

1.) $3 \csc^2 x = 4$

$$\sqrt{\csc^2 x} = \sqrt{\frac{4}{3}}$$

$$\csc x = \pm \frac{2}{\sqrt{3}}$$

$$\sin x = \pm \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

2.) $\sec x \tan x - \cos x \cot x = \sin x$

$$\frac{\sin x}{\cos^2 x} - \frac{\cos^2 x}{\sin x} = \sin x$$

$$\frac{\sin x}{\cos^2 x} = \frac{\sin x}{\sin x} + \frac{\cos^2 x}{\sin x}$$

$$\frac{\sin x}{\cos^2 x} = \frac{1}{\sin x}$$

3.) $2 \sin^2 x - \cos x = 1$ $\sin^2 x + \cos^2 x = 1$

$$2 \sin^2 x - \cos x - 1 = 0$$

$$2(1 - \cos^2 x) - \cos x - 1 = 0$$

$$2 - 2\cos^2 x - \cos x - 1 = 0$$

$$0 = 2\cos^2 x + \cos x - 1$$

$$0 = (2\cos x - 1)(\cos x + 1)$$

$$\sin^2 x = \cos^2 x$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

CHECK

$$\frac{2}{\sqrt{2}} \cdot 1 - \frac{\sqrt{2}}{2} \cdot 1 = \frac{\sqrt{2}}{2}$$

$$\frac{2\sqrt{2}}{2} \rightarrow \sqrt{2} - \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} \checkmark$$

$$-\sqrt{2} - \frac{-\sqrt{2}}{2} \rightarrow -\sqrt{2} + \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2} \checkmark$$

$$\cos x = \frac{1}{2} \quad \cos x = -1$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3} \quad x = \pi$$

$$4 \cdot 3 = \frac{12}{4,3} \rightarrow 7$$

$$4.) 3 \sec^2 x + 4 \cos^2 x = 7$$

$$\frac{3}{\cos^2 x} + \frac{4 \cos^2 x}{\cos^2 x} = 7 \frac{\cos^2 x}{\cos^2 x}$$

$$3 + 4 \cos^2 x = 7 \cos^2 x$$

$$4 \cos^2 x - 7 \cos^2 x + 3 = 0$$

$$(\cos^2 x - 1)(4 \cos^2 x - 3) = 0$$

$$5.) 2 \cos^2 x - 5 \cos x = -2$$

$$2 \cos^2 x - 5 \cos x + 2 = 0$$

$$(2 \cos x - 1)(\cos x - 2)$$

$$\cos x = \frac{1}{2} \quad \cos x = 2$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$6.) (1 - \cos x)^2 = (\sin x)^2$$

$$1 - 2 \cos x + \cos^2 x = \sin^2 x$$

$$1 - 2 \cos x + \cos^2 x = 1 - \cos^2 x$$

$$-2 \cos x + 2 \cos^2 x = 0$$

$$2 \cos x (-1 + \cos x) = 0$$

$$\cos x = 0$$

$$\cos x = 1$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, 0$$

$$7.) \frac{4 \sin x \cos x}{4} = \frac{\sqrt{3}}{4}$$

$$(\sin x \cos x)^2 = \left(\frac{\sqrt{3}}{4}\right)^2$$

$$\sin^2 x \cos^2 x = \frac{3}{16}$$

$$\sin^2 x (1 - \sin^2 x) = \frac{3}{16}$$

$$\sin^2 x - \sin^4 x = \frac{3}{16}$$

$$0 = \sin^4 x - \sin^2 x + \frac{3}{16}$$

$$0 = 16 \sin^4 x - 16 \sin^2 x + 3$$

$$0 = (4 \sin^2 x - 1)(4 \sin^2 x - 3)$$

$$\sin^2 x = \frac{1}{4} \quad \sin^2 x = \frac{3}{4}$$

$$\sin x = \pm \frac{1}{2} \quad \sin x = \pm \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{6}, \frac{7\pi}{6} \quad x = \frac{\pi}{3}, \frac{4\pi}{3}$$

$$\frac{4 \sin x \cos x}{2} = \frac{\sqrt{3}}{2}$$

$$2 \sin x \cos x = \frac{\sqrt{3}}{2}$$

$$\sin(2x) = \frac{\sqrt{3}}{2}$$

$$\frac{2x}{2} = \frac{\pi}{3} + \frac{2\pi n}{2}, \frac{2\pi}{3} + \frac{2\pi n}{2}$$

$$x = \frac{\pi}{6} + \pi n, \frac{\pi}{3} + \pi n$$

$$x = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{\pi}{3}, \frac{4\pi}{3}$$

$$\cos^2 x = 1$$

$$\cos^2 x = \frac{3}{4}$$

$$\cos x = \pm 1$$

$$\cos x = \pm \frac{\sqrt{3}}{2}$$

$$x = 0, \pi$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\sin a \cos b + \sin b \cos a$$

$$8.) \sin 2x \cos 64 + \sin 64 \cos 2x = \frac{\sqrt{3}}{2}$$

Domain $[0, 360)$

$$\sin(2x+64) = \frac{\sqrt{3}}{2}$$

$$2x+64 = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$2x+64 = 60 + 360n, 120 + 360n$$

$$\frac{2x}{2} = \frac{-4 + 360n}{2}, \frac{56 + 360n}{2}$$

$$x = -2 + 180n, 28 + 180n$$

$$x = \{178^\circ, 358^\circ, 28^\circ, 208^\circ\}$$

$$9.) (3 \sin x)^2 = (7 \cos x)^2$$

$$9 \sin^2 x = 49 \cos^2 x$$

$$9(1 - \cos^2 x) = 49 \cos^2 x$$

$$9 - 9 \cos^2 x = 49 \cos^2 x$$

$$\frac{9}{58} = \frac{58 \cos^2 x}{58}$$

$$\frac{9}{58} = \cos^2 x \rightarrow \cos x = \pm \sqrt{\frac{9}{58}}$$

$$\cos^{-1} \sqrt{\frac{9}{58}}$$

$$x = 66.8^\circ, 246.8^\circ$$

$$10.) \tan^4 x - 13 \tan^2 x + 36 = 0$$

$$(\tan^2 x - 4)(\tan^2 x - 9) = 0$$

$$(\tan x - 2)(\tan x + 2)(\tan x - 3)(\tan x + 3) = 0$$

$$\tan x = 2 \quad \tan x = -2 \quad \tan x = 3 \quad \tan x = -3$$

Inverse
tan all

$$x = \{63.4^\circ, 243.4^\circ, 116.6^\circ, 296.6^\circ, 71.6^\circ, 251.6^\circ, 108.4^\circ, 288.4^\circ\}$$

