AB Calculus Euler's Method Day 2 Homework

Name: Key

1. Evaluate each integral.

a) $\int \frac{1}{\sqrt{64-36x^2}}dx \rightarrow \sqrt{64(1-\frac{36}{64}x^2)}$

$\int \frac{1}{8\sqrt{1-(\frac{3}{4}x)^2}}dx$ $\quad (\frac{6}{8}x)^2 \rightarrow (\frac{3}{4}x)^2$

$u=\frac{3}{4}x$ $\quad \frac{du}{dx}=\frac{3}{4}$ $\quad \frac{4}{3}du=dx$

$\frac{4}{3}\cdot\frac{1}{8}\int\frac{1}{\sqrt{1-u^2}}du$

$\rightarrow \frac{1}{6}\sin^{-1}u+C \rightarrow \boxed{\frac{1}{6}\sin^{-1}(\frac{3}{4}x)+C}$

b) $\int \frac{1}{2x\sqrt{x^2-16}}dx \rightarrow 2x\sqrt{16(\frac{1}{16}x^2-1)}$

$2x\cdot 4\sqrt{(\frac{x}{4})^2-1}$

$\frac{1}{2}\cdot\frac{1}{4}\int\frac{1}{x\sqrt{(\frac{x}{4})^2-1}}dx \rightarrow u=\frac{x}{4} \rightarrow 4u=x$

$\frac{du}{dx}=\frac{1}{4}$ $\quad 4\,du=dx$

$\rightarrow \frac{1}{2}\cdot\frac{1}{4}\cdot 4\int\frac{1}{4u\sqrt{u^2-1}}du$

$\rightarrow \frac{1}{8}\int\frac{1}{u\sqrt{u^2-1}}du$

$\rightarrow \frac{1}{8}\sec^{-1}u+C \rightarrow \boxed{\frac{1}{8}\sec^{-1}(\frac{x}{4})+C}$

2. The curve passing through (2, 0) satisfies the differential equation $\frac{dy}{dx}=4x+y$. Find an approximation to $y(3)$ using Euler's Method with two equal steps.

$\frac{(3-2)}{2}=\frac{1}{2}=dx$

old pt	dx	$m=\frac{dy}{dx}$	$dy=m\cdot dx$	new pt
(2,0)	.5	8	8(.5) = 4	(2.5, 4)
(2.5, 4)	.5	14	14(.5) = 7	(3, 11)

$y(3)\approx 11$

3. Assume that f and its derivative f' have the values given in the table. Use Euler's Method with two equal steps to approximate the value of $f(4.4)$

x	4	4.2	4.4
$f'(x)$	-0.5	-0.3	-0.1
$f(x)$	2	1.9	1.84

old pt	dx	$m=\frac{dy}{dx}$	$dy=m\cdot dx$	new pt.
(4,2)	.2	-.5	(-.5)(.2) = -.1	(4.2, 1.9)
(4.2, 1.9)	.2	-.3	(-.3)(.2) = -.06	(4.4, 1.84)

$\boxed{f(4.4)\approx 1.84}$

4. Solve each of the following:

a) Find the general solution for $\frac{dy}{dx}=\frac{(1-x)}{y^2}$ $\quad \int y^2\,dy=\int(1-x)\,dx$

$\frac{y^3}{3}=x-\frac{x^2}{2}+C \rightarrow y^3=3x-\frac{3}{2}x^2+c$

$\boxed{y=\sqrt[3]{3x-\frac{3}{2}x^2+c}}$

b) Find the ~~particular~~ general solution for $\frac{dy}{dx}=xy^2$

$\int\frac{dy}{y^2}=\int x\,dx$

$-\frac{1}{y}=\frac{x^2}{2}+C$

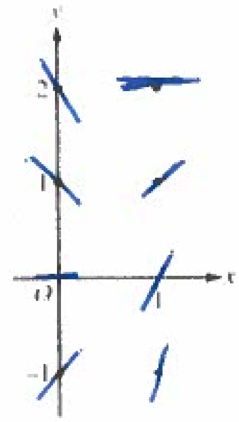
$-1=(\frac{x^2}{2}+C)y$

$\rightarrow \boxed{y=\frac{-1}{\frac{x^2}{2}+C}}$

5.

Consider the differential equation $\frac{dy}{dx} = 2x - y$.

(a) On the axes provided, sketch a slope field for the given differential equation at the ~~six~~ 8 points indicated.



(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

(c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.

(d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.

(b) $\frac{dy}{dx} = 2x - y$

$\frac{d^2y}{dx^2} = 2 - \frac{dy}{dx}$

$\frac{d^2y}{dx^2} = 2 - 2x + y$

II $\rightarrow (-,+) \rightarrow 2 - 2(-) + (+) = +$

all concave up since $\frac{d^2y}{dx^2} > 0$

(c) $(2,3) \rightarrow \frac{dy}{dx} = 2(2) - 3 = 4 - 3 = 1$

neither b/c $\frac{dy}{dx} \neq 0$ nor is undefined

(d) $y = mx + b$ or $\frac{dy}{dx} = m = 2x - y$

$y = (2x - y)x + b$

so $m = 2x - (mx + b)$

$m = 2x - mx - b$

or $m = (2 - m)x - b$

$0x + m = (2 - m)x - b$

$0 = 2 - m$

$m = 2$

$m = -b$

$2 = -b$

$b = -2$