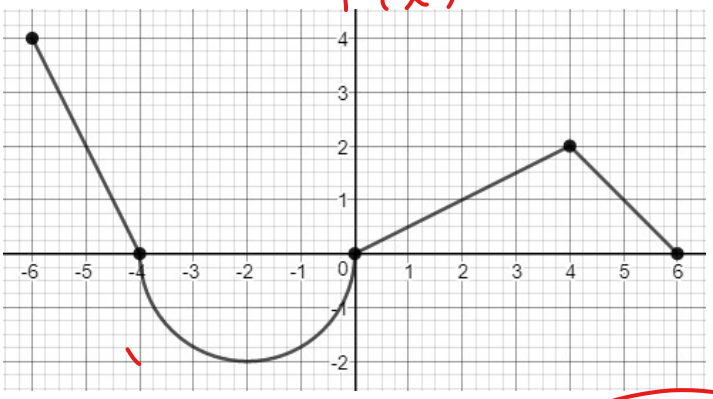
AB Calculus: Information About f(x) Given f'(x) **Name:** _____

In this section, we are going to look at the relationships we have previously learned about a function, its derivative, and its second derivative through another perspective. We have already been given either the graph of the original function or the equation for the original function and been able to find information about the original function such as: intervals the function is increasing or decreasing, x-coordinates of relative extrema, intervals the function is concave up or concave down, and x-coordinates of points of inflection. Today, we will learn how to find this same information given the graph of the derivative.

Example 1: The function f is defined and differentiable on the closed interval $[-6, 6]$. The graph of $y = f'(x)$, the derivative of f, consists of a semicircle and three line segments, as shown in the figure below.



a) Find the x-coordinate of each critical point of $y = f(x)$ on the interval $-6 < x < 6$. Justify your answer.

CP $f'(x) = 0$ or $f'(x)$ is undefined

$x = -4$ $x = 0$

b) Find the x-coordinate of each relative extrema of $y = f(x)$ on the interval $-6 < x < 6$. Label each as a minimum or maximum and justify each response.

relative maximum at $x = -4$ $f'(x)$ changes + to −

relative minimum at $x = 0$ $f'(x)$ changes − to +

c) Find the open intervals over which the function $y = f(x)$ is increasing and decreasing on the interval $-6 < x < 6$. Justify your answer.

$f(x)$ inc $(-6,-4)$ $(0,6)$ → $f'(x) > 0$ / $f(x)$ dec $(-4,0)$ $f'(x) < 0$

d) Find the x-coordinate of each point of inflection for $y = f(x)$ on the interval $-6 < x < 6$. Justify your answer.

$x = -2$ $x = 4$

f'' changes sign (+ to − or − to +)

or

f' changes from inc to dec or dec to inc

e) Find the intervals over which the function $y = f(x)$ is concave up and concave down on the interval $-6 < x < 6$. Justify your answer.

Concave up $(-2,0)$ $(0,4)$ $f'' +$ f' inc

CC down $(-6,-4)$ $(-4,-2)$ $(4,6)$ $f'' -$

f) If $f(-2) = 4$, find the equation of the tangent line to $y = f(x)$ at $x = -2$.

$y - 4 = -2(x + 2)$

Guidelines for Finding Information From the Graph of the Derivative of a Function

Critical points for the function $y = f(x)$ occur when the graph of $y = f'(x)$:

$= 0$ or undefined

Relative maxima for the function $y = f(x)$ occur when the graph of $y = f'(x)$:

+ to − (dips below the x-axis)

Relative minima for the function $y = f(x)$ occur when the graph of $y = f'(x)$:

− to + (rise above the x-axis)

The function $y = f(x)$ is increasing when the graph of $y = f'(x)$:

positive (above x-axis)

The function $y = f(x)$ is decreasing when the graph of $y = f'(x)$:

negative (below x-axis)

Points of inflection for the function $y = f(x)$ occur when the graph of $y = f'(x)$:

changes from inc to dec or dec to inc

The function $y = f(x)$ is concave up when the graph of $y = f'(x)$:

increasing

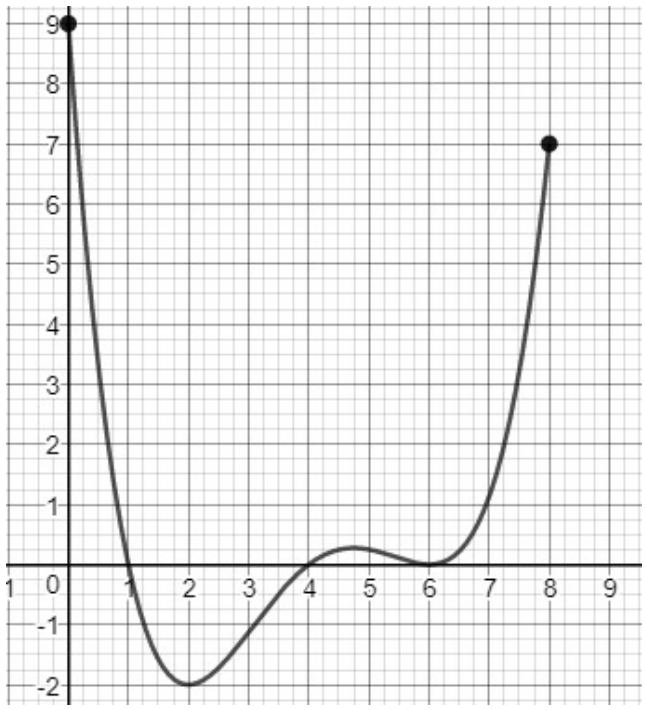
The function $y = f(x)$ is concave down when the graph of $y = f'(x)$:

decreasing

The slope of the original function at a point is:

$f'(x)$ (so if I'm looking at a graph of $f'(x)$ already → find the y-value)

Example 2: The function f is defined and differentiable on the closed interval $[0, 8]$. The graph of $y = f'(x)$, the derivative of f, is shown in the figure below and has horizontal tangents at $x = 2$, $x = 4.75$, and $x = 6$.



a) Find the x-coordinate of each critical point of $y = f(x)$ on the interval $0 < x < 8$. Justify your answer.

$f'(x) = 0$

$x = 1 \quad x = 4 \quad x = 6$

b) Find the open intervals over which the function $y = f(x)$ is increasing and decreasing on the interval $0 < x < 8$. Justify your answer.

INC $(0,1)$ $(4,6)$ $(6,8)$

Dec $(1,4)$

c) Find the x-coordinate of each relative extrema of $y = f(x)$ on the interval $0 < x < 8$. Label each as a minimum or maximum and justify each response.

rel maximum at $x = 1$ f' + to −

rel minimum at $x = 4$ f' − to +

d) Find the x-coordinate of each point of inflection for $y = f(x)$ on the interval $0 < x < 8$. Justify your answer.

$x = 2, 4.75, 6$

f' changes dec to inc (2 and 6) or inc to dec (4.75)

e) Find the intervals over which the function $y = f(x)$ is concave up and concave down on the interval $0 < x < 8$. Justify your answer.

concave up $(2, 4.75)$ $(6, 8)$

concave down $(0, 2)$ $(4.75, 6)$

f) Let $g(x) = x^2 - f(x)$. Find $g'(2)$.

$g'(x) = 2x - f'(x)$

$g'(2) = 2(2) - f'(2) = 4 - -2 = \boxed{6}$