A.P. Calculus No Calculator Name Key

2 ____ 1. Determine the derivative, dy/dx of $y = 3x^5 - 5x^2 + 6$

$\frac{dy}{dx} = 15x^4 - 10x$

3 ____ 2. What is the instantaneous rate of change of $f(x) = x^3 - 3x^2 + x - 1$ at $x = 2$?

$f'(x) = 3x^2 - 6x + 1$

$f'(2) = 3(2)^2 - 6(2) + 1 = 1$

2 ____ 3. $\lim_{x\to\infty}\left(\frac{7x^2 + 5x - 3}{2 + 3x - 11x^2}\right) =$ $\frac{-7}{11}$

2 ____ 4. If $f(x) = 2(3x+5)^3$, then $f'(x) =$

$f'(x) = 6(3x+5)^2(3)$

$18(3x+5)^2$ or $18(9x^2 + 30x + 25)$ or $162x^2 + 540x + 45$

4 ____ 5. What is the equation, in point slope form of the normal line to the function $y = 3x^2 - 32\sqrt{x+2}$ at $x = 2$?

$y(2) = 3(2)^2 - 32\sqrt{4}$

$12 - 64$

$= -52$

$\frac{dy}{dx} = 6x - \frac{32}{2\sqrt{x+2}}(1)$

$\boxed{y + 52 = \frac{-1}{4}(x-2)}$

$\frac{dy}{dx}\Big|_{x=2}$ $12 - \frac{32}{2\sqrt{4}} \to 12 - \frac{32}{4} \to 12 - 8$

$= 4$

$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ $\lim_{h\to 0} \frac{4(x+h)^8 - 4x^8}{h}$

$f(x)=4x^8$

$f'(x)=32x^7$

$f'(\frac{1}{2})=32(\frac{1}{2})^7$

$= \frac{32}{2^7} = \frac{32}{2^5 \cdot 32}$

$= \frac{1}{4}$

3 ______ 6. Evaluate the limit: $\lim_{h\to 0}\left(\frac{4\left(\frac{1}{2}+h\right)^8 - 4\left(\frac{1}{2}\right)^8}{h}\right) =$

$f(x)=4x^8$ $f'(x)=32x^7$

$f'(\frac{1}{2})=32(\frac{1}{2})^7=\frac{32}{128}=\frac{1}{4}$

3 ______ 7. If $f(x)=\frac{\sqrt{x-5}}{3x^2}$ then $f'(x) = ?$

*Simplify your answer to one single rational expression. No need to rationalize the denominator.

$f'(x) = \frac{(3x^2)\frac{1}{2\sqrt{x-5}} - \sqrt{x-5}(6x)}{(3x^2)^2}$

$= \frac{\frac{x}{2\sqrt{x-5}} - \frac{2\sqrt{x-5}}{1}\cdot\frac{2\sqrt{x-5}}{2\sqrt{x-5}}}{3x^3}$ $\frac{x-4(x-5)}{6x^3\sqrt{x-5}} \rightarrow \frac{-3x+20}{6x^3\sqrt{x-5}}$

2

8. Determine the 38th derivative of $y=\sin x$.

$\frac{38}{4} = 9 \text{ R}2$

$\frac{d}{dx}\sin x = \cos(x)$ (1)

$\frac{d}{dx}\cos x = -\sin(x)$ (2)

so $-\sin x$

$\frac{d}{dx}(-\sin x) = -\cos(x)$ (3)

$\frac{d}{dx}(-\cos x) = \sin(x)$ (4)

9. Determine the second derivative of $y=\tan x$.

2

$\frac{d}{dx}\tan x = \sec^2 x \rightarrow (\sec x)^2$

$\frac{d}{dx}(\sec x)^2 = 2\sec x \sec x \tan x = 2\sec^2 x \tan x$

2 10. Evaluate the limit: $\lim_{x\to 0} \frac{2\sin 10x}{x} =$

$$\lim_{x\to 0} 2\cdot \frac{\sin \cancel{10x}}{\cancel{10x}} \cdot 10 = 20$$

3 11. Given $f(1)=1$, $g(1)=-2$, $f'(1)=3$, and $g'(1)=-1$,

Evaluate $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)\Big|_{x=1}$

$$\frac{g(x)f'(x)-g'(x)f(x)}{(g(x))^2} \rightarrow \frac{(-2)(3)-(-1)(1)}{(-2)^2}$$

$$\frac{-6+1}{4} = -\frac{5}{4}$$

2 12. Determine the derivative of $h(x) = 4x^2 \tan x$.

$$h'(x) = (4x^2)(\sec^2 x) + (8x)\tan x$$

3 13. For $x \neq 4$, the function h(x) is equal to $\frac{x^2+x-20}{x-4}$. What value should be assigned to h(4) to make h(x) continuous at x = 4?

$$\frac{(x+5)\cancel{(x-4)}}{\cancel{x-4}} \qquad x+5 \quad \uparrow 4 \qquad h(4)=9$$

Determine the limits of the following functions

2 14. $\lim_{x\to\infty} \frac{2x^2-3}{5-x} =$ $\quad \frac{2x^2}{-x} = -2x \quad$ $-\infty$

3 15. $\lim_{x\to 3} \frac{2x^2-5x-3}{x^3-27}$

$\frac{(2x+1)\cancel{(x-3)}}{\cancel{(x-3)}(x^2+3x+9)} \qquad \frac{2(3)+1}{3^2+3(3)+9} = \frac{7}{27}$

3 16. $\lim_{h\to 0} \frac{(x+h)^2-x^2}{h} =$ $\quad f'(x) = 2x$

$f(x) = x^2$

17. If $s(t) = -2t^3 + 15t^2 + 144t - 24$ and $t \geq 0$

3 i.) what is $v(t)$? $\quad v(t) = -6t^2 + 30t + 144$

$6\overline{)144}$ = 24; 12, 24

3 ii.) when is the particle moving in the positive direction?

$v(t) = -6(t^2-5t-24)$

$-6(t-8)(t+3)$

$- \quad + \quad + \quad -$

$-3 \quad 0 \quad 8$

3 iii.) when is the particle slowing down? $\quad [0,8)$

$a(t) = -12t + 30$

$\frac{-30}{-12} = \frac{15}{6} = \frac{5}{2} \qquad (\frac{5}{2}, 8)$

$+ \quad -$

$0 \quad \frac{5}{2}$

AP Calculus Name________________________

Fall Break HW to help improve grade and understanding :-) Date______________

Differentiate each function with respect to the given variable. Don't simplify.

1) $g(t) = \sqrt{\dfrac{2t-5}{3t^3+1}}$ $\rightarrow \left(\dfrac{2t-5}{3t^3+1}\right)^{\frac{1}{2}}$

$$g'(t) = \frac{1}{2}\left(\frac{2t-5}{3t^3+1}\right)^{\frac{-1}{2}}\left(\frac{(3t^3+1)(2)-(2t-5)(9t^2)}{(3t^3+1)^2}\right)$$

2) $g(x) = (-3x^4+5)^4$

$$g'(x) = 4(-3x^4+5)^3(-12x^3)$$

3) $f = ((-3t^4-2)^3+1)^{\frac{1}{4}}$

$x^{\frac{1}{4}} \rightarrow \frac{1}{4}x^{-\frac{3}{4}}$

$x^3+1 \rightarrow 3x^2$

$-3t^4-2 \rightarrow -12t^3$

$$f'(t) = \frac{1}{4}((-3t^4-2)^3+1)^{\frac{-3}{4}} \cdot 3(-3t^4-2)^2 \cdot -12t^3$$

4) $f = (5r+1)(-r^5+3)^2$

$$f'(r) = (5r+1) \cdot 2(-r^5+3)^1 \cdot -5r^4 + (5)(-r^5+3)^2$$

Differentiate each function with respect to x.

5) $f(x) = \cos 2x^5$

$$f'(x) = -\sin(2x^5) \cdot 10x^4$$

6) $y = \sec \dfrac{4x^3}{x^4 - 3}$

$$\frac{dy}{dx} = \sec\left(\frac{4x^3}{x^4-3}\right)\tan\left(\frac{4x^3}{x^4-3}\right)\left(\frac{(x^4-3)(12x^2)-(4x^3)(4x^3)}{(x^4-3)^2}\right)$$

7) $f(x) = \tan(\cos 5x^2)$

$\tan x \rightarrow \sec^2 x$

$\cos x \rightarrow -\sin x$

$5x^2 \rightarrow 10x$

$$f'(x) = \sec^2(\cos(5x^2)) \cdot -\sin(5x^2) \cdot 10x$$

For each problem, find the equation of the line tangent to the function at the given point.

8) $y = +\frac{-x^2}{2x-2}$ at $x = -3$

$y(-3) = \frac{-(-3)^2}{2(-3)-2} = \frac{-9}{-8} = \frac{9}{8}$

$\frac{dy}{dx} = \frac{(2x-2)(-2x) - (-x^2)(2)}{(2x-2)^2} \rightarrow \frac{dy}{dx}\Big|_{x=-3} = \frac{(-8)(6)-(-9)(2)}{(-8)^2}$

$= \frac{-48+18}{64} = \frac{-30}{64} = \frac{-15}{32}$

$y - \frac{9}{8} = \frac{-15}{32}(x+3)$

Find the equation of the line normal to the function at the given point. Evaluate the trig value.

9) $y = 2\sin(x)$ at $x = \frac{2\pi}{3}$

$y\left(\frac{2\pi}{3}\right) = 2\sin\left(\frac{2\pi}{3}\right) = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$

$\frac{dy}{dx} = 2\cos x$

$\frac{dy}{dx}\Big|_{x=\frac{2\pi}{3}} = 2\cos\left(\frac{2\pi}{3}\right)$

$= 2\left(-\frac{1}{2}\right)$

$= -1$

$-1 \perp 1$

$y - \sqrt{3} = 1\left(x - \frac{2\pi}{3}\right)$

A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the velocity function $v(t)$, the acceleration function $a(t)$, the times t when the particle changes directions, the intervals of time when the particle is moving left and moving right, the times t when the acceleration is 0, and the intervals of time when the particle is slowing down and speeding up.

10) $s(t) = -t^3 + 22t^2 - 105t$

$s'(t) = -3t^2 + 44t - 105$

$s''(t) = -6t + 44$

$v(t)$: $-$ | 0 | 3 | $+$ | $\frac{35}{3}$ | $-$

$a(t)$: $+$ | $\frac{22}{3}$ | $-$

$-3t^2 + 44t - 105 = 0$

$-(3t^2 - 44t + 105) = 0$

$-(3t - 35)(t - 3) = 0 \qquad t = \frac{35}{3}, 3$

$105 \to 21 \cdot 5 \to 7 \cdot 3 \cdot 5$

$-6t + 44 = 0$

$6t = 44 \to \div 6 \to t = \frac{44}{6} = \frac{22}{3}$

$v(t)$	$-3t^2 + 44t - 105$	Moving right	$(3, \frac{35}{3})$
$a(t)$	$-6t + 44$	Acceleration is 0	$t = \frac{22}{3}$
t when particle changes direction	$3, \frac{35}{3}$	Speeding up $v(t)/a(t)$ same	$(3, \frac{22}{3})$ $(\frac{35}{3}, \infty)$
Moving left	$(0, 3)$ $(\frac{35}{3}, \infty)$	Slowing down $v(t)/a(t)$ opposite	$(0, 3)$ $(\frac{22}{3}, \frac{35}{3})$

For each problem, you are given a table containing some values of differentiable functions $f(x)$, $g(x)$ and their derivatives. Use the table data and the rules of differentiation to solve each problem.

11)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	5	−1	6	−1
2	4	−1	5	−1
3	3	−1	4	−1
4	2	−1	3	$-\frac{3}{2}$
5	1	0	1	0
6	2	1	3	2

Part 1) Given $h_1(x) = f(x) + g(x)$, find $h_1'(5)$

Part 2) Given $h_2(x) = f(x) - g(x)$, find $h_2'(1)$

Part 3) Given $h_3(x) = f(x) \cdot g(x)$, find $h_3'(3)$

Part 4) Given $h_4(x) = \dfrac{f(x)}{g(x)}$, find $h_4'(6)$

Part 5) Given $h_5(x) = (f(x))^2$, find $h_5'(3)$

Part 6) Given $h_6(x) = f(g(x))$, find $h_6'(3)$

$2 - 1 =$

$\frac{d}{dx}(f(x))^2 = 2(f(x))^1 f'(x$

$\frac{d}{dx} f(g(x))$

$f(x) \to f'(x)$

$g(x) \to g'(x)$

$\to f'(g(x)) \cdot g'(x)$

1.) $f'(5) + g'(5) = 0 + 0 = 0$

2) $f'(1) - g'(1) = -1 - -1 = 0$

3) $f(3) \cdot g'(3) + f'(3) \cdot g(3) = 3 \cdot (-1) + (-1) \cdot (4) = -3 - 4 = -7$

4) $\dfrac{g(6) \cdot f'(6) - f(6) \cdot g'(6)}{(g(6))^2} = \dfrac{3 \cdot 1 - 2 \cdot 2}{3^2} = \dfrac{3-4}{9} = -\dfrac{1}{9}$

5.) $2 \cdot f(3) \cdot f'(3) = 2(3)(-1) = -6$

6) $f'(g(3)) \cdot g'(3)$

$= f'(4) \cdot (-1) = (-1) \cdot (-1)$

$= 1$

Evaluate each limit. Evaluate the trig value. Don't simplify your answer for the radical problem.

12) $\displaystyle\lim_{h \to 0} \frac{\sin\left(\frac{\pi}{3} + h\right) - \sin\frac{\pi}{3}}{h}$

Limit Def. of Derivative

$f(x) = \sin x$

$f'(x) = \cos x$

$x = \frac{\pi}{3}$

$f'(\frac{\pi}{3}) = \cos\frac{\pi}{3} = \frac{1}{2}$

13) $\displaystyle\lim_{h \to 0} \frac{\sqrt[3]{2+h} - \sqrt[3]{2}}{h}$

$f(x) = \sqrt[3]{x} \to x^{\frac{1}{3}}$

$f'(x) = \frac{1}{3} x^{\frac{-2}{3}}$

$= \dfrac{1}{3\sqrt[3]{x^2}}$

$x = 2$

$f'(2) = \dfrac{1}{3\sqrt[3]{2^2}}$ or $\dfrac{1}{3\sqrt[3]{4}}$