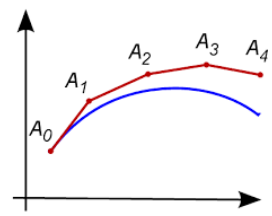
AB Calculus Euler's Method Notesheet **Name: ____**

For differential equations that cannot be solved, a slope field provides a graphical solution to the differential equation. The problem with this approach is that a slope field is only really good for getting general trends in solutions and for long-term behavior of solutions. There are times when we will need something more. For example, it is sometimes useful to know how a specific solution behaves, including some values that the solution will take.

In these cases, we must resort to numerical methods that will allow us to **approximate** the solutions to differential equations. There are many different methods that can be used to approximate solutions to a differential equation. We have already seen one example, linearization, otherwise known as tangent line approximations, which can give us a decent approximation of a solution near a point of tangency. Today, we are going to look at a method devised by 18$^{\text{th}}$ century mathematician Leonhard Euler that expands upon this idea.



Euler's method basically involves walking out along a tightrope from an initial point along its tangent line. Instead of walking along the same line the whole time as in a tangent line approximation, we change tangent lines with each step (of length dx). This involves recalculating the point and slope after each step. This will produce a much more accurate approximation than simply using the original tangent line. The process itself is pretty easy and repetitive, and it is easier demonstrated with an example rather than a complicated formula. First, a bit of background information.

- We will need to designate the number of equal steps we would like to take. Call this number n.
- To find dx, the length of each step, we calculate how far we want to go and divide by how many steps we want to take. Assume $x = a$ is our initial x-value and $x = b$ is the x-value we are trying to get to. Then $dx = \frac{b-a}{n}$.
- Recall slope $m = \frac{dy}{dx}$. If you solve for dy, we get $dy = m(dx)$.

We can now proceed. The following chart will make things easier to organize. Commit it to memory.

x	y	$m = \left.\frac{dy}{dx}\right\|_{(x,y)}$	$dy = m(dx)$	y_{new}
a	$y(a)$			

Example 1 Given the differential equation $\frac{dy}{dx} = x - 2$ and $y(0) = 5$.

a) Find an approximation for $y(0.8)$ by using Euler's method with two equal steps.

b) Solve the differential equation $\frac{dy}{dx} = x - 2$ with the initial condition $y(0) = 5$, and use your solution to find $y(0.8)$.

Example 2 If $\frac{dy}{dx} = 2x - y$ and $y = 3$ when $x = 2$, use Euler's method with 3 equal steps to approximate y when $x = 1.7$.

Example 3 Assume that f and f' have the values given in the table. Use Euler's method with two equal steps to approximate the value of $f(2.6)$.

x	3	2.8	2.6
$f'(x)$	0.4	0.7	0.9
$f(x)$	2		

Example 4 Consider the differential equation $\frac{dy}{dx} = y^2(2x + 2)$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(0) = -1$.

a) Find $\lim_{x \to 0} \frac{f(x)+1}{\sin x}$. Show the work that leads to your answer.

b) Use Euler's Method, starting at $x = 0$ with two steps of equal size, to approximate $f\left(\frac{1}{2}\right)$.

c) Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = -1$.