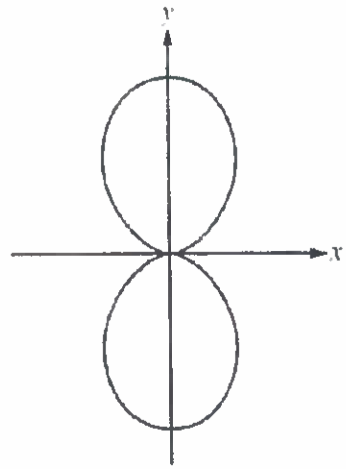
23+14 out of /37

AP Review Polar and Parametric HW **Name:** Key

Non-Calculator Multiple Choice

1.	D	For $0 \le t \le 13$, an object travels along an elliptical path given by the parametric equations $x = 3\cos t$ and $y = 4\sin t$. At the point where $t = 13$, the object leaves the path and travels along the line tangent to the path at that point. What is the slope of the line on which the object travels? (A) $-\frac{4}{3}$ (B) $-\frac{3}{4}$ (C) $\frac{4\tan 13}{3}$ (D) $-\frac{4}{3\tan 13}$ (E) $-\frac{3}{4\tan 13}$ $dy/dx = \frac{4\cos t}{-3\sin t} \to \frac{-4\cos 13}{3\sin 13}$
2.	C	The position of a particle moving in the xy-plane is given by the parametric equations $x = t^3 - 3t^2$ and $y = 2t^3 - 3t^2 - 12t$. For what values of t is the particle at rest? (A) -1 only (B) 0 only (C) 2 only (D) -1 and 2 only (E) -1, 0, and 2 $\frac{dx}{dt} = 3t^2 - 6t$ $3t(t-2)$ $t = 0, 2$ $dy/dt = 6t^2 - 6t - 12$ $6(t^2 - t - 2)$ $6(t-2)(t+1)$ $2, -1$
3.	A	A curve C is defined by the parametric equations $x = t^2 - 4t + 1$ and $y = t^3$. Which of the following is an equation of the line tangent to the graph of C at the point $(-3, 8)$? (A) $x = -3$ (B) $x = 2$ (C) $y = 8$ (D) $y = -\frac{27}{10}(x+3) + 8$ (E) $y = 12(x+3) + 8$ $\frac{dy}{dt} = 3t^2$ $\frac{dx}{dt} = 2t - 4$ $\frac{3t^2}{2t-4} \to \frac{12}{0}$ und. $x = -3$ $8 = t^3$ $t = 2$
4.	B	At time $t \ge 0$, a particle moving in the xy-plane has velocity vector given by $v(t) = \langle t^2, 5t \rangle$. What is the acceleration vector of the particle at time $t = 3$? (A) $\langle 9, \frac{45}{2} \rangle$ (B) $\langle 6, 5 \rangle$ (C) $\langle 2, 0 \rangle$ (D) $\sqrt{306}$ (E) $\sqrt{61}$ $\langle 2t, 5 \rangle$ $\langle 6, 5 \rangle$
5.	C	Which of the following gives the length of the path described by the parametric equations $x = \sin(t^3)$ and $y = e^{5t}$ from $t = 0$ to $t = \pi$? (A) $\int_0^\pi \sqrt{\sin^2(t^3) + e^{10t}}\,dt$ (B) $\int_0^\pi \sqrt{\cos^2(t^3) + e^{10t}}\,dt$ (C) $\int_0^\pi \sqrt{9t^2\cos^2(t^3) + 25e^{10t}}\,dt$ (D) $\int_0^\pi \sqrt{3t^2\cos(t^3) + 5e^{5t}}\,dt$ (E) $\int_0^\pi \sqrt{\cos^2(3t^2) + e^{10t}}\,dt$ $\int_0^\pi \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$ $\frac{dx}{dt} = \cos(t^3) \cdot 3t^2$ $\frac{dy}{dt} = e^{5t} \cdot 5$

6. D



Which of the following expressions gives the total area enclosed by the polar curve $r = \sin^2\theta$ shown in the figure above?

$0 = \sin^2\theta$

$\sin\theta = 0$

$\theta = 0, \pi, \text{etc.}$

(A) $\frac{1}{2}\int_0^{\pi} \sin^2\theta \, d\theta$

(B) $\int_0^{\pi} \sin^2\theta \, d\theta$

(C) $\frac{1}{2}\int_0^{\pi} \sin^4\theta \, d\theta$

(D) $\int_0^{\pi} \sin^4\theta \, d\theta$

$2 \cdot \frac{1}{2}\int_0^{\pi} (\sin^2\theta)^2 \, d\theta$

(E) $2\int_0^{\pi} \sin^4\theta \, d\theta$

7. D

In the xy-plane, a particle moves along the parabola $y = x^2 - x$ with a constant speed of $2\sqrt{10}$ units per second. If $\frac{dx}{dt} > 0$, what is the value of $\frac{dy}{dt}$ when the particle is at the point $(2, 2)$?

$\frac{dy}{dx} = 2x - 1 = \frac{dy/dt}{dx/dt}$

(A) $\frac{2}{3}$ (B) $\frac{2\sqrt{10}}{3}$ (C) 3 (D) 6 (E) $6\sqrt{10}$

$3 = \frac{dy/dt}{dx/dt}$

$\text{speed} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$3\,dx/dt = dy/dt$

$2\sqrt{10} =$

$40 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2$

$40 = a^2 + 9a^2$

$40 = 10a^2$

$a^2 = 4$

$a = 2$

Calculator Multiple Choice

8. C

A particle moves in the xy-plane so that its position at any time t is given by $x(t) = t^2$ and $y(t) = \sin(4t)$. What is the speed of the particle when $t = 3$?

(A) 2.909 (B) 3.062 (C) 6.884 (D) 9.016 (E) 47.393

$\text{speed} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$dx/dt = 2t \rightarrow 6$ $dy/dt = 4\cos 4t \rightarrow 4\cos 12$

9. E

If f is a vector-valued function defined by $f(t) = \left(e^{-t}, \cos t\right)$, then $f''(t) =$

$-e^{-t}, -\sin t$ $e^{-t}, -\cos t$

(A) $-e^{-t} + \sin t$ (B) $e^{-t} - \cos t$ (C) $\left(-e^{-t}, -\sin t\right)$

(D) $\left(e^{-t}, \cos t\right)$ (E) $\left(e^{-t}, -\cos t\right)$

10.	D	The area of the region inside the polar curve $r = 4\sin\theta$ and outside the polar curve $r = 2$ is given by Find int. $2=4\sin\theta$ $\sin\theta = 1/2$ $\pi/6$ $5\pi/6$ (A) $\frac{1}{2}\int_0^{\pi}(4\sin\theta-2)^2\,d\theta$ (B) $\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}(4\sin\theta-2)^2\,d\theta$ (C) $\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}(4\sin\theta-2)^2\,d\theta$ (D) $\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}\left(16\sin^2\theta-4\right)d\theta$ (E) $\frac{1}{2}\int_0^{\pi}\left(16\sin^2\theta-4\right)d\theta$ $\int R^2-r^2$
11.	C	The length of the path described by the parametric equations $x=\frac{1}{3}t^3$ and $y=\frac{1}{2}t^2$, where $0\le t\le 1$, is given by (A) $\int_0^1\sqrt{t^2+1}\,dt$ (B) $\int_0^1\sqrt{t^2+t}\,dt$ (C) $\int_0^1\sqrt{t^4+t^2}\,dt$ (D) $\frac{1}{2}\int_0^1\sqrt{4+t^1}\,dt$ (E) $\frac{1}{6}\int_0^1 t^2\sqrt{4t^2+9}\,dt$ $L=\int_0^1\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$ $\frac{dx}{dt}=t^2$ $\frac{dy}{dt}=t$

Free Response #1 Calculator Active

At time $t \ge 0$, a particle moving along a curve in the xy-plane has position $(x(t), y(t))$ with velocity vector $v(t) = \left(\cos\left(t^2\right), e^{0.5t}\right)$. At $t = 1$, the particle is at the point $(3, 5)$.

(a) Find the x-coordinate of the position of the particle at time $t = 2$.

(b) For $0 < t < 1$, there is a point on the curve at which the line tangent to the curve has a slope of 2. At what time is the object at that point?

(c) Find the time at which the speed of the particle is 3.

(d) Find the total distance traveled by the particle from time $t = 0$ to time $t = 1$.

a $x(1)=3$ $\quad 3+\int_1^2\cos(t^2)\,dt = x(2) = 2.557$

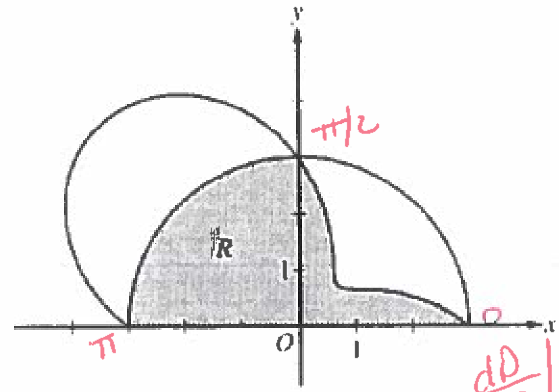
b $\frac{dy}{dx}=2$ $\quad \frac{dy}{dx}=\frac{e^{.5t}}{\cos t^2}$ $\quad t\approx .840$

c $\sqrt{(\cos t^2)^2+(e^{.5t})^2}=3$ $\quad t\approx 2.196$

d $d=\int_0^1\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt \approx 1.595$

Free Response Non-Calculator

The graphs of the polar curves $r = 3$ and $r = 3 - 2\sin(2\theta)$ are shown in the figure above for $0 \le \theta \le \pi$.



(a) Let R be the shaded region that is inside the graph of $r = 3$ and inside the graph of $r = 3 - 2\sin(2\theta)$. Find the area of R.

(b) For the curve $r = 3 - 2\sin(2\theta)$, find the value of $\frac{dx}{d\theta}$ at $\theta = \frac{\pi}{6}$.

(c) The distance between the two curves changes for $0 < \theta < \frac{\pi}{2}$. Find the rate at which the distance between the two curves is changing with respect to θ when $\theta = \frac{\pi}{3}$.

$\frac{dD}{d\theta}\Big|_{\theta=\frac{\pi}{3}}$ $= -2$

(c) $D = 3 - (3 - 2\sin(2\theta)) = 2\sin 2\theta$ $\quad \frac{dD}{d\theta} = 4\cos 2\theta$

(d) A particle is moving along the curve $r = 3 - 2\sin(2\theta)$ so that $\frac{d\theta}{dt} = 3$ for all times $t \ge 0$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{6}$.

(d) $\frac{dr}{d\theta} = \frac{dr/dt}{d\theta/dt}$ $\quad \frac{dr}{d\theta} = -4\cos 2\theta$ $\quad -4\cos 2\theta = \frac{\frac{dr}{dt}}{3}$ $\quad -12\cos 2\theta = \frac{dr}{dt}$

$-12\cos\frac{\pi}{3} = \frac{dr}{dt}\Big|_{\theta=}$ $= -6$

(a) $3 = 3 - 2\sin 2\theta$

$0 = -2\sin 2\theta$

$\sin 2\theta = 0$

$2\theta = \pi n$

$\theta = \frac{\pi}{2} n$

$\frac{1}{2}\int_0^{\pi/2} (3 - 2\sin 2\theta)^2 \, d\theta \approx 2.639$

$\frac{1}{2}\int_0^{\pi/2} (9 - 12\sin 2\theta + 4\sin^2(2\theta)) \, d\theta$

annoying. do it w/ a calculator

$\frac{1}{2}\int_{\pi/2}^{\pi} 3^2 \, d\theta \approx 7.069$

$A_{Total} \approx 9.708$

(b) $\frac{dr}{d\theta} = -2\cos(2\theta)\cdot 2$ $\quad x = r\cos\theta$ $\quad \frac{dx}{d\theta} = r'\cos\theta - r\sin\theta$

$\frac{dx}{d\theta}\Big|_{\theta=\frac{\pi}{6}} = -4\cos\left(\frac{\pi}{3}\right)\cos\frac{\pi}{6} - \left(3 - 2\sin\frac{\pi}{3}\right)\left(\sin\frac{\pi}{6}\right)$

$= -4\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(3 - 2\left(\frac{\sqrt{3}}{2}\right)\right)\left(\frac{1}{2}\right)$

$= -\sqrt{3} - \frac{1}{2}(3 - \sqrt{3}) \approx -2.366$

or $-\frac{\sqrt{3}}{2} - \frac{3}{2}$

AP Review Advanced Integration Methods HW **Name:** Key

Non-Calculator Multiple Choice

1. $\int \frac{1}{x^2-6x+8}\,dx =$

$\frac{1}{(x-4)(x-2)}$

$\frac{A}{x-4}+\frac{B}{x-2}=\frac{1}{(x-4)(x-2)}$

$A(x-2)+B(x-4)=1$

$x=2 \quad -2B=1 \quad B=-\frac{1}{2}$

$x=4 \quad 2A=1 \quad A=1/2$

$\int \frac{1/2}{x-4}+\int\frac{-1/2}{x-2}$

$\frac{1}{2}\ln|x-4|-\frac{1}{2}\ln|x-2|+C$

(A) $\frac{1}{2}\ln\left|\frac{x-4}{x-2}\right|+C$

(B) $\frac{1}{2}\ln\left|\frac{x-2}{x-4}\right|+C$

(C) $\frac{1}{2}\ln|(x-2)(x-4)|+C$

(D) $\frac{1}{2}\ln|(x-4)(x+2)|+C$

(E) $\ln|(x-2)(x-4)|+C$

2. $\int x\cos x\,dx =$

$u=x \quad dv=\cos x\,dx$

$du=dx \quad v=\sin x$

$x\sin x-\int \sin x\,dx$

$x\sin x+\cos x+C$

(A) $x\sin x-\cos x+C$

(B) $x\sin x+\cos x+C$

(C) $-x\sin x+\cos x+C$

(D) $x\sin x+C$

(E) $\frac{1}{2}x^2\sin x+C$

3. $\int_0^\infty x^2e^{-x^3}\,dx$ is

$\lim_{b\to\infty}\int_0^b x^2e^{-x^3}\,dx$

$u=-x^3 \quad \frac{du}{dx}=-3x^2 \quad \frac{du}{-3}=x^2dx \to$

$\to -\frac{1}{3}\int e^u\,du \to -\frac{1}{3}e^u \to$

(A) $-\frac{1}{3}$ (B) 0 (C) $\frac{1}{3}$ (D) 1 (E) divergent

$\lim_{b\to\infty}\left[-\frac{1}{3}e^{-x^3}\right]_0^b \to \lim_{b\to\infty} -\frac{1}{3}e^{-b^3}+\frac{1}{3}e^0 \to \cancel{-\frac{1}{3}\cdot\frac{1}{\infty}}^{0}+\frac{1}{3}$

4. The population $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt}=P\left(2-\frac{P}{5000}\right)$, where the initial population $P(0)=3{,}000$ and t is the time in years. What is $\lim_{t\to\infty}P(t)$?

(A) 2,500 (B) 3,000 (C) 4,200 (D) 5,000 (E) 10,000

$\frac{dy}{dt}=ky(L-y)$

$\frac{dP}{dt}=\frac{1}{5000}P(10{,}000-P)$

5. $\int_1^{\infty} \frac{x}{(1+x^2)^2}\,dx$ is

$u = 1+x^2$
$du = 2x\,dx$
$\frac{du}{2} = x\,dx$

$\frac{1}{2}\int \frac{1}{u^2}\,du$

$\frac{1}{2}\int u^{-2}\,du \rightarrow \frac{\frac{1}{2}u^{-1}}{-1}$

(A) $-\frac{1}{2}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) divergent

$\lim_{b\to\infty} \int_1^b \frac{1}{2}u^{-2}\,du \rightarrow \lim_{b\to\infty} -\frac{1}{2}u^{-1}\Big|_1^b \rightarrow \lim_{b\to\infty} -\frac{1}{2}(1+x^2)^{-1} + \frac{1}{2}(1+(1)^2)^{-1}$

$\rightarrow 0 + \frac{1}{2}(2)^{-1}$

$0 + \frac{1}{2}\cdot\frac{1}{2} = 1/4$

Calculator Multiple Choice

6. $\int x^2 \sin x\,dx =$

$u = x^2$ + $dv = \sin x\,dx$
$du = 2x$ − $v = -\cos x$
again → 2 → $-\sin x$ ← again
+ $\cos x$ → again

(A) $-x^2\cos x - 2x\sin x - 2\cos x + C$

(B) $-x^2\cos x + 2x\sin x - 2\cos x + C$

(C) $-x^2\cos x + 2x\sin x + 2\cos x + C$

(D) $-\frac{x^3}{3}\cos x + C$

(E) $2x\cos x + C$

$-x^2\cos x + 2x\sin x + 2\cos x + C$

7. $\int \frac{dx}{(x-1)(x+3)} =$

$\frac{A}{x-1} + \frac{B}{x+3} = \frac{1}{(x-1)(x+3)}$

(A) $\frac{1}{4}\ln\left|\frac{x-1}{x+3}\right| + C$

$A(x+3) + B(x-1) = 1$

(B) $\frac{1}{4}\ln\left|\frac{x+3}{x-1}\right| + C$

$x \to -3$ $-4B = 1$ $B = -1/4$
$x \to 1$ $4A = 1$ $A = 1/4$

(C) $\frac{1}{2}\ln\left|(x-1)(x+3)\right| + C$

(D) $\frac{1}{2}\ln\left|\frac{2x+2}{(x-1)(x+3)}\right| + C$

$\frac{\frac{1}{4}}{x-1}$ $\frac{-\frac{1}{4}}{x+3} \rightarrow \frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+3| + C$

(E) $\ln\left|(x-1)(x+3)\right| + C$

8. $\int_4^{\infty} \frac{-2x}{\sqrt[3]{9-x^2}}\,dx$ is

$u = 9-x^2$ $du = -2x\,dx$

$\int \frac{du}{\sqrt[3]{u}} \rightarrow \int u^{-\frac{1}{3}}\,du \rightarrow \frac{3}{2}u^{\frac{2}{3}} + C$

↳ $x = \pm 3$ is discontinuity

But not a problem

(A) $7^{\frac{2}{3}}$ (B) $\frac{3}{2}\left(7^{\frac{2}{3}}\right)$ (C) $9^{\frac{2}{3}} + 7^{\frac{2}{3}}$ (D) $\frac{3}{2}\left(9^{\frac{2}{3}} + 7^{\frac{2}{3}}\right)$ (E) nonexistent

$\frac{3}{2}(9-x^2)^{\frac{2}{3}}\Big|_4^{\infty}$ $\lim_{b\to\infty} \frac{3}{2}(9-b^2)^{\frac{2}{3}} - \frac{3}{2}(9-4^2)^{\frac{2}{3}}$ divergent

$-\infty$ − something

9. $\int x f(x)\,dx =$

$u = f(x) \quad dv = x\,dx$

$du = f'(x)\,dx \quad v = \frac{x^2}{2}$

(A) $x f(x) - \int x f'(x)\,dx$

(B) $\frac{x^2}{2} f(x) - \int \frac{x^2}{2} f'(x)\,dx$

$\frac{x^2}{2} f(x) - \int \frac{x^2}{2} f'(x)\,dx$

(C) $x f(x) - \frac{x^2}{2} f(x) + C$

(D) $x f(x) - \int f'(x)\,dx$

(E) $\frac{x^2}{2} \int f(x)\,dx$

$u = x \quad dv = \sec^2 x\,dx \quad x\tan x - \int \tan x\,dx$

10. $\int x \sec^2 x\,dx =$

$du = dx \quad v = \tan x \quad \int \tan x \to \int \frac{\sin x}{\cos x}\,dx$

(A) $x \tan x + C$

(B) $\frac{x^2}{2} \tan x + C$

(C) $\sec^2 x + 2\sec^2 x \tan x + C$

$u = \cos x$

$du = -\sin x\,dx$

(D) $x \tan x - \ln|\cos x| + C$

(E) $x \tan x + \ln|\cos x| + C$

$-\int \frac{1}{u}\,du \to -\ln|u|$

$\to -\ln|\cos x|$

So $- \cdot - \to +$

11. $\int_2^{+\infty} \frac{dx}{x^2}$ is

$\lim_{b \to \infty} \int_2^b x^{-2}\,dx \to -x^{-1} \Big|_2^b$

(A) $\frac{1}{2}$

(B) $\ln 2$

(C) 1

(D) 2

(E) nonexistent

$-b^{-1} + 2^{-1} \to -\frac{1}{\infty} + \frac{1}{2} \to \frac{1}{2}$

12. $\int x e^{2x}\,dx =$

(A) $\frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + C$

(B) $\frac{xe^{2x}}{2} - \frac{e^{2x}}{2} + C$

(C) $\frac{xe^{2x}}{2} + \frac{e^{2x}}{4} + C$

(D) $\frac{xe^{2x}}{2} + \frac{e^{2x}}{2} + C$

(E) $\frac{x^2 e^{2x}}{4} + C$

$u = x \quad dv = e^{2x}\,dx$

$du = dx \quad v = \frac{1}{2} e^{2x}$

$\frac{1}{2} x e^{2x} - \int \frac{1}{2} e^{2x}\,dx$

$\frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} + C$

Free Response #1 Calculator Active

$\frac{dy}{dt} = ky(L-y)$

A certain rumor spreads through a community at the rate $\frac{dy}{dt} = 2y(1-y)$, where y is the proportion of the population that has heard the rumor at time t. $L = 1$

(a) What proportion of the population has heard the rumor when it is spreading the fastest? $\frac{1}{2}$ (50%)

(b) If at time $t = 0$ ten percent of the people have heard the rumor, find y as a function of t.

$y = \frac{L}{1+Ce^{-Lkt}}$ $\quad .10 = \frac{1}{1+Ce^{-(1)(2)(0)}}$ $\quad .10 = \frac{1}{1+C}$

$.10 + .10C = 1$

$.10C = .90$

$C = 9$

$\boxed{y = \frac{1}{1+9e^{-2t}}}$

(c) At what time t is the rumor spreading the fastest?

$\frac{1}{2} = \frac{1}{1+9e^{-2t}} \rightarrow$ calc (graph)

$t \approx 1.099$

or $(\ln(1/9))/-2$

Free Response #2 Non-Calculator

Let f be the function satisfying $f'(x) = -3xf(x)$, for all real numbers x, with $f(1) = 4$ and $\lim_{x\to\infty} f(x) = 0$.

(a) Evaluate $\int_1^{\infty} -3xf(x)\,dx$. Show the work that leads to your answer.

(b) Use Euler's method, starting at $x = 1$ with a step size of 0.5, to approximate $f(2)$.

(c) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = -3xy$ with the initial condition $f(1) = 4$.

(a) $\int_1^{\infty} f'(x)\,dx \quad \lim_{b\to\infty}\int_1^b f'(x)\,dx \quad \lim_{b\to\infty} f(x)\Big|_1^b$

$\boxed{\lim_{b\to\infty} f(b) - f(1) \rightarrow 0 - 4 = -4}$

(b)

old pt	dx	$\frac{dy}{dx} = m$	$dy = m\,dx$	new pt
(1,4)	.5	$-3(1)\cdot 4 = -12$	$(-12)(.5) = -6$	(1.5, -2)
(1.5,-2)	.5	$-3(1.5)\cdot -2 = +9$	$(+9)(.5) = +4.5$	(2, 2.5)

$\boxed{f(2) \approx 2.5}$

(c) $\int \frac{dy}{y} = \int -3x\,dx$

$\ln|y| = -\frac{3}{2}x^2 + C$

$\ln|4| = -\frac{3}{2} + C$

$C = \ln 4 + \frac{3}{2}$

$\ln y = -\frac{3}{2}x^2 + \ln 4 + \frac{3}{2}$

$y = e^{-\frac{3}{2}x^2 + \ln 4 + \frac{3}{2}}$ or $\boxed{y = 4e^{-\frac{3}{2}x^2 + \frac{3}{2}}}$