Name Key Date________ Period______

Worksheet 7.3—Separable Differential Equations
Show all work on a separate sheet of paper. No Calculator unless specified.

Multiple Choice

$.01 = e^{K(199)} \rightarrow \frac{\ln .01}{199} = \frac{199K}{199}$ $K \approx .0231...$

1. (OK, so you can use your calculator right away on a non-calculator worksheet. Use it on this one.) A sample of Kk-1234 (an isotope of Kulmakorpium) loses 99% of its radioactive matter in 199 hours. What is the half-life of Kk-1234?
   (A) 4 hours (B) 6 hours (C) 30 hours (D) 100.5 hours (E) 143 hours

   $\frac{1}{2} = e^{-.0231t}$ $\frac{\ln \frac{1}{2}}{-.0231} = \frac{-.0231t}{-.0231}$

2. In which of the following models is $\frac{dy}{dt}$ directly proportional to y?

   I. $y = e^{kt} + C$
   II. $y = Ce^{kt}$
   III. $y = 28^{kt}$
   IV. $y = 3\left(\frac{1}{2}\right)^{3t+1}$

   I. $\frac{dy}{dt} = ke^{kt}$ X
   II. $\frac{dy}{dt} = k \cdot Ce^{kt}$ ✓
   III. $\frac{dy}{dt} = K \cdot \ln 28 \cdot 28^{kt}$ ✓
   IV. $\frac{dy}{dt} = 3 \cdot 3 \cdot \ln(\frac{1}{2}) \cdot (\frac{1}{2})^{3t+1}$ ✓

   (A) I only (B) II only (C) I and II only (D) II and III only (E) II, III, and IV (F) all of them

3. (Use your calculator on this one, too, but get the exact answer first.) The rate at which acreage is being consumed by a plot of kudzu is proportional to the number of acres already consumed at time t. If there are 2 acres consumed when $t = 1$ and 3 acres consumed when $t = 5$, how many acres will be consumed when $t = 8$?
   (A) 3.750 (B) 4.000 (C) 4.066 (D) 4.132 (E) 4.600

   ① $2 = Ce^{K(1)}$
   $3 = Ce^{5K}$
   $\frac{3}{2} = e^{4K}$ $\frac{\ln(\frac{3}{2})}{4} = \frac{4K}{4}$
   Back into equation ① $C = 1.807...$ stored in calc
   $y = Ce^{K(8)}$

Free Response

For problems 4 – 13, find the general solution to the following differential equations, then find the particular solution using the initial condition.

4. $\frac{dy}{dx} = \frac{x}{y}$, $y(1) = -2$
5. $\frac{dy}{dx} = -\frac{x}{y}$, $y(4) = 3$
6. $\frac{dy}{dx} = \frac{y}{x}$, $y(2) = 2$
7. $\frac{dy}{dx} = 2xy$, $y(0) = -3$
8. $\frac{dy}{dx} = (y+5)(x+2)$, $y(0) = -1$
9. $\frac{dy}{dx} = \cos^2 y$, $y(0) = 0$
10. $\frac{dy}{dx} = (\cos x)e^{y+\sin x}$, $y(0) = 0$
11. $\frac{dy}{dx} = e^{x-y}$, $y(0) = 2$
12. $\frac{dy}{dx} = -2xy^2$, $y(1) = 0.25$
13. $\frac{dy}{dx} = \frac{4\sqrt{y}\ln x}{x}$, $y(e) = 1$

For problems 14 – 17, find the solution of the differential equation $\frac{dy}{dt} = ky$ that satisfies the given conditions.

14. $k = 1.5$, $y(0) = 100$

15. $k = -0.5$, $y(0) = 200$

16. $y(0) = 50$, $y(5) = 100$

17. $y(1) = 55$, $y(10) = 30$ (divide one by the other)

18. Let $v(t)$ be the velocity, in feet per second, of a skydiver at time t seconds, $t \geq 0$. After her parachute opens, her velocity satisfies the differential equation $\frac{dv}{dt} = -2v - 32$, with initial condition $v(0) = -50$.

a) Use separation of variables to find an expression for v in terms of t, where t is measured in seconds.

b) Terminal velocity is defined as $\lim_{t\to\infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second.

c) It is safe to land when her speed is 20 feet per second. At what time t does she reach this speed?

d) If the skydiver's parachute opens at an altitude of 2,000 feet, how long will it take her to reach the ground, and will it be safe for her to land when she does reach the ground?

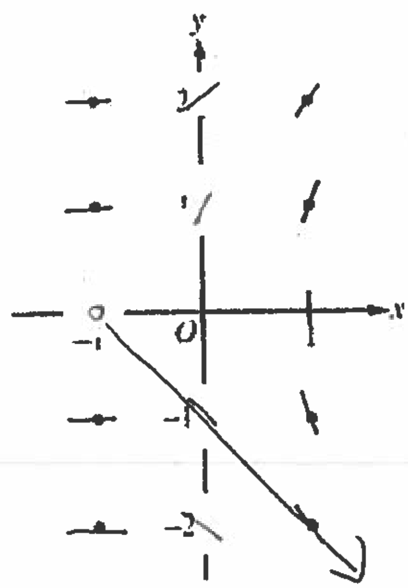
19. AP 2010B-5 (No Calculator)

Consider the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and for $-1 < x < 1$, sketch the solution curve that passes through the point $(0, -1)$.

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane for which $y \neq 0$. Describe all points in the xy-plane, $y \neq 0$, for which $\frac{dy}{dx} = -1$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -2$.



$$\frac{dy}{dx} = -1$$

(b) $\int dy = \int -1\, dx$

$y = -x + C$

$-1 = 0 + C$

$\boxed{y = -x - 1}$

(c) $y \cdot dy = (x+1)\, dx$

$\frac{y^2}{2} = \frac{x^2}{2} + x + C$

$y^2 = x^2 + 2x + 2C$

$(-2)^2 = 2C$

$4 = 2C \rightarrow C = 2$

$y^2 = x^2 + 2x + 4$

$\boxed{y = -\sqrt{x^2 + 2x + 4}}$

4.) $\frac{dy}{dx} = \frac{x}{y} \rightarrow \int y\,dy = \int x\,dx \rightarrow \frac{y^2}{2} = \frac{x^2}{2} + C$

$y^2 = x^2 + 2C \rightarrow y = \pm\sqrt{x^2+2C}$ G.S.

$(-2)^2 = 1^2 + 2C \rightarrow 4 = 1+2C \quad 2C = 3 \rightarrow y = -\sqrt{x^2+3}$ P.S.

5.) $\frac{dy}{dx} = \frac{-x}{y} \rightarrow \int y\,dy = \int -x\,dx \rightarrow \frac{y^2}{2} = \frac{-x^2}{2} + C$

$y^2 = -x^2 + 2C \rightarrow y = \pm\sqrt{-x^2+2C}$ G.S.

$3^2 = -4^2 + 2C \rightarrow 9 = -16 + 2C \rightarrow 2C = 25 \rightarrow y = \sqrt{-x^2+25}$ P.S.

6.) $\frac{dy}{dx} = \frac{y}{x} \rightarrow \int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx \rightarrow \ln|y| = \ln|x| + C$

$y = Ce^{\ln|x|}$ G.S. $\rightarrow \ln|2| = \ln|2| + C \quad C = 0 \rightarrow$ but my "C" is really e^C so $e^0 = 1$

$y = e^{\ln|x|}$ P.S. (or $y = |x|$)

7.) $\frac{dy}{dx} = 2xy \rightarrow \int \frac{dy}{y} = \int 2x\,dx \rightarrow \ln|y| = x^2 + C$

$y = Ce^{x^2}$ G.S. $\quad \ln|-3| = C \rightarrow$ old C so "new" $e^C = e^{\ln|-3|} = 3$

$y = -3e^{x^2}$ P.S.

8.) $\frac{dy}{dx} = (y+5)(x+2) \rightarrow \frac{dy}{y+5} = (x+2)\,dx$

$\ln|y+5| = \frac{x^2}{2} + 2x + c$

$y+5 = ce^{\frac{x^2}{2}+2x} \rightarrow y = ce^{\frac{x^2}{2}+2x} - 5$ G.S.

$\ln|-1+5| = c$ new $c = e^{\ln 4} = 4$ $y = 4e^{\frac{x^2}{2}+2x} - 5$ P.S.

9.) $\frac{dy}{dx} = \cos^2 y \rightarrow \frac{dy}{\cos^2 y} = dx \rightarrow \int \sec^2 y\,dy = \int dx$

$\tan y = x + c$ $\tan 0 = 0 + c$ $c = 0$

$y = \tan^{-1}(x+c)$ G.S. $y = \tan^{-1} x$ P.S.

10.) $\frac{dy}{dx} = (\cos x)e^{y+\sin x} \rightarrow \frac{dy}{dx} = \cos x \cdot e^y \cdot e^{\sin x}$

$\frac{dy}{e^y} = \cos x \cdot e^{\sin x}\,dx \rightarrow \int e^{-y}\,dy = \int \cos x \cdot e^{\sin x}\,dx$

$u = \sin x$
$du = \cos x\,dx$
$\int e^u\,du = e^u + c$

$-e^{-y} = e^{\sin x} + c$ $\searrow$

$e^{-y} = -e^{\sin x} - c$ $-e^{-0} = e^0 + c$

$-1 = 1 + c$

$-y = \ln(-e^{\sin x} - c)$ $c = -2$

$y = -\ln(-e^{\sin x} - c)$ $y = -\ln(-e^{\sin x} + 2)$

11.) $\frac{dy}{dx} = e^{x-y} \rightarrow \frac{dy}{dx} = \frac{e^x}{e^y} \rightarrow \int e^y dy = \int e^x dx$

$e^y = e^x + C \rightarrow y = \ln(e^x + c)$ G.S.

$e^2 = e^0 + c \rightarrow e^2 = 1 + c \quad c = e^2 - 1 \rightarrow y = \ln(e^x + e^2 - 1)$ P.S.

12.) $\frac{dy}{dx} = -2xy^2 \rightarrow \int \frac{dy}{y^2} = \int -2x\,dx$

$\frac{-1}{y} = -x^2 + c \rightarrow y = \frac{-1}{-x^2 + c}$ G.S.

$\frac{-1}{1/4} = -1^2 + c \rightarrow -4 = -1 + c \rightarrow c = -3 \qquad y = \frac{-1}{-x^2 - 3}$ P.S.

or $y = \frac{1}{x^2 + 3}$

13.) $\frac{dy}{dx} = \frac{4\sqrt{y}\ln x}{x} \rightarrow \int \frac{dy}{\sqrt{y}} = \int \frac{4\ln x}{x} dx$

$2\sqrt{y} = \frac{4(\ln x)^2}{2} + C \qquad \rightarrow 2\sqrt{1} = \frac{4}{2}(\ln e)^2 + C$

$2 = 2 + c \quad c = 0$

$\sqrt{y} = (\ln x)^2 + \frac{c}{2}$

$y = (\ln x)^4$ P.S.

$y = \left((\ln x)^2 + \frac{c}{2}\right)^2$ G.S.

14.) $\frac{dy}{dt} = ky$ $\quad$ $y = ce^{kt}$

$100 = ce^{1.5(0)}$

$y = 100e^{1.5t}$

$\int \frac{dy}{y} = \int k\,dt$

$\ln|y| = kt + c$

15.) $200 = ce^{-.5(0)}$ $\quad$ $y = 200e^{-.5t}$

16.) $100 = ce^{5k}$ $\rightarrow$ $2 = e^{5k}$

$50 = ce^{0k}$ $\quad$ $\ln 2 = 5k$

$c = 50$ $\quad$ $y = 50e^{\frac{\ln 2}{5}t}$

17.) $30 = ce^{10k}$ $\quad$ $\frac{30}{55} = e^{9k}$

$55 = ce^{k}$ $\quad$ $\ln\left(\frac{6}{11}\right) = 9k$

$30 = ce^{10\left(\frac{\ln(6/11)}{9}\right)}$

$y = 30\left(\frac{6}{11}\right)^{-10/9} e^{\frac{\ln \frac{6}{11}}{9} t}$

$\frac{30}{e^{10\left(\frac{\ln(6/11)}{9}\right)}} = c$

$e^{\frac{10}{9} \cdot \ln \frac{6}{11}}$

$\left(e^{\ln \frac{6}{11}}\right)^{\frac{10}{9}}$

$\left(\frac{6}{11}\right)^{\frac{10}{9}} \rightarrow \frac{30}{\left(\frac{6}{11}\right)^{10/9}} = c$

18.) $\frac{dv}{dt} = -2v - 32$

a

$$\int \frac{dv}{-2v-32} = \int dt$$

$$-\frac{1}{2}\int \frac{dv}{v+16} = \int dt$$

$$-\frac{1}{2}\ln|v+16| = t + c$$

$$\ln|v+16| = -2t - 2c$$

$$v+16 = ce^{-2t}$$

new "c"

$$v = ce^{-2t} - 16$$

$$-50 = ce^{0} - 16$$

$$-34 = c$$

$$\boxed{v = -34e^{-2t} - 16}$$

b $\lim_{t\to\infty} v \Rightarrow \frac{-34}{e^{\infty}} - 16$

$\to 0 - 16$

$\boxed{= -16 \text{ ft/s}}$

c $-20 = -34e^{-2t} - 16$

$-4 = -34e^{-2t}$

$\frac{-4}{-34} \to \frac{2}{17} = e^{-2t} \to \frac{\ln\left(\frac{2}{17}\right)}{-2} = \frac{-2t}{-2}$ $\boxed{\to t \approx 1.07 \text{ sec}}$

d $\int v = \int(-34e^{-2t} - 16)\,dt$

$S(t) = 17e^{-2t} - 16t + c$

$s(0) = 17e^{0} - 16(0) + c$

2000

$C = 1983$

$S(t) = 17e^{-2t} - 16t + 1983$

$0 = 17e^{-2t} - 16t + 1983$ (calc)

$\boxed{t \approx 123.9375 \text{ seconds}}$

$v(123.9375) = -16$ ft/seconds

So yes, safe!