AB Calculus Extreme Values of a Function Homework Name: Key

1. For each of the following, find all points of relative minima and maxima

a) $y = 2x^2 - 16x + 28$

$4x - 16 = 0$

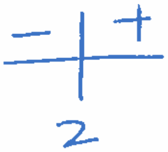
$x = 4$

-4 is an rel/ abs. min @ 4

$32 - 64 + 28 = -32 + 28$

b) $y = 2x^2 - 8x + 8$

$4x - 8$



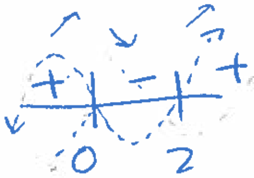
0 is abs./rel min@2

$8 - 16 + 8 = 0$

c) $y = x^3 - 3x^2 + 3$

$3x^2 - 6x$

$3x(x-2)$



3 is rel max @ 0

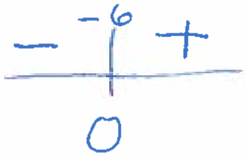
-1 is rel min @ 2

d) $y = (3x + 18)^{\frac{2}{3}}$

$\frac{2}{3}(3x+18)^{-\frac{1}{3}}(3)$

$\frac{2}{\sqrt[3]{3x+18}}$

$x = -6$

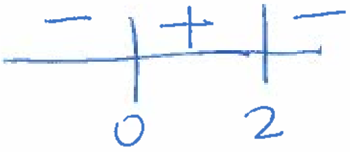


0 is rel/abs min

e) $f(x) = -x^3 + 3x^2 - 5$

$-3x^2 + 6x = 0$

$-3x(x-2)$



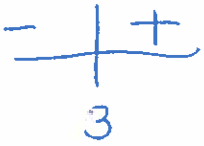
rel. min -5 @ 0

rel. max -1 @ 2

$-8 + 12 = 4 - 5$

f) $f(x) = x^2 - 6x + 8$

$2x - 6$



abs/rel. min

-1 @ 3

2. For each of the following, find all points of relative minima and maxima over the given interval

absolute

a) $y = -\sin x$ over $\left[\frac{\pi}{4}, \frac{\pi}{3}\right]$ → exclusively decreasing

$-\cos x = 0$

$\cos x = 0$

$x = \frac{\pi}{2}$ → not in domain

Not a relative b/c endpoint

abs. min $= -\frac{\sqrt{3}}{2}$ @ $x = \frac{\pi}{3}$

$-\sin\frac{\pi}{4} \rightarrow -\frac{\sqrt{2}}{\sqrt{2}}$

$-\sin\frac{\pi}{3} \rightarrow -\frac{\sqrt{3}}{2}$

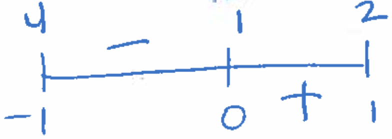
abs max $= -\frac{\sqrt{2}}{2}$ @ $x = \frac{\pi}{4}$ → not rel. b/c endpoint

b) $y = -x^3 + 2x^2 + 1$ over $[-1, 1]$

$-3x^2 + 4x = 0$

$-x(3x - 4) = 0$

$1, -1, 0, \frac{4}{3}$ outside



abs. max=4 @ −1

abs min=1 @ x=0 /rel

3. Find $\frac{dy}{dx}$ for each of the following

a) $y = e^{\cos^2 3x}$

$e^x \quad x^2 \quad \cos x \quad 3x$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$e^x \quad 2x \quad -\sin x \quad 3$

$e^{\cos^2 3x} \cdot 2(\cos 3x) \cdot -\sin(3x) \cdot 3$

b) $y = 3^{4x}$

$3^x \quad 4x$

$\downarrow \quad \downarrow$

$3^x \cdot \ln 3 \quad 4$

$\ln 3 \cdot 3^{4x} \cdot 4$

c) $y = \tan^{-1}(e^x)$

$y' = \frac{1}{1+e^{2x}} \cdot e^x$

d) $y = \ln(\csc x)$

$\ln x \quad \csc x$

$\frac{1}{x} \quad -\cot x \csc x$

$\frac{1}{\csc x} \cdot -\cot x \csc x = -\cot x$

e) $y = \log_5(5x^2 - 3x + 1)$

$5^y = 5x^2 - 3x + 1$

$5^y \cdot \ln 5 \cdot y' = 10x - 3$

$y' = \frac{10x - 3}{(5x^2 - 3x + 1) \cdot \ln 5}$

f) $f(x) = x^2 \ln x$

$2x \ln x + \frac{x^2}{x}$

$= 2x \ln x + x$

g) $y = (\tan x)^x$

$\ln y = x \ln \tan x$

$\frac{y'}{y} = \frac{x \sec^2 x}{\tan x} + \ln \tan x$

$y' = (\tan x)^x \left(\frac{x \sec^2 x}{\tan x} + \ln(\tan x) \right)$

h) $y^2 - x^3 y = 4$

$2yy' - x^3 y' - 3x^2 y = 0$

$y' = \frac{3x^2 y}{2y - x^3}$

i) $f(x) = x \cos^{-1} x$

$1 \cos^{-1} x + \frac{x(-1)}{\sqrt{1-x^2}}$

j $e^y + y^3 - x^2 + \cos y = 3$

$e^y y' + 3y^2 y' - 2x - \sin y \cdot y' = 0$

$y' = \frac{2x}{e^y + 3y^2 - \sin y}$

4. Find the equation of the tangent line to the graph of $y^3 + (xy + 2)^2 = 0$ at the point $(3, -1)$

$3y^2 y' + 2(xy+2)(y + xy') = 0$

$3y^2 y' + (2xy + 4)(y + xy')$

$3y^2 y' + 2xy^2 + 2x^2 yy' + 4y + 4xy' = 0$

$y' = \frac{-2xy^2 - 4y}{3y^2 + 2x^2 y + 4x}$

$y' = \frac{2}{3}$

$y + 1 = \frac{2}{3}(x - 3)$