theta) = \frac{2\left(\frac{-8}{15}\right)}{1-\left(\frac{-8}{15}\right)^2}$$

$$= \frac{\frac{-16}{15}\ \frac{15}{15}}{\frac{225}{225} - \frac{64}{225}} = \frac{-240}{161}$$

8) $\cos\theta = -\frac{3\sqrt{22}}{22}$ and $\pi < \theta < \frac{3\pi}{2}$

Find $\sin 2\theta$

$$-\frac{3}{\sqrt{22}}$$



$$\sin(2\theta)$$

$$= 2\left(\frac{-\sqrt{13}}{\sqrt{22}}\right)\left(\frac{-3}{\sqrt{22}}\right)$$

$$= \frac{6\sqrt{13}}{22}$$

$$= \frac{3\sqrt{13}}{11}$$

$\cos(2\theta) = 1 - 2\sin^2\theta \rightarrow \cos(2\theta) - 1 = -2\sin^2\theta$

$\boxed{\sin^2\theta = \frac{1-\cos 2\theta}{2}}$

Use a half-angle identity to find the exact value of each expression.

9) $\sin 22\frac{1}{2}°$

$\sin^2\left(22\tfrac{1}{2}\right) = \frac{1 - \cos(45)}{2}$

$= \frac{\frac{2}{2} - \frac{\sqrt{2}}{2}}{2}$

$= \frac{\sqrt{2-\sqrt{2}}}{\sqrt{4}}$

$\boxed{= \frac{\sqrt{2-\sqrt{2}}}{2}}$

10) $\cos 165°$

$\cos(2\theta) = 2\cos^2\theta - 1$

$\boxed{\frac{1+\cos 2\theta}{2} = \cos^2\theta}$

$\cos^2(165°) = \frac{1+\cos(330°)}{2}$

$= \frac{\frac{2}{2} + \frac{\sqrt{3}}{2}}{2}$

$\sqrt{\cos^2(165°)} = \frac{\sqrt{2+\sqrt{3}}}{\sqrt{4}}$

$\cos(165°) = -\frac{\sqrt{2+\sqrt{3}}}{2}$

11) $\tan \frac{11\pi}{12}$

$\tan^2(\theta) = \frac{\sin^2\theta}{\cos^2\theta} = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$

$\tan^2\left(\frac{11\pi}{12}\right) = \frac{1-\cos\left(\frac{11\pi}{6}\right)}{1+\cos\left(\frac{11\pi}{6}\right)}$

$= \frac{\frac{2}{2} - \frac{\sqrt{3}}{2}}{\frac{2}{2} + \frac{\sqrt{3}}{2}}$

$= \frac{2-\sqrt{3}}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \frac{\sqrt{(2-\sqrt{3})^2}}{\sqrt{4-3=1}} = -(2-\sqrt{3})$

$\downarrow$ tan in Q2 is negative

12) $\cos \frac{\pi}{8}$

$\cos^2\left(\frac{\pi}{8}\right) = \frac{1+\cos\left(\frac{\pi}{4}\right)}{2}$

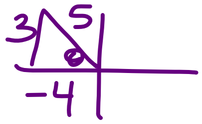
$= \frac{\frac{2}{2} + \frac{\sqrt{2}}{2}}{2}$

$\cos^2\left(\frac{\pi}{8}\right) = \frac{2+\sqrt{2}}{4}$

$\cos\left(\frac{\pi}{8}\right) = \frac{\sqrt{2+\sqrt{2}}}{2}$

13) $\cos\theta = -\frac{4}{5}$ and $90° < \theta < 180°$

Find $\sin\frac{\theta}{2}$



$$\sin^2\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{2}$$

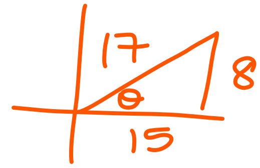
$$= \frac{\frac{5}{5}+\frac{4}{5}}{2}$$

$$\sqrt{\sin^2\left(\frac{\theta}{2}\right)} = \frac{\sqrt{9}}{\sqrt{10}}$$

$$\sin\left(\frac{\theta}{2}\right) = \frac{3}{\sqrt{10}} \text{ or } \frac{3\sqrt{10}}{10}$$

14) $\sin\theta = \frac{8}{17}$ and $0° < \theta < 90°$

Find $\sin\frac{\theta}{2}$



$$\sin^2\left(\frac{\theta}{2}\right) = \frac{1-\cos(\theta)}{2}$$

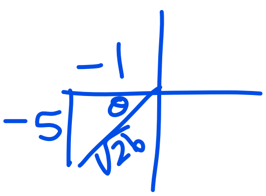
$$= \frac{\frac{17}{17}-\frac{15}{17}}{2}$$

$$\sin^2\left(\frac{\theta}{2}\right) = \frac{1}{17}$$

$$\sin\left(\frac{\theta}{2}\right) = \frac{\sqrt{17}}{17}$$

15) $\cos\theta = -\frac{\sqrt{26}}{26}$ and $180° < \theta < 270°$

Find $\tan\frac{\theta}{2}$ $\quad -\frac{1}{\sqrt{26}} \quad \frac{(a)}{h}$



$$\tan^2\left(\frac{\theta}{2}\right) = \frac{1-\cos(\theta)}{1+\cos(\theta)}$$

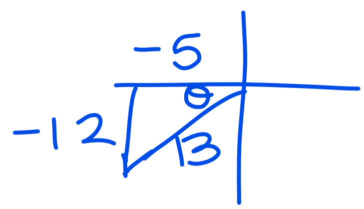
$$= \frac{\frac{\sqrt{26}}{\sqrt{26}}+\frac{1}{\sqrt{26}}}{\frac{\sqrt{26}}{\sqrt{26}}-\frac{1}{\sqrt{26}}}$$

$$= \frac{\sqrt{26}+1}{\sqrt{26}-1} \cdot \frac{\sqrt{26}+1}{\sqrt{26}+1}$$

$$\tan^2\left(\frac{\theta}{2}\right) = \frac{(\sqrt{26}+1)^2}{26-1} \rightarrow \tan\left(\frac{\theta}{2}\right) = \frac{-(\sqrt{26}+1)}{5}$$

16) $\csc\theta = -\frac{13}{12}$ and $180° < \theta < 270°$

Find $\cos\frac{\theta}{2}$



$$\cos^2\left(\frac{\theta}{2}\right) = \frac{1+\cos(\theta)}{2}$$

$$= \frac{\frac{13}{13}-\frac{5}{13}}{2} = \frac{\frac{8}{13}}{2}$$

$$\sqrt{\cos^2\left(\frac{\theta}{2}\right)} = \frac{\sqrt{4}}{\sqrt{13}} \rightarrow \cos\left(\frac{\theta}{2}\right) = \frac{2}{\sqrt{13}}$$

$$\cos\left(\frac{\theta}{2}\right) = -\frac{2\sqrt{13}}{13}$$