Pre Calculus Name_____

Double Angle and Half Angle Notes

Date_____ Period____

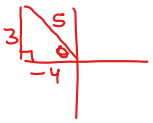
$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

Use a double-angle identity to find the exact value of each expression.

$\cos(\theta+\theta) = \cos\theta\cos\theta - \sin\theta\sin\theta$

1) $\sin\theta = \frac{3}{5}$ and $90° < \theta < 180°$

Find $\cos 2\theta$



$= \left(\frac{-4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

Check

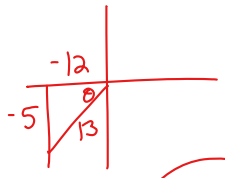
$1-2\sin^2\theta$

$1-2\left(\frac{3}{5}\right)^2$

$1-2\frac{9}{25} \rightarrow 1-\frac{18}{25} \rightarrow \frac{25}{25}-\frac{18}{25} = \frac{7}{25}$

2) $\sec\theta = -\frac{13}{12}$ and $180° < \theta < 270°$

Find $\sin 2\theta$

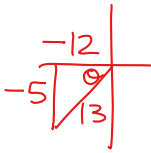


$\sin 2\theta = 2\left(\frac{-5}{13}\right)\left(\frac{-12}{13}\right) = \frac{120}{169}$

3) $\cot\theta = \frac{12}{5}$ and $180° < \theta < 270°$

Find $\tan 2\theta$

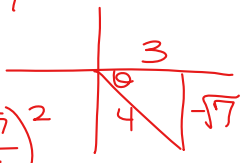
$= \frac{2\left(\frac{5}{12}\right)}{1-\left(\frac{5}{12}\right)^2}$



$= \frac{\frac{10}{12}\cdot\frac{12}{12}}{\frac{144}{144}-\frac{25}{144}} \rightarrow \frac{120}{119}$

4) $\cot\theta = -\frac{3\sqrt{7}}{7}$ and ~~$270° < \theta < 360°$~~ $\frac{a}{o}$ $\frac{\sqrt{7}}{\sqrt{7}}$ $\frac{-3}{\sqrt{7}}$ $\frac{7}{\sqrt{7}}$

Find $\cos 2\theta$



$= \left(\frac{3}{4}\right)^2 - \left(\frac{-\sqrt{7}}{4}\right)^2$

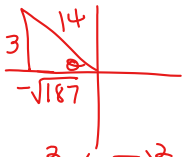
$= \frac{9}{16} - \frac{7}{16} = \frac{2}{16} = \frac{1}{8}$

$* \; 2\left(\frac{3}{4}\right)^2 - 1 = 2\left(\frac{9}{16}\right) - 1 = \frac{18}{16} - \frac{16}{16} = \frac{2}{16} = \frac{1}{8}$ ✓

5) $\tan\theta = -\frac{3\sqrt{187}}{187}$ and $90° < \theta < 180°$

Find $\cos 2\theta$

$\frac{-3}{\sqrt{187}}$



$3^2 + (-\sqrt{187})^2$
$= 9 + 187$
$= 196 \rightarrow \sqrt{196} = 14$

$1 - 2\sin^2\theta$

$1 - 2\left(\frac{3}{14}\right)^2$

$1 - 2\left(\frac{9}{196}\right)$

$\frac{98}{98} - \frac{9}{98} = \boxed{\frac{89}{98}}$

6) $\sec\theta = -\frac{\sqrt{10}}{3}$ and $90° < \theta < 180°$

Find $\tan 2\theta$

$(\sqrt{10})^2 = y^2 + (-3)^2$

$10 = y^2 + 9$

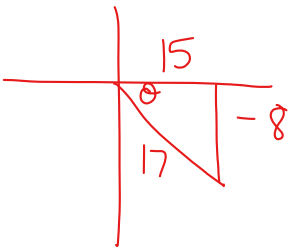
$y^2 = 1 \rightarrow y = 1$

$\tan 2\theta = \frac{2\left(-\frac{1}{3}\right)}{1 - \left(-\frac{1}{3}\right)^2}$

$\frac{-\frac{2}{3}\,\frac{3}{3}}{\frac{9}{9} - \frac{1}{9}} \rightarrow \frac{-6}{8} = \boxed{-3/4}$

7) $\sin\theta = -\frac{8}{17}$ and $\frac{3\pi}{2} < \theta < 2\pi$

Find $\tan 2\theta$



$\frac{2\left(\frac{-8}{15}\right)}{1 - \left(\frac{-8}{15}\right)^2}$

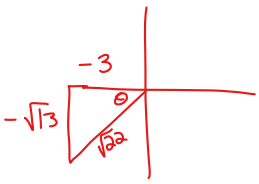
$\frac{-16}{15}\,\frac{15}{15} = \boxed{\frac{-240}{161}}$

$\frac{225}{225} - \frac{64}{225}$

8) $\cos\theta = -\frac{3\sqrt{22}}{22}$ and $\pi < \theta < \frac{3\pi}{2}$

Find $\sin 2\theta$

$-\frac{3}{\sqrt{22}}$



$2\left(\frac{-\sqrt{13}}{\sqrt{22}}\right)\left(\frac{-3}{\sqrt{22}}\right)$

$= \frac{6\sqrt{13}}{22}$

$= \boxed{\frac{3\sqrt{13}}{11}}$

Use a half-angle identity to find the exact value of each expression.

9) $\sin 22\frac{1}{2}°$

$\sin\left(22\frac{1}{2}\right) = \sin\left(\frac{45°}{2}\right)$

$\sin^2\left(\frac{45°}{2}\right) = \frac{1-\cos 45°}{2/1}$

$= \frac{\frac{2}{2} - \frac{\sqrt{2}}{2}}{2/1} \rightarrow \frac{2-\sqrt{2}}{2} \cdot \frac{1}{2}$

$\sin^2\left(\frac{45°}{2}\right) = \frac{2-\sqrt{2}}{4}$

$\sin(22.5) = \frac{\sqrt{2-\sqrt{2}}}{\sqrt{4}} = \frac{\sqrt{2-\sqrt{2}}}{2}$

10) $\cos 165°$

$\cos^2\left(\frac{330°}{2}\right) = \frac{\cos 330° + 1}{2}$

$= \frac{\frac{\sqrt{3}}{2} + \frac{2}{2}}{2}$

$= \frac{\sqrt{3}+2}{2} \cdot \frac{1}{2}$

$\cos^2(165°) = \frac{\sqrt{3}+2}{4}$

$\cos 165° = \frac{-\sqrt{\sqrt{3}+2}}{2}$

11) $\tan \frac{11\pi}{12}$

$\tan^2\left(\frac{11\pi/6}{2}\right) = \frac{1-\cos\frac{11\pi}{6}}{1+\cos\frac{11\pi}{6}}$

$= \frac{\frac{2}{2} - \frac{\sqrt{3}}{2}}{\frac{2}{2} + \frac{\sqrt{3}}{2}}$

$= \frac{2-\sqrt{3}}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \frac{(2-\sqrt{3})^2}{4-3} = \sqrt{(2-\sqrt{3})^2}$

$= -2+\sqrt{3}$

12) $\cos \frac{\pi}{8}$

$\cos^2\left(\frac{\pi/4}{2}\right) = \frac{1+\cos\left(\frac{\pi}{4}\right)}{2}$

$\frac{\frac{2}{2} + \frac{\sqrt{2}}{2}}{2}$

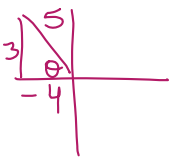
$\cos^2\left(\frac{\pi}{8}\right) = \frac{2+\sqrt{2}}{4}$

$\cos\frac{\pi}{8} = \frac{\sqrt{2+\sqrt{2}}}{2}$

13) $\cos\theta = -\frac{4}{5}$ and $90° < \theta < 180°$

Find $\sin\frac{\theta}{2}$

$45° < \frac{\theta}{2} < 90°$



$= \frac{3}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} \rightarrow \frac{3\sqrt{10}}{10}$

$\sin^2\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{2}$

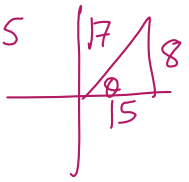
$= \frac{\frac{5}{5} + \frac{4}{5}}{2} = \frac{\frac{9}{5}}{2}$

$\sin^2\left(\frac{\theta}{2}\right) = \frac{9}{10}$

14) $\sin\theta = \frac{8}{17}$ and $0° < \theta < 90°$

Find $\sin\frac{\theta}{2}$

$0 < \frac{\theta}{2} < 45$



$\sin^2\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{2}$

$= \frac{\frac{17}{17} - \frac{15}{17}}{2} = \frac{\frac{2}{17}}{2} = \frac{1}{17}$

$\sin\left(\frac{\theta}{2}\right) = \frac{\sqrt{1}}{\sqrt{17}} \cdot \frac{\sqrt{17}}{\sqrt{17}} = \frac{\sqrt{17}}{17}$

15) $\cos\theta = -\frac{\sqrt{26}}{26}$ and $180° < \theta < 270°$

Find $\tan\frac{\theta}{2}$

$-\frac{1}{\sqrt{26}}$

$90° < \frac{\theta}{2} < 135°$

$\tan^2\left(\frac{\theta}{2}\right) = \frac{1 + \frac{1}{\sqrt{26}}}{1 - \frac{1}{\sqrt{26}}}$

$= \frac{(\sqrt{26}+1)}{(\sqrt{26}-1)} \cdot \frac{(\sqrt{26}+1)}{(\sqrt{26}+1)}$

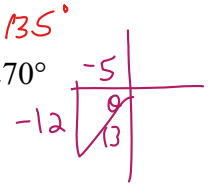
$\sqrt{\tan^2\left(\frac{\theta}{2}\right)} = \frac{\sqrt{(\sqrt{26}+1)^2}}{\sqrt{25}}$

$\tan\left(\frac{\theta}{2}\right) = \frac{\sqrt{26}+1}{5}$

16) $\csc\theta = -\frac{13}{12}$ and $180° < \theta < 270°$

Find $\cos\frac{\theta}{2}$

$90° < \theta/2 < 135°$



$\cos^2\left(\frac{\theta}{2}\right) = \frac{1+\cos\theta}{2}$

$= \frac{\frac{13}{13} - \frac{5}{13}}{2}$

$= \frac{8}{13} \cdot \frac{1}{2} = \frac{4}{13}$

$\cos\left(\frac{\theta}{2}\right) = \frac{\sqrt{4}}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}}$

$= \frac{2\sqrt{13}}{13}$

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Use a double-angle identity to find the exact value of each expression.

1) $\sin\theta = \frac{3}{5}$ and $90° < \theta < 180°$

Find $\cos 2\theta$

$\frac{7}{25}$

2) $\sec\theta = -\frac{13}{12}$ and $180° < \theta < 270°$ $\frac{120}{169}$

Find $\sin 2\theta$

3) $\cot\theta = \frac{12}{5}$ and $180° < \theta < 270°$

Find $\tan 2\theta$

$\frac{120}{119}$

4) $\cot\theta = -\frac{3\sqrt{7}}{7}$ and $270° < \theta < 360°$ $\frac{1}{8}$

Find $\cos 2\theta$

5) $\tan\theta = -\frac{3\sqrt{187}}{187}$ and $90° < \theta < 180°$

Find $\cos 2\theta$

$\frac{89}{98}$

6) $\sec\theta = -\frac{\sqrt{10}}{3}$ and $90° < \theta < 180°$ $-\frac{3}{4}$

Find $\tan 2\theta$

7) $\sin\theta = -\frac{8}{17}$ and $\frac{3\pi}{2} < \theta < 2\pi$

Find $\tan 2\theta$

$-\frac{240}{161}$

8) $\cos\theta = -\frac{3\sqrt{22}}{22}$ and $\pi < \theta < \frac{3\pi}{2}$ $\frac{3\sqrt{13}}{11}$

Find $\sin 2\theta$

Use a half-angle identity to find the exact value of each expression.

9) $\sin 22\frac{1}{2}°$

$\frac{\sqrt{2-\sqrt{2}}}{2}$

10) $\cos 165°$ $-\frac{\sqrt{2+\sqrt{3}}}{2}$

11) $\tan \frac{11\pi}{12}$

$-2+\sqrt{3}$

12) $\cos \frac{\pi}{8}$ $\frac{\sqrt{2+\sqrt{2}}}{2}$

13) $\cos\theta = -\frac{4}{5}$ and $90° < \theta < 180°$

Find $\sin\frac{\theta}{2}$

$\frac{3\sqrt{10}}{10}$

14) $\sin\theta = \frac{8}{17}$ and $0° < \theta < 90°$ $\frac{\sqrt{17}}{17}$

Find $\sin\frac{\theta}{2}$

15) $\cos\theta = -\frac{\sqrt{26}}{26}$ and $180° < \theta < 270°$

Find $\tan\frac{\theta}{2}$

$\frac{-\sqrt{26}-1}{5}$

16) $\csc\theta = -\frac{13}{12}$ and $180° < \theta < 270°$

Find $\cos\frac{\theta}{2}$

$-\frac{2\sqrt{13}}{13}$