

1.)  $3 \csc^2 x = 4$

$$\csc^2 x = \frac{4}{3}$$

$$\sqrt{\sin^2 x} = \frac{\sqrt{3}}{\sqrt{4}}$$

$$\sin x = \pm \frac{\sqrt{3}}{2}$$

$$X = \left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \right\}$$

2.)  $\sec x \tan x - \cos x \cot x = \sin x$

$$\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} - \frac{\cos x}{1} \frac{\cos x}{\sin x}$$

$$\frac{\sin x}{\sin x} \frac{\sin x}{\cos^2 x} - \frac{\cos^2 x \cos^2 x}{\sin x \cos^2 x} \sin x \frac{\sin x \cos^2 x}{\sin x \cos^2 x} = 0$$

$$\sin^2 x - \cos^4 x - \sin^2 x \cos^2 x = 0$$

$$1 - \cos^2 x - \cos^4 x - (1 - \cos^2 x) \cos^2 x = 0$$

$$1 - \cos^2 x - \cos^4 x - \cos^2 x + \cos^4 x = 0$$

3.)  $2 \sin^2 x - \cos x = 1$

$$2(1 - \cos^2 x) - \cos x = 1$$

$$2 - 2\cos^2 x - \cos x - 1 = 0$$

$$2\cos^2 x + \cos x - 1 = 0$$

$$(2\cos x - 1)(\cos x + 1) = 0$$

$$\cos x = \frac{1}{2} \quad \cos x = -1$$

$$1 - 2\cos^2 x = 0$$

$$2\cos^2 x = 1$$

$$\sqrt{\cos^2 x} = \frac{\sqrt{1}}{\sqrt{2}} \rightarrow \frac{1}{\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}}$$

$$\cos x = \pm \frac{\sqrt{2}}{2}$$

$$X = \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\}$$

$$X = \left\{ \frac{\pi}{3}, \frac{5\pi}{3}, \pi \right\}$$

$$4.) 3 \sec^2 x + 4 \cos^2 x = 7 \cos^2 x$$

$$\frac{3}{\cos^2 x} + \frac{4 \cos^2 x}{\cos^2 x} = \frac{7 \cos^2 x}{\cos^2 x}$$

$$\sqrt{\cos^2 x} = \frac{\sqrt{3}}{\sqrt{4}}$$

$$\sqrt{\cos^2 x} = \sqrt{1}$$

$$3 + 4 \cos^4 x = 7 \cos^2 x$$

$$\cos x = \pm \frac{\sqrt{3}}{2}$$

$$\cos x = \pm 1$$

$$4 \cos^4 x - 7 \cos^2 x + 3 = 0$$

$$(4 \cos^2 x - 3)(\cos^2 x - 1) = 0$$

$$x = \left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, 0, \pi \right\}$$

$$5.) 2 \cos^2 x - 5 \cos x = -2$$

$$2 \cos^2 x - 5 \cos x + 2 = 0$$

$$(2 \cos x - 1)(\cos x - 2) = 0$$

$$\cos x = \frac{1}{2}$$

$$\cancel{\cos x = 2}$$
  
Not possible

$$x = \left\{ \frac{\pi}{3}, \frac{5\pi}{3} \right\}$$

$$6.) 1 - \cos x = \sin^2 x$$

$$1 - 2 \cos x + \cos^2 x = \sin^2 x$$

$$1 - 2 \cos x + \cos^2 x = 1 - \cos^2 x$$

$$2 \cos^2 x - 2 \cos x = 0$$

$$2 \cos x (\cos x - 1) = 0$$

$$\cos x = 0 \quad \cos x = 1$$

$$x = \left\{ \frac{\pi}{2}, \frac{3\pi}{2}, 0 \right\}$$

extraneous  
(Doesn't satisfy original equation)

$$7.) 4 \sin x \cos x = \sqrt{3}$$

$$\frac{4 \sin x \cos x}{4} = \frac{\sqrt{3}}{4}$$

$$x = \left\{ \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3} \right\}$$

$$\sin x = \pm \frac{\sqrt{3}}{2} \quad \cos x = \pm \frac{1}{2}$$

or

$$\sin x = \pm \frac{1}{2} \quad \cos x = \pm \frac{\sqrt{3}}{2}$$

$$\sin a \cos b + \sin b \cos a = \sin(a+b)$$

$$8.) \sin 2x \cos 64 + \sin 64 \cos 2x = \frac{\sqrt{3}}{2}$$

Domain  $[0, 360)$

$$\sin^{-1} \left[ \sin(2x + 64) = \frac{\sqrt{3}}{2} \right]$$

$$2x + 64 = \{60, 120\} + 360n$$

-64                      -64, -64

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$$\frac{2x}{2} = \left\{ \frac{-4}{2}, \frac{56}{2} \right\} + \frac{360n}{2}$$

$$x = \{-2, 28\} + 180n$$

$$x = \{28, 178, 208, 358\}$$

$$9.) 3 \sin x = 7 \cos x$$

$$\textcircled{1} \quad 9 \sin^2 x = 49 \cos^2 x$$

$$9 \sin^2 x - 49 \cos^2 x = 0$$

$$9(1 - \cos^2 x) - 49 \cos^2 x = 0$$

$$9 - 9 \cos^2 x - 49 \cos^2 x = 0$$

$$9 - 58 \cos^2 x = 0$$

$$9 = 58 \cos^2 x$$

$$\frac{9}{58} = \cos^2 x \rightarrow \pm \frac{\sqrt{9}}{\sqrt{58}} = \cos x \rightarrow x = \cos^{-1} \left( \pm \frac{\sqrt{9}}{\sqrt{58}} \right)$$

$$\textcircled{2} \quad \frac{3 \sin x}{7 \cos x} = 1$$

$$\frac{3}{7} \tan x = 1$$

$$\tan x = \frac{7}{3}$$

$$x = \tan^{-1} \left( \frac{7}{3} \right) \text{ Q1,3}$$

$$x = \{1166, 4307\}$$

$$x = \{1166, 4307\}$$

only Q1,3

$$\textcircled{3} \quad y_1 = 3 \sin x$$

$$y_2 = 7 \cos x$$

Calculate intersections

$$10.) \tan^4 x - 13 \tan^2 x + 36 = 0$$

$[0, 2\pi)$

$$(\tan^2 x - 9)(\tan^2 x - 4) = 0$$

$$(\tan x - 3)(\tan x + 3)(\tan x - 2)(\tan x + 2) = 0$$

$$\tan x = 3 \quad \tan x = -3 \quad \tan x = 2 \quad \tan x = -2$$

$$x = \{1249, 4391, 1893, 5034\} \text{ from } \tan x = \pm 3$$

$$x = \{1107, 4249, 2034, 5176\}$$

From  $\tan x = \pm 2$