AB Calculus Euler's Method Homework — Name: Key

1. Find the particular solution y = f(x) using the given initial condition.

a) $\frac{dy}{dx} = -\frac{1}{x^2} - \frac{3}{x^4} + 12$ and $y = 3$ when $x = 1$.

$\int dy = \int \left(\frac{-1}{x^2} - \frac{3}{x^4} + 12\right) dx$

$y = \frac{1}{x} + \frac{1}{x^3} + 12x + C$ → plug in (1,3)

$\boxed{y = \frac{1}{x} + \frac{1}{x^3} + 12x - 11}$

b) $\frac{dx}{dt} = \frac{1}{t} - \frac{1}{t^2} + 6$ and $x = 0$ when $t = 1$.

pt: (1, 0)

$\int dx = \int \left(\frac{1}{t} - \frac{1}{t^2} + 6\right) dt$

$x = \ln|t| + \frac{1}{t} + 6t + C$

plug in (1, 0) → $C = -7$

$\boxed{x = \ln|t| + \frac{1}{t} + 6t - 7}$

c) $\frac{du}{dx} = 7x^6 - 3x^2 + 5$ and $u = 1$ when $x = 1$.

$\int du = \int (7x^6 - 3x^2 + 5) dx$

$u = x^7 - x^3 + 5x + C$

(1,1) $\quad 1 = 1 - 1 + 5 + C \quad C = -4$

(x, u)

$\boxed{u = x^7 - x^3 + 5x - 4}$

d) $\frac{dv}{dt} = 4\sec t \tan t + e^t + 6t$ and $v = 5$ when $t = 0$.

$\int dv = \int (4\sec t \tan t + e^t + 6t) dt$

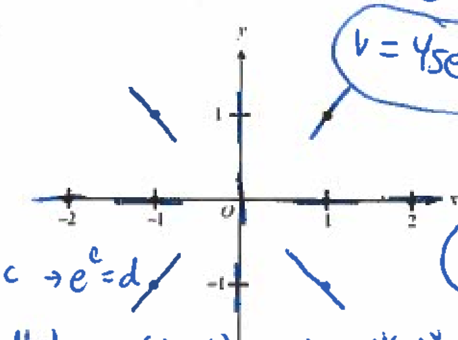
(t, v) $\quad v = 4\sec t + e^t + 3t^2 + C$

$(0, 5)$ $\quad 5 = 4\sec(0) + e^0 + 3(0)^2 + C \rightarrow C = 0$

$\boxed{v = 4\sec t + e^t + 3t^2}$

2. [No Calculator] Consider the differential equation $\frac{dy}{dx} = \frac{y}{x}$, where $x \neq 0$.

a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.



b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

$\int \frac{dy}{y} = \int \frac{dx}{x}$

$\ln|y| = \ln|x| + C \qquad e^{\ln|y|} = e^{\ln|x| + C} \rightarrow e^C = d$

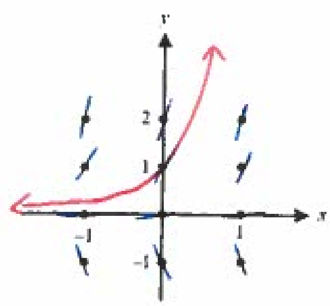
$y = d|x| \rightarrow (-1, +1) \rightarrow 1 = d|-1| \quad d = +1$

or $\ln|1| = \ln|-1| + C$
$0 = 0 + C \Rightarrow C = 0$

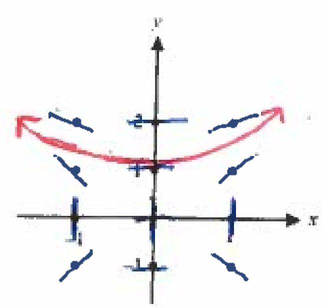
$\boxed{y = |x|}$

3. Construct a slope field for each differential equation. Draw tiny segments through the twelve lattice points shown in the graph.

a) $\frac{dy}{dx} = 2y$



b) $\frac{dy}{dx} = \frac{x}{2y}$



For each slope field above, sketch the solution curve that passes through the point (0, 1).

4. Answer the following questions. $(0,3)$ $dx = \frac{1-0}{2} = \frac{1}{2}$

a) Given the differential equation $\frac{dy}{dx} = x+2$ and $y(0) = 3$. Find an approximation for $y(1)$ by using Euler's method with two equal steps. Sketch you solution.

b) Solve the differential equation $\frac{dy}{dx} = x+2$ with the initial condition $y(0) = 3$, and use your solution to find $y(1)$.

c) The error in using Euler's Method is the difference between the approximate value and the exact value. What was the error in your answer? How could you produce a smaller error using Euler's Method?

a

old pt	dx	$m = \frac{dy}{dx}$	$dy = m\,dx$	new pt
(0, 3)	.5	0 + 2 = 2	2(.5) = 1	(.5, 4)
(.5, 4)	.5	.5 + 2 = 2.5	2.5(.5) = 1.25	(1, 5.25)

$\rightarrow \boxed{y(1) \approx 5.25}$

b

$\int dy = \int (x+2)\,dx$

$y = \frac{x^2}{2} + 2x + C$

$3 = 0 + 0 + C \rightarrow C = 3$

$\boxed{y = \frac{1}{2}x^2 + 2x + 3}$

$y = \frac{1}{2}(1)^2 + 2(1) + 3$

$= 5.5 \rightarrow \boxed{y(1) = 5.5}$

c

Actual - predicted

$5.5 - 5.25 = .25$

use more (smaller) steps (dx)

5. Suppose a continuous function f and its derivative f' have values that are given in the following table. Given that $f(2) = 5$, use Euler's Method with two steps of size $\Delta x = 0.5$ to approximate the value of $f(3)$.

x	2.0	2.5	3.0
$f'(x)$	0.4	0.6	0.8
$f(x)$	5	5.2	5.5

$\boxed{f(3) \approx 5.5}$

old pt.	dx	$m = \frac{dy}{dx}$	$dy = m \cdot dx$	new pt
(2, 5)	.5	.4	(.4)(.5) = .2	(2.5, 5.2)
(2.5, 5.2)	.5	.6	(.6)(.5) = .3	(3, 5.5)