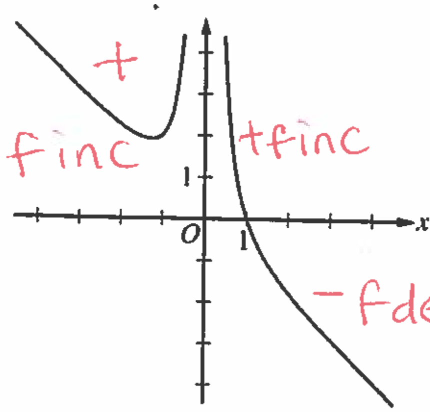
1.)

$\lim_{x\to 0} \frac{4x^2}{e^{4x} - 4x - 1}$ is $\frac{0}{0}$ L'H' $\lim_{x\to 0} \frac{8x}{4e^{4x}-4} \to \frac{0}{0} \to$ L'H

(A) 0   (B) $\frac{1}{2}$ (C) 8   (D) nonexistent $\lim_{x\to 0} \frac{8}{16e^{4x}} \Rightarrow \frac{8}{16}$

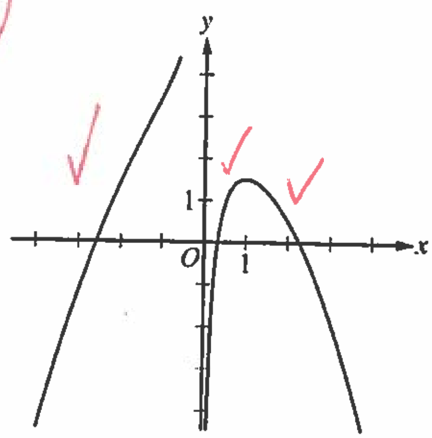
2.)



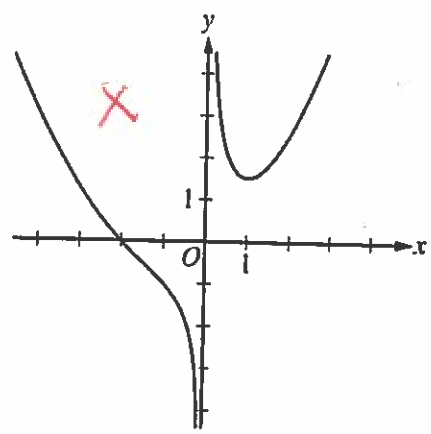
Graph of f'

The graph of f', the derivative of the function f, is shown above. Which of the following could be the graph of f?

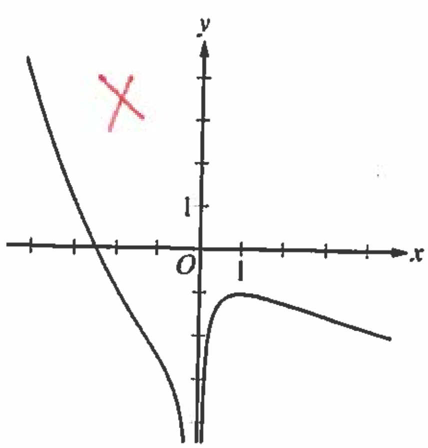
(A)



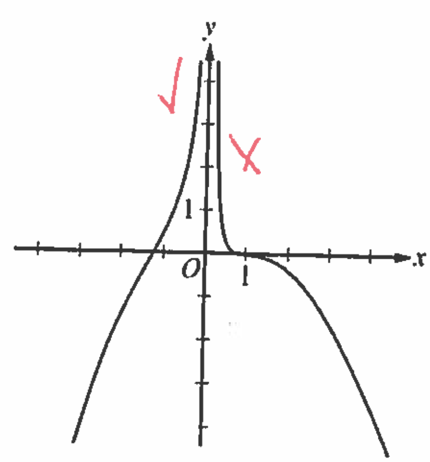
(B)



(C)



(D)



3)

Let f be the function given by $f(x) = x^3 - 6x^2 - 15x$. What is the maximum value of f on the interval [0, 6] ?

(A) 0 (B) 5 (C) 6 (D) 8

$f'(x) = 3x^2 - 12x - 15$

$0 = 3(x^2 - 4x - 5)$

$0 = 3(x-5)(x+1)$ $x = 5, -1$

0	
0	0 biggest!
5	$5^3 - 6(5^2) - 15(5) \to 125 - 150 - 75 \to - \#$
6	$6^3 - 6(6)^2 - 15(6) \to 216 - 216 - 80 \to - \#$

4) Let f be the function defined by $f(x) = \sqrt[3]{x}$. What is the approximation for $f(10)$ found by using the line tangent to the graph of f at the point (8, 2) ?

Need: $x, y_1, \frac{dy}{dx}$

(A) $\frac{11}{6}$ (B) $\frac{25}{12}$ (C) $\frac{13}{6}$ (D) $\frac{7}{3}$

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$

$f'(8) = \frac{1}{3}(8)^{-\frac{2}{3}}$

$= \frac{1}{3}(2^3)^{-\frac{2}{3}} = \frac{1}{3}(2)^{-2} = \frac{1}{3 \cdot 4} = \frac{1}{12}$

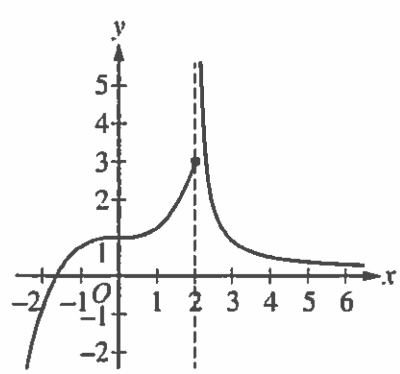
$y - 2 = \frac{1}{12}(x - 8)$

$y = \frac{1}{12}(10 - 8) + 2$

$\frac{1}{12}(2) + 2$

$2\frac{1}{6} = \frac{13}{6}$

5)



Graph of f

The graph of the function f is shown in the figure above. Which of the following statements must be false?

(A) $\lim_{x \to 2^-} f(x) = 3$ T

(B) $\lim_{x \to 2^+} f(x) = \infty$ T

(C) $\lim_{x \to 2} f(x) = f(2)$ X

(D) $\lim_{x \to \infty} f(x) = 0$ T

6.) If $f(x) = (x^2+1)^3$, what is $\lim_{x \to -1} \frac{f(x) - f(-1)}{x+1}$? AH. Def of Derivative or L'H'

(A) -24 (B) -8 (C) 0 (D) 12

$f'(x) = 3(x^2+1)^2(2x)$

$f'(-1) = 3((-1)^2+1)^2)(2(-1)) = 3(2)^2(-2)$

$3(4)(-2)$

7.) If $y = x^2(e^x - 1)$, then $\frac{dy}{dx} =$ Product rule

(A) $2xe^x$

(B) $2xe^x - 2x$

(C) $x^2e^x + 2xe^x - 2x$

(D) $x^2e^x + 2xe^x - x^2 - 2x$

$\frac{dy}{dx} = x^2(e^x) + 2x(e^x - 1)$

$= x^2e^x + 2xe^x - 2x$

8.) When $x = 2e$, $\lim_{h \to 0} \frac{\ln(x+h) - \ln(x)}{h}$ is Limit Def of Der. or L'H'

(A) $\frac{1}{2e}$ (B) 1 (C) $\ln(2e)$ (D) nonexistent

$f(x) = \ln x$ $f'(2e) = \frac{1}{2e}$

$f'(x) = \frac{1}{x}$

9.) A spherical snowball is melting in such a way that it maintains its shape. The snowball is decreasing in volume at a constant rate of 8 cubic centimeters per hour. At what rate, in centimeters per hour, is the radius of the snowball decreasing at the instant when the radius is 10 centimeters? (The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.)

$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$

(A) $\frac{1}{50\pi}$ (B) $\frac{3}{50\pi}$ (C) 400π (D) 3200π

$-8 = 4\pi(10)^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{-8}{4\pi(100)} = \frac{-2}{\pi(100)} = \frac{-1}{50\pi}$

10.) Let f be the function given by $f(x) = 2\cos x + 1$. What is the approximation for $f(1.5)$ found by using the line tangent to the graph of f at $x = \frac{\pi}{2}$?

$f(\frac{\pi}{2}) = 2\cos(\frac{\pi}{2}) + 1 = 0 + 1 = 1$

(A) -2 (B) 1 (C) $\pi - 2$ (D) $4 - \pi$

$f'(x) = -2\sin x$

$f'(\frac{\pi}{2}) = -2\sin\frac{\pi}{2}$

$-2(1) = -2$

$y - 1 = -2(x - \frac{\pi}{2})$

$y = -2(1.5 - \frac{\pi}{2}) + 1$

$y = -3 + \pi + 1$

11.) If $f(x) = \sin^{-1} x$, then $f'\left(\frac{\sqrt{3}}{2}\right) =$

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{4}{7}$ (D) 2

$f'(x) = \frac{1}{\sqrt{1-x^2}}$ $f'(\frac{\sqrt{3}}{2}) = \frac{1}{\sqrt{1-(\frac{\sqrt{3}}{2})^2}} = \frac{1}{\sqrt{1-\frac{3}{4}}} = \frac{1}{\sqrt{1/4}} = \frac{1}{1/2}$

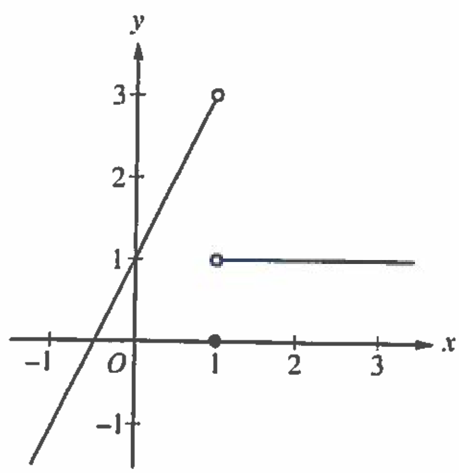
12.) If $f(x) = \ln x$, then $\lim_{x\to 3} \frac{f(x) - f(3)}{x - 3}$ is

Alt. Def Der. or L'H'

(A) $\frac{1}{3}$ (B) e^3 (C) $\ln 3$ (D) nonexistent

$f'(x) = \frac{1}{x}$ $f'(3) = \frac{1}{3}$

13.)



Graph of f

LHL = 3

RHL = 1

$3 \neq 1$

The graph of $y = f(x)$ is shown above. What is $\lim_{x\to 1} f(x)$?

(A) 0 (B) 1 (C) 3 (D) The limit does not exist.

Jump (non-removable)

14.)

x	3	7
$h(x)$	7	22
$h'(x)$	5	10

→ this means h & g are Inverses!

Selected values of the increasing function h and its derivative h' are shown in the table above. If g is a differentiable function such that $h(g(x)) = x$ for all x, what is the value of $g'(7)$?

(A) $-\frac{1}{10}$ (B) $\frac{1}{10}$ (C) $\frac{1}{5}$ (D) $\frac{7}{5}$

$g'(7) = \frac{1}{h'(?)} \rightarrow \frac{1}{h'(3)} = \frac{1}{5}$

7 is input of g so output of h

15.)

Double sided Limit

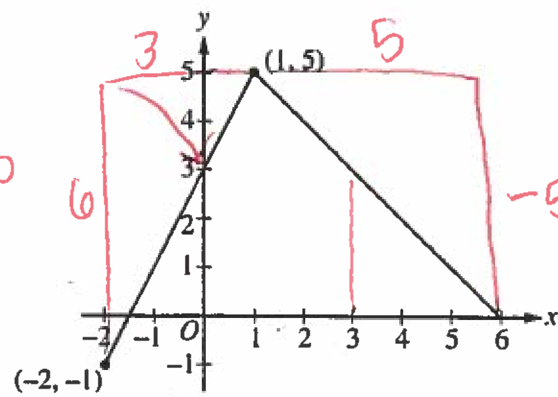
$\lim_{x\to -7} \frac{x+7}{|x+7|}$ is

$\frac{x+7}{x+7}$ or $\frac{x+7}{-(x+7)}$

(A) -1 (B) 0 (C) 1 (D) nonexistent

1 or -1 (Jump at $x=-7$) (one-sided limit would either be 1 or -1)

16.)



Graph of g

$g'(0)$ is slope of g at $x=0$ ie $\frac{6}{3} = 2$

Slope at $x=3$ $\frac{-5}{5} = -1$

Chain rule!

The graph of the function g is shown above. If f is the function given by $f(x) = g(g(x))$, what is the value of $f'(0)$?

(A) -2 (B) -1 (C) 2 (D) 3

$f'(x) = g'(g(x)) \cdot g'(x)$

$f'(0) = g'(g(0)) \; g'(0)$

$= g'(3) \cdot (2)$

$= (-1)(2)$

17.) Let g be a twice-differentiable, increasing function of t. If $g(0) = 20$ and $g(10) = 220$, which of the following must be true on the interval $0 < t < 10$?

(A) $g'(t) = 0$ for some t in the interval.

(B) $g'(t) = 20$ for some t in the interval.

(C) $g''(t) = 0$ for some t in the interval.

(D) $g''(t) > 0$ for all t in the interval.

$\frac{g(10)-g(0)}{10-0} = \frac{220-20}{10} = \frac{200}{10} = 20$

→ MVT A.R.O.C = I.R.O.C

18.) Let $y = f(x)$ be a differentiable function such that $\frac{dy}{dx} = \frac{x}{y}$ and $f(8) = 2$. What is the approximation of $f(8.1)$ using the line tangent to the graph of f at $x = 8$?

(A) 0.4 (B) 2.025 (C) 2.4 (D) 6

$\frac{dy}{dx} = \frac{8}{2} = 4$

$y - 2 = 4(x - 8)$

$y \cancel{-2} = 4(8.1 - 8) + 2 \rightarrow 4(.1) + 2$

$+2$

19.) The number of gallons of water in a storage tank at time t, in minutes, is modeled by $w(t) = 25 - t^2$ for $0 \le t \le 5$. At what rate, in gallons per minute, is the amount of water in the tank changing at time $t = 3$ minutes?

slope (r.o c) → derivative

(A) 66 (B) 16 (C) −3 (D) −6

$w'(t) = -2t$

$w'(3) = -2(3)$

20.) For $t \ge 0$, the velocity of a particle moving along the x-axis is given by $v(t) = t^3 - 6t^2 + 10t - 4$. At what time t does the direction of motion of the particle change from right to left?

(A) 0.586 (B) 1.184 (C) 2.000 (D) 2.816

$0 = t^3 - 6t^2 + 10t - 4$

$t = 2?$ $2^3 - 6(2)^2 + 10(2) - 4 = 0$

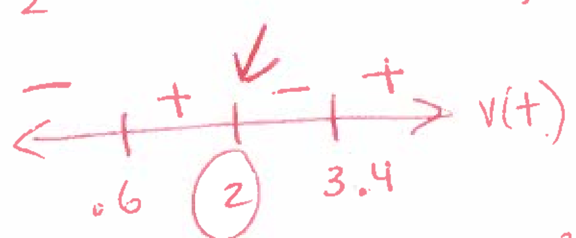
$$\begin{array}{r|rrrr} 2 & 1 & -6 & 10 & -4 \\ & & 2 & -8 & 4 \\ \hline & 1 & -4 & 2 & 0 \end{array}$$

$(t-2)(t^2 - 4t + 2)$ → Quadratic formula

$\frac{+4 \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)} = \frac{4 \pm \sqrt{8}}{2}$

$\frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2} \approx 2 \pm 1.4$

$\approx 3.4, .6$



21.) If $f'(x) = 3x^2 + 2x$ and $f(2) = 3$, then $f(1) =$

(A) −10 (B) −7 (C) 10 (D) 13

$f'(x) = 3x^2$ means $f(x) = x^3$

" $= 2x$ means " $= x^2$

constant

$f(x) = x^3 + x^2 + C$

$3 = 2^3 + 2^2 + C \rightarrow 3 = 12 + C \rightarrow C = -9$

$f(x) = x^3 + x^2 - 9$

$f(1) = 1 + 1 - 9$

22.)

What is the absolute minimum value of $y = -\cos x - \sin x$ on the closed interval $\left[0, \frac{\pi}{2}\right]$?

(A) $-2\sqrt{2}$ (B) -2 (C) $-\sqrt{2}$ (D) -1

$\frac{dy}{dx} = \sin x - \cos x$

$0 = \sin x - \cos x$

$\cos x = \sin x$

this happens at $\pi/4$ and $3\pi/4$ (only $\pi/4$ between $0, \pi/2$

	y
0	$-1-0 \to -1$
$\pi/4$	$-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \to -\sqrt{2}$ smalle
$\pi/2$	$-0-1 \to -1$

23.)

A particle moves along the x-axis so that at time $t > 0$ its position is given by $x(t) = 12e^{-t}\sin t$. What is the first time t at which the velocity of the particle is zero?

$\to x'(t) = 12e^{-t}\cos t + (-1)12e^{-t}\sin t$

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{3\pi}{4}$ (D) π

$x'(t) = \underbrace{12e^{-t}}_{\text{never zero}}(\cos t - \sin t) = 0$? at $t = \pi/4, 3\pi/4,$ etc

never zero :)

24.)

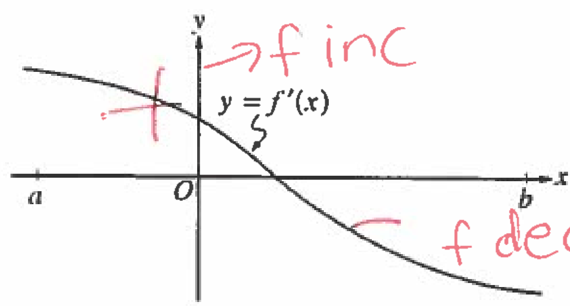
$$\lim_{x\to\infty} \frac{\ln\left(e^{3x}+x\right)}{x} =$$

$\frac{\infty}{\infty}$ L'H $\lim_{x\to\infty} \frac{\frac{1}{e^{3x}+x}\left(e^{3x}(3)+1\right)}{1} \to \frac{3e^{3x}+1}{e^{3x}+x}$

EBM $\frac{3e^{3x}}{e^{3x}} = 3$

(A) 0 (B) 1 (C) 3 (D) ∞

25.)



$f'(x)$ is decreasing, so $f''(x)$ is negative, so $f(x)$ is CCDown

The graph of f', the derivative of the function f, is shown in the figure above. Which of the following statements must be true?

→ derivative exists so it's differentiable ↳ f(x) that is

I. f is continuous on the open interval (a, b). ✓

II. f is decreasing on the open interval (a, b). False

III. The graph of f is concave down on the open interval (a, b). ✓

(A) I only

(B) I and II only

(C) I and III only

(D) II and III only

26.)

Which of the following is a solution to the differential equation $y'' - 4y = 0$?

(A) $y = e^{2x}$ (B) $y = 2e^x$ (C) $y = \sin(2x)$ (D) $y = \cos(2x)$

$y' = 2e^{2x}$ $y' = 2e^x$ $y' = 2\cos(2x)$ $y' = -2\sin(2x)$

$y'' = 4e^{2x}$ $y'' = 2e^x$ $y'' = -4\sin(2x)$ $y'' = -4\cos(2x)$

$-4\cos(2x) - 4\cos(2x) \neq 0$

$4e^{2x} - 4(e^{2x}) = 0$

$-4\sin(2x) - 4\sin(2x) \neq 0$

True!

$2e^x - 4(2e^x) \neq 0$

27.)

The function f has first derivative given by $f'(x) = x^4 - 6x^2 - 8x - 3$. On what intervals is the graph of f concave up?

$f''(x) = 4x^3 - 12x - 8$ (Answer choices give me idea of x-values!)

$0 = 4(x^3 - 3x - 2)$

(A) $(2, \infty)$ only

(B) $(0, \infty)$

(C) $(-1, 2)$

(D) $(-\infty, -1)$ and $(3, \infty)$

Try $x = -1$ $4((-1)^3 - 3(-1) - 2) = 0$!

Try $x = 2$ $2^3 - 3(2) - 2 = 0$!

$-1 \mid 4 \;\; 0 \;\; -12 \;\; -8$

$\quad -4 \;\; 4 \;\; 8$

$4 \;\; -4 \;\; -8 \;\; 0$

$4(x+1)(x^2 - x - 2)$

$4(x+1)(x-2)(x+1)$

$+ \quad -1 \quad - \quad 2 \quad +$

28.)

x	1	2	3	4	5
$f(x)$	9	4	0	−3	−5

Decreasing but slower and slower

The table above gives values of a function f at selected values of x. If f is twice-differentiable on the interval $1 \le x \le 5$, which of the following statements could be true?

slope $= -5$ $(1 \to 2)$

(A) ✓ f' is negative and ~~decreasing~~ for $1 \le x \le 5$.

$= -4$ $(2 \to 3)$

(B) ✓ f' is negative and increasing for $1 \le x \le 5$.

$= -3$ $(3 \to 4)$

(C) ✗ f' is positive and ~~decreasing~~ for $1 \le x \le 5$.

$= -2$ $(4 \to 5)$

(D) ✗ f' is positive and ~~increasing~~ for $1 \le x \le 5$.

Increasing!

29.)

A tire that is leaking air has an initial air pressure of 30 pounds per square inch (psi). The function $t = f(p)$ models the amount of time t, in hours, it takes for the air pressure of the tire to reach p psi. What are the units for $f'(p)$?

$y = t =$ hours $x = p =$ psi

(A) hours (B) psi (C) psi per hour (D) hours per psi

$f'(p)$ is $\frac{dy}{dx} \to$ units $\to \frac{t}{\text{psi}} \to \frac{\text{hours}}{\text{psi}}$

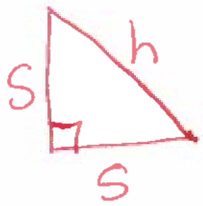
30.)

An isosceles right triangle with legs of length s has area $A = \frac{1}{2}s^2$. At the instant when $s = \sqrt{32}$ centimeters, the area of the triangle is increasing at a rate of 12 square centimeters per second. At what rate is the length of the hypotenuse of the triangle increasing, in centimeters per second, at that instant?

(A) $\frac{3}{4}$ (B) 3 (C) $\sqrt{32}$ (D) 48

$\frac{dA}{dt} = s\frac{ds}{dt}$ $\frac{ds}{dt} = \frac{12}{\sqrt{32}}$

$12 = \sqrt{32}\frac{ds}{dt}$



$s^2 + s^2 = h^2$

$2s^2 = h^2$

$4s\frac{ds}{dt} = 2h\frac{dh}{dt} \rightarrow 4\left(\frac{12}{\sqrt{32}}\right)(\sqrt{32}) = 2(8)\frac{dh}{dt}$

$4(12) = 16\frac{dh}{dt}$

$(\sqrt{32})^2 + (\sqrt{32})^2 = h^2 \Rightarrow 64 = h^2 \Rightarrow h = 8$ $\frac{dh}{dt} = 3$

31.)

The graph of which of the following functions has exactly one horizontal asymptote and no vertical asymptotes?

H.A. (1 only!) (den $\neq 0$)

(A) $y = \frac{1}{x^2+1}$ $\rightarrow \neq 0$ $y = 0$

(B) $y = \frac{1}{x^3+1}$ $= 0$ at $x = -1$

(C) $y = \frac{1}{e^x - 1}$ $= 0$ at $x = 0$

(D) $y = \frac{1}{e^x + 1}$ $\neq 0$

if $x \to \infty$ H.A. $y = 0$ (2)!
if $x \to -\infty$ H.A. $y = 1$

32.)

The continuous function f is positive and has domain $x > 0$. If the asymptotes of the graph of f are $x = 0$ and $y = 2$, which of the following statements must be true?

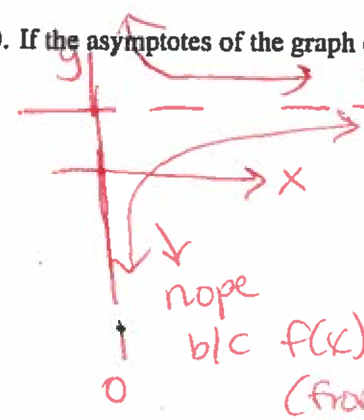
(A) $\lim_{x\to0^+} f(x) = \infty$ and $\lim_{x\to2} f(x) = \infty$ No idea

(B) $\lim_{x\to0^+} f(x) = 2$ and $\lim_{x\to\infty} f(x) = 0$ X 2!

(C) $\lim_{x\to0^+} f(x) = \infty$ and $\lim_{x\to\infty} f(x) = 2$

(D) $\lim_{x\to2} f(x) = \infty$ and $\lim_{x\to\infty} f(x) = 2$

No idea



only possible scenario

nope b/c $f(x)$ is > 0 (from given)

33.)

Which of the following limits are equal to -1?

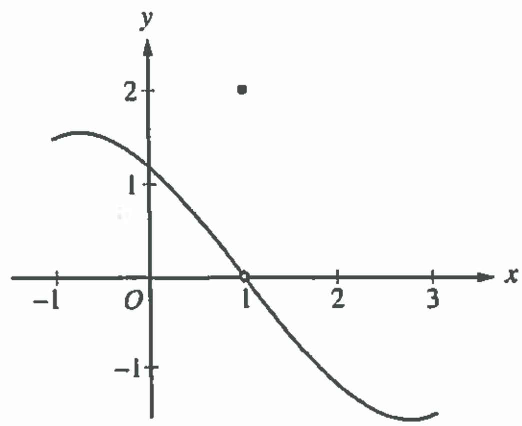
I. $\lim_{x\to0^-} \frac{|x|}{x}$ $\frac{-x \text{ from left}}{x} = -1$ ✓

L'H' $\frac{0}{0}$ II. $\lim_{x\to3} \frac{x^2 - 7x + 12}{3 - x}$ $\frac{2x-7}{-1}$ at $x=3$ $\frac{2(3)-7}{-1} = \frac{6-7}{-1} = 1$

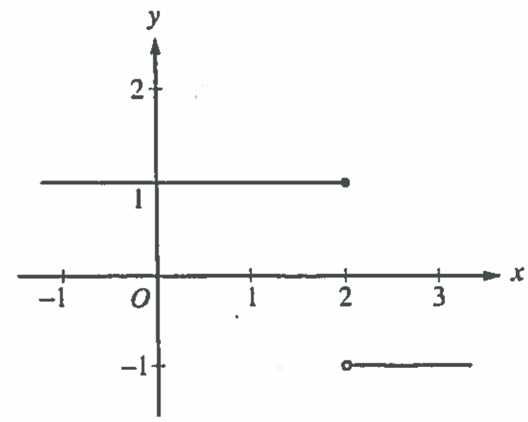
III. $\lim_{x\to\infty} \frac{1-x}{1+x}$ $= -1$ ☺

(A) I only (B) I and III only (C) II and III only (D) I, II, and III

34.)



Graph of f



Graph of g

The graphs of the functions f and g are shown in the figures above. Which of the following statements is false?

(A) $\lim_{x \to 1} f(x) = 0$ True

(B) $\lim_{x \to 2} g(x)$ does not exist. True (Jump

(C) $\lim_{x \to 1} (f(x)g(x+1))$ does not exist. $f(1) \cdot g(2)$ MIGHT exist

(D) $\lim_{x \to 1} (f(x+1)g(x))$ exists. True $f(2) \cdot g(1)$ exist exist

35.)

The positive variables p and c change with respect to time t. The relationship between p and c is given by the equation $p^2 = (20 - c)^3$. At the instant when $\frac{dp}{dt} = 41$ and $c = 15$, what is the value of $\frac{dc}{dt}$?

(A) $-\frac{82}{75}$ (B) $-\frac{2\sqrt{5}}{3}$ (C) $-\frac{3\sqrt{5}}{2}$ (D) $-\frac{82\sqrt{5}}{15}$

$p^2 = (20-15)^3$

$p^2 = 5^3$

$p^2 = 125$

$p = \sqrt{125} = 5\sqrt{5}$

$2p\frac{dp}{dt} = 3(20-c)^2(-1)\frac{dc}{dt}$

$2(5\sqrt{5})(41) = 3(5)^2(-1)\frac{dc}{dt}$

$\frac{82 \cdot 5\sqrt{5}}{-3 \cdot 5^2} = \frac{dc}{dt}$

$\to \frac{82\sqrt{5}}{-15}$

36.)

$\lim_{x \to -\infty} \frac{3 + 2^x}{4 - 5^x}$ is

$3 + 2^{-\infty} \to 3 + 0 \to 3$

(A) $-\frac{2}{5}$ (B) 0 (C) $\frac{3}{4}$ (D) nonexistent

$4 - 5^{-\infty} \to 4 - 0 \to 4$

37.)

For time $t \ge 1$, the position of a particle moving along the x-axis is given by $p(t) = \sqrt{t} - 2$. At what time t in the interval $1 \le t \le 16$ is the instantaneous velocity of the particle equal to the average velocity of the particle over the interval $1 \le t \le 16$?

$p'(t) = \frac{1}{2\sqrt{t}}$ $\quad \frac{p(16)-p(1)}{16-1}$

(A) 1 (B) $\frac{121}{25}$ (C) $\frac{25}{4}$ (D) 25

$\sqrt{16}-2=2$

$\sqrt{1}-2=-1$

$\frac{2--1}{15} = \frac{3}{15} = \frac{1}{5} \rightarrow \frac{1}{2\sqrt{t}} = \frac{1}{5}$

$(5=2\sqrt{t})^2$

$25=4t$

38.)

If f is a differentiable function and $y = \sin\left(f\left(x^2\right)\right)$, what is $\frac{dy}{dx}$ when $x = 3$?

(A) $\cos(f'(9))$

(B) $6\cos(f(9))$

(C) $f'(9)\cos(f(9))$

(D) $6f'(9)\cos(f(9))$

$\sin x \rightarrow \cos x$

$f(x) \rightarrow f'(x)$

$x^2 \rightarrow 2x$

$\frac{dy}{dx} = \cos f(x^2) \cdot f'(x^2) \cdot 2x$

at $x=3 \Rightarrow \cos f(9) \cdot f'(9) \cdot 6$

39.)

The number of insects in a certain population at time t days is modeled by the function P with first derivative $P'(t) = 0.3t^2 + 12t + 210$. At time $t = 0$, the number of insects in the population is 40. Which of the following statements are true? $P'(10) = .3(100) + 12(10) + 210 = 30 + 120 + 210 = 360$

I. At time $t = 10$, the number of insects in the population is 2840.

II. At time $t = 10$, the number of insects in the population is increasing at a rate of 360 insects per day.

III. At time $t = 10$, the rate of change of the number of insects in the population is increasing at a rate of 18 insects per day per day. $P''(t) = .6t + 12$ $\quad P''(10) = 6 + 12 = 18$

(A) ~~I only~~ (B) ~~II only~~ (C) ~~III only~~ (D) I, II, and III

(no need to check I, but I could if needed)

40.)

If $f(x) = \cos^2(3x - 5)$, then $f'(x) =$

$x^2 \rightarrow 2x$

$\cos x \rightarrow -\sin x$

$3x-5 \rightarrow 3$

(A) $6\cos(3x - 5)$

(B) $-3\sin^2(3x - 5)$

(C) $-2\sin(3x - 5)\cos(3x - 5)$

(D) $-6\sin(3x - 5)\cos(3x - 5)$

$f'(x) = 2\cos(3x-5)(-\sin(3x-5)) \cdot 3$

41.)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	-0.1054	-0.0101	-0.001	0.001	0.0099	0.0953

The function f is continuous and increasing for $x > -1$. The table above gives values of f at selected values of x. Of the following, which is the best approximation for $\lim_{x\to 0} e^{-2f(x)}$? $\rightarrow e^{-2(0)} = 1$

$\lim_{x\to 0} f(x) = 0$

(A) -2

(B) 0

(C) 1

(D) The limit does not exist.

42.)

x	10	11	12	13	14
$f(x)$	5	2	3	6	5

→ not necessarly differentiable

The table above gives values of the continuous function f at selected values of x. If f has exactly two critical points on the open interval $(10, 14)$, which of the following must be true?

(A) $f(x) > 0$ for all x in the open interval $(10, 14)$. not necessarily

(B) $f'(x)$ exists for all x in the open interval $(10, 14)$. not nec. $f'(x)$ undefined could be C.P.

(C) $f'(x) < 0$ for all x in the open interval $(10, 11)$. not necessarly

(D) $f'(12) \neq 0$ C.P. can't be at $x=12$ b/c it would change direction too many times

43.) Let f be the function with $f(0) = \frac{1}{\pi^2}$, $f(2) = \frac{1}{\pi^2}$, and derivative given by $f'(x) = (x+1)\cos(\pi x)$. How many values of x in the open interval $(0, 2)$ satisfy the conclusion of the Mean Value Theorem for the function f on the closed interval $[0, 2]$?

$$\frac{f(2)-f(0)}{2-0} = \frac{\frac{1}{\pi^2}-\frac{1}{\pi^2}}{2-0} = \frac{0}{2} = 0$$

(A) None

(B) One

(C) Two

(D) More than two

$0 = (x+1)\cos(\pi x)$

$x = -1$ ↓ not in $[0,2]$

$\cos(\pi x) = 0$

$\frac{\pi x}{\pi} = \frac{\frac{\pi}{2}, \frac{3\pi}{2}}{\pi}$

$x = \frac{1}{2}, \frac{3}{2}$

44.) Let f be a twice-differentiable function for all real numbers x. Which of the following additional properties guarantees that f has a relative minimum at $x = c$?

If $f'(c)=0$ and $f''(c) > 0$ (concave up)

(A) $f'(c) = 0$

(B) $f'(c) = 0$ and $f''(c) < 0$

(C) $f'(c) = 0$ and $f''(c) > 0$

(D) $f'(x) > 0$ for $x < c$ and $f'(x) < 0$ for $x > c$

45.) The rate at which water leaks from a tank, in gallons per hour, is modeled by R, a differentiable function of the number of hours after the leak is discovered. Which of the following is the best interpretation of $R'(3)$?

R

input

I.R.O.C. of R

(A) The amount of water, in gallons, that has leaked out of the tank during the first three hours after the leak is discovered

(B) The amount of change, in gallons per hour, in the rate at which water is leaking during the three hours after the leak is discovered

(C) The rate at which water leaks from the tank, in gallons per hour, three hours after the leak is discovered

(D) The rate of change of the rate at which water leaks from the tank, in gallons per hour per hour, three hours after the leak is discovered

I.R.O.C is slope (derivative)

$\frac{\text{Function}}{\text{input}}$ $\frac{\text{gallons/hr}}{\text{hr}}$

46.) Let f be a twice-differentiable function such that $f''(x) < 0$ for all x. The graph of $y = S(x)$ is the secant line passing through the points $(3, f(3))$ and $(5, f(5))$. The graph of $y = T(x)$ is the line tangent to the graph of f at $x = 4$. Which of the following is true?

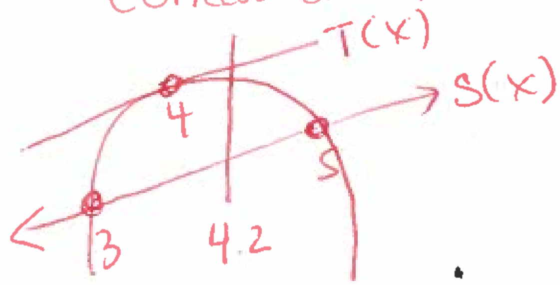
Concave down

(A) $f(4.2) < S(4.2) < T(4.2)$

(B) $f(4.2) < T(4.2) < S(4.2)$

(C) $S(4.2) < f(4.2) < T(4.2)$

(D) $T(4.2) < f(4.2) < S(4.2)$



S(4.2) <

47.) If $f(x) = 3x^2 + 2x$, then $f'(x) =$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(A) $\lim\limits_{h \to 0} \dfrac{\left(3x^2 + 2x + h\right) - \left(3x^2 + 2x\right)}{h}$

Nope

(B) $\lim\limits_{x \to 0} \dfrac{\left(3x^2 + 2x + h\right) - \left(3x^2 + 2x\right)}{h}$

nope

(C) $\lim\limits_{h \to 0} \dfrac{\left(3(x+h)^2 + 2(x+h)\right) - \left(3x^2 + 2x\right)}{h}$

(D) $\lim\limits_{x \to 0} \dfrac{\left(3(x+h)^2 + 2(x+h)\right) - \left(3x^2 + 2x\right)}{h}$

nope

48.) The function f is increasing on the interval $[1, 3]$ and nowhere else. The first derivative of f, f', is continuous for all real numbers. Which of the following could be a table of values for $f'(x)$?

(A)

x	$f'(x)$
0	−1
1	0
2	2
3	0
4	−2

(B)

x	$f'(x)$
0	−1
1	1
2	2
3	1
4	−2

(C)

x	$f'(x)$
0	1
1	0
2	1
3	2
4	0

(D)

x	$f'(x)$
0	1
1	0
2	2
3	0
4	−2

$f'(x) = 0$ at $x = 1, 3$ and $f'(x) > 0$ between $(1,3)$ only!

49.) On a certain day, the total number of pieces of candy produced by a factory since it opened is modeled by C, a differentiable function of the number of hours since the factory opened. Which of the following is the best interpretation of $C'(3) = 500$?

(A) The factory produces 500 pieces of candy during its 3rd hour of operation.

(B) The factory produces 500 pieces of candy in the first 3 hours after it opens.

(C) The factory is producing candy at a rate of 500 pieces per hour, 3 hours after it opens.

(D) The rate at which the factory is producing candy is increasing at a rate of 500 pieces per hour per hour, 3 hours after it opens.

$C' \Rightarrow \dfrac{\text{candy}}{\text{hours}}$

50.)
$$f(x) = \begin{cases} x^2 \sin(\pi x) & \text{for } x < 2 \\ x^2 + cx - 18 & \text{for } x \geq 2 \end{cases}$$

$2^2 \sin(2\pi) = 2^2 + 2c - 18$

$0 = -14 + 2c$

Let f be the function defined above, where c is a constant. For what value of c, if any, is f continuous at $x = 2$?

$14 = 2c \quad c = 7$

(A) 2 (B) 7 (C) 9 (D) $4\pi - 4$ (E) There is no such value of c.