

2. 1. Determine the derivative,  $dy/dx$  of  $y = 3x^5 - 5x^2 + 6$

$$\frac{dy}{dx} = 15x^4 - 10x$$

3. 2. What is the instantaneous rate of change of  $f(x) = x^3 - 3x^2 + x - 1$  at  $x = 2$ ?

$$f'(x) = 3x^2 - 6x + 1$$

$$f'(2) = 3(2)^2 - 6(2) + 1 = 1$$

2. 3.  $\lim_{x \rightarrow \infty} \left( \frac{7x^3 + 5x - 3}{2 + 3x - 11x^3} \right) = \frac{-7}{11}$

2. 4. If  $f(x) = 2(3x + 5)^3$ , then  $f'(x) =$

$$f'(x) = 6(3x + 5)^2(3)$$

$$18(3x + 5)^2 \text{ or } 18(9x^2 + 30x + 25) \text{ or } 162x^2 + 540x + 450$$

4. 5. What is the equation, in point slope form of the normal line to the function  $y = 3x^2 - 32\sqrt{x+2}$  at  $x = 2$ ?

$$y(2) = 3(2)^2 - 32\sqrt{4}$$

$$12 - 64 = -52$$

$$\frac{dy}{dx} = 6x - \frac{32}{2\sqrt{x+2}} \quad (1)$$

$$y + 52 = -\frac{1}{4}(x - 2)$$

$$\frac{dy}{dx} \Big|_{x=2} = 12 - \frac{32}{2\sqrt{4}} \rightarrow 12 - \frac{32}{4} \rightarrow 12 - 8 = 4$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \rightarrow 0} \frac{4(x+h)^8 - 4x^8}{h}$$

$$\begin{aligned} f(x) &= 4x^8 \\ f'(x) &= 32x^7 \\ f'\left(\frac{1}{2}\right) &= 32\left(\frac{1}{2}\right)^7 \\ &= \frac{32}{2^7} = \frac{32}{2^5 \cdot 2^2} \\ &= \frac{32}{32 \cdot 4} \\ &= \frac{1}{4} \end{aligned}$$

3 \_\_\_\_\_ 6. Evaluate the limit:

$$\lim_{h \rightarrow 0} \left( \frac{4\left(\frac{1}{2}+h\right)^8 - 4\left(\frac{1}{2}\right)^8}{h} \right) =$$

$$f(x) = 4x^8 \quad f'(x) = 32x^7$$

$$f'\left(\frac{1}{2}\right) = 32\left(\frac{1}{2}\right)^7 = \frac{32}{128} = \left(\frac{1}{4}\right)$$

3 \_\_\_\_\_ 7. If  $f(x) = \frac{\sqrt{x-5}}{3x^2}$  then  $f'(x) = ?$

\*Simplify your answer to one single rational expression. No need to rationalize the denominator.

$$f'(x) = \frac{(3x^2) \frac{1}{2\sqrt{x-5}} - \sqrt{x-5}(6x)}{(3x^2)^2}$$

$$= \frac{\frac{x}{2\sqrt{x-5}} - \frac{2\sqrt{x-5} \cdot 2\sqrt{x-5}}{1 \cdot 2\sqrt{x-5}}}{3x^3} = \frac{x - 4(x-5)}{6x^3\sqrt{x-5}} \rightarrow \frac{-3x + 20}{6x^3\sqrt{x-5}}$$

2 \_\_\_\_\_ 8. Determine the 38<sup>th</sup> derivative of  $y = \sin x$ .

$$\frac{d}{dx} \sin x = \cos(x) \quad (1)$$

$$\frac{d}{dx} \cos x = -\sin(x) \quad (2)$$

$$\frac{d}{dx} (-\sin x) = -\cos(x) \quad (3)$$

$$\frac{d}{dx} (-\cos x) = \sin(x) \quad (4)$$

$$\frac{38}{4} = 9 \text{ R } 2$$

so

$$-\sin x$$

9. Determine the second derivative of  $y = \tan x$ .

$$\frac{d}{dx} \tan x = \sec^2 x \rightarrow (\sec x)^2$$

$$\frac{d}{dx} (\sec x)^2 = 2\sec x \sec x \tan x = 2\sec^2 x \tan x$$

2

10. Evaluate the limit:  $\lim_{x \rightarrow 0} \frac{2 \sin 10x}{x} = 1$

$$\lim_{x \rightarrow 0} 2 \cdot \frac{\sin 10x}{10x} \cdot 10 = 20$$

3

11. Given  $f(1)=1$ ,  $g(1)=-2$ ,  $f'(1)=3$ , and  $g'(1)=-1$ ,

Evaluate  $\frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) \Big|_{x=1}$

$$\frac{g(x)f'(x) - g'(x)f(x)}{(g(x))^2} \rightarrow \frac{(-2)(3) - (-1)(1)}{(-2)^2}$$

$$\frac{-6+1}{4} = -\frac{5}{4}$$

2

12. Determine the derivative of  $h(x) = 4x^2 \tan x$ .

$$h'(x) = (4x^2)(\sec^2 x) + (8x)\tan x$$

3

13. For  $x \neq 4$ , the function  $h(x)$  is equal to  $\frac{x^2 + x - 20}{x - 4}$ . What value should be assigned to  $h(4)$  to make  $h(x)$  continuous at  $x = 4$ ?

$$\frac{(x+5)(x-4)}{x-4}$$

$$x+5$$

$$\uparrow 4$$

$$h(4) = 9$$

Determine the limits of the following functions

2 14.  $\lim_{x \rightarrow \infty} \frac{2x^2 - 3}{5 - x} =$   $\frac{2x^2}{-x} = -2x$

$-\infty$

3 15.  $\lim_{x \rightarrow 3} \frac{2x^2 - 5x - 3}{x^3 - 27}$

$\frac{(2x+1)(x-3)}{(x-3)(x^2+3x+9)}$

$\frac{2(3)+1}{3^2+3(3)+9} = \frac{7}{27}$

3 16.  $\lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} =$

$f(x) = x^2$

$f'(x) = 2x$

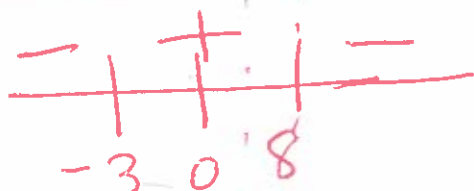
17. If  $s(t) = -2t^3 + 15t^2 + 144t - 24$  and  $t \geq 0$

3 i.) what is  $v(t)$ ?  $v(t) = -6t^2 + 30t + 144$

$6 \sqrt{144} = \frac{12}{24}$

3 ii.) when is the particle moving in the positive direction?

$v(t) = -6(t^2 - 5t - 24)$   
 $-6(t-8)(t+3)$

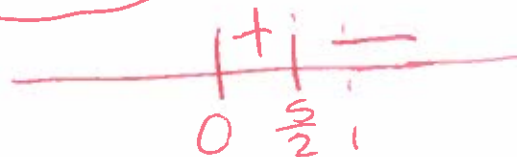


3 iii.) when is the particle slowing down?

$[0, 8)$

$a(t) = -12t + 30$

$\frac{-30}{-12} = \frac{15}{6} = \frac{5}{2} \quad (\frac{5}{2}, 8)$



Fall Break HW to help improve grade and understanding :-)

Date \_\_\_\_\_

Differentiate each function with respect to the given variable. Don't simplify.

$$1) g(t) = \sqrt{\frac{2t-5}{3t^3+1}} \rightarrow \left(\frac{2t-5}{3t^3+1}\right)^{\frac{1}{2}}$$

$$g'(t) = \frac{1}{2} \left(\frac{2t-5}{3t^3+1}\right)^{-\frac{1}{2}} \left(\frac{(3t^3+1)(2) - (2t-5)(9t^2)}{(3t^3+1)^2}\right)$$

$$2) g(x) = (-3x^4 + 5)^4$$

$$g'(x) = 4(-3x^4 + 5)^3 (-12x^3)$$

$$3) f = ((-3t^4 - 2)^3 + 1)^{\frac{1}{4}}$$

$$x^{\frac{1}{4}} \rightarrow \frac{1}{4} x^{-\frac{3}{4}}$$

$$x^3 + 1 \rightarrow 3x^2$$

$$-3t^4 - 2 \rightarrow -12t^3$$

$$f'(t) = \frac{1}{4} ((-3t^4 - 2)^3 + 1)^{-\frac{3}{4}} \cdot 3(-3t^4 - 2)^2 \cdot -12t^3$$

$$4) f = (5r+1)(-r^5+3)^2$$

$$f'(r) = (5r+1) \cdot 2(-r^5+3)^1 \cdot -5r^4 + (5)(-r^5+3)^2$$

Differentiate each function with respect to  $x$ .

5)  $f(x) = \cos 2x^5$

$$f'(x) = -\sin(2x^5) \cdot 10x^4$$

6)  $y = \sec \frac{4x^3}{x^4 - 3}$

$$\frac{dy}{dx} = \sec\left(\frac{4x^3}{x^4 - 3}\right) \tan\left(\frac{4x^3}{x^4 - 3}\right) \left( \frac{(x^4 - 3)(12x^2) - (4x^3)(4x^3)}{(x^4 - 3)^2} \right)$$

7)  $f(x) = \tan(\cos 5x^2)$

$$\tan x \rightarrow \sec^2 x$$

$$\cos x \rightarrow -\sin x$$

$$5x^2 \rightarrow 10x$$

$$f'(x) = \sec^2(\cos(5x^2)) \cdot -\sin(5x^2) \cdot 10x$$

For each problem, find the equation of the line tangent to the function at the given point.

8)  $y = \frac{-x^2}{2x-2}$  at  $x = -3$

$$y(-3) = \frac{-(-3)^2}{2(-3)-2} = \frac{-9}{-8} = \frac{9}{8}$$

$$\frac{dy}{dx} = \frac{(2x-2)(-2x) - (-x^2)(2)}{(2x-2)^2} \rightarrow \frac{dy}{dx} \Big|_{x=-3} = \frac{(-8)(6) - (-9)(2)}{(-8)^2}$$

$$= \frac{-48 + 18}{64} = \frac{-30}{64} = \frac{-15}{32}$$

$$y - \frac{9}{8} = \frac{-15}{32}(x + 3)$$

Find the equation of the line normal to the function at the given point. Evaluate the trig value.

9)  $y = 2\sin(x)$  at  $x = \frac{2\pi}{3}$

$$y\left(\frac{2\pi}{3}\right) = 2\sin\left(\frac{2\pi}{3}\right) = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$$

$$\frac{dy}{dx} = 2\cos x$$

$$\frac{dy}{dx} \Big|_{x=\frac{2\pi}{3}} = 2\cos\left(\frac{2\pi}{3}\right)$$

$$= 2\left(-\frac{1}{2}\right)$$

$$= -1$$

$$-1 \perp 1$$

$$y - \sqrt{3} = 1\left(x - \frac{2\pi}{3}\right)$$

A particle moves along a horizontal line. Its position function is  $s(t)$  for  $t \geq 0$ . For each problem, find the velocity function  $v(t)$ , the acceleration function  $a(t)$ , the times  $t$  when the particle changes directions, the intervals of time when the particle is moving left and moving right, the times  $t$  when the acceleration is 0, and the intervals of time when the particle is slowing down and speeding up.

10)  $s(t) = -t^3 + 22t^2 - 105t$

$$s'(t) = -3t^2 + 44t - 105$$

$$s''(t) = -6t + 44$$

$$-3t^2 + 44t - 105 = 0$$

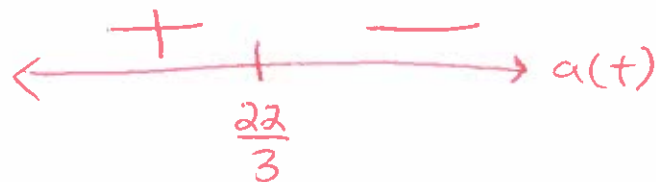
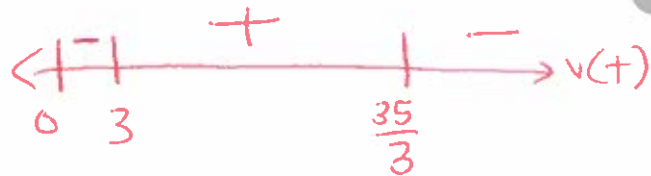
$$-(3t^2 - 44t + 105) = 0$$

$$-(3t - 35)(t - 3) = 0 \quad t = \frac{35}{3}, 3$$

$$105 \rightarrow 21 \cdot 5 \rightarrow 7 \cdot 3 \cdot 5$$

$$-6t + 44 = 0$$

$$6t = 44 \rightarrow t = \frac{44}{6} = \frac{22}{3}$$



$v(t)$	$-3t^2 + 44t - 105$	Moving right	$(3, \frac{35}{3})$
$a(t)$	$-6t + 44$	Acceleration is 0	$t = \frac{22}{3}$
$t$ when particle changes direction	$3, \frac{35}{3}$	Speeding up $v(t)/a(t)$ Same	$(3, \frac{22}{3})$ $(\frac{35}{3}, \infty)$
Moving left	$(0, 3)$ $(\frac{35}{3}, \infty)$	Slowing down $v(t)/a(t)$ opposite	$(0, 3)$ $(\frac{22}{3}, \frac{35}{3})$



For each problem, you are given a table containing some values of differentiable functions  $f(x)$ ,  $g(x)$  and their derivatives. Use the table data and the rules of differentiation to solve each problem.

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	5	-1	6	-1
2	4	-1	5	-1
3	3	-1	4	-1
4	2	-1	3	$-\frac{3}{2}$
5	1	0	1	0
6	2	1	3	2

Part 1) Given  $h_1(x) = f(x) + g(x)$ , find  $h_1'(5)$

Part 2) Given  $h_2(x) = f(x) - g(x)$ , find  $h_2'(1)$

Part 3) Given  $h_3(x) = f(x) \cdot g(x)$ , find  $h_3'(3)$

Part 4) Given  $h_4(x) = \frac{f(x)}{g(x)}$ , find  $h_4'(6)$

Part 5) Given  $h_5(x) = (f(x))^2$ , find  $h_5'(3)$

Part 6) Given  $h_6(x) = f(g(x))$ , find  $h_6'(3)$

$$\frac{d}{dx} (f(x))^2 = 2(f(x))' f'(x)$$

$$\frac{d}{dx} f(g(x))$$

$$f(x) \rightarrow f'(x)$$

$$g(x) \rightarrow g'(x)$$

$$\rightarrow f'(g(x)) \cdot g'(x)$$

$$1.) f'(5) + g'(5) = 0 + 0 = 0$$

$$2.) f'(1) - g'(1) = -1 - (-1) = 0$$

$$3.) f(3) \cdot g'(3) + f'(3) \cdot g(3) = 3 \cdot (-1) + (-1) \cdot (4) = -3 - 4 = -7$$

$$4.) \frac{g(6) \cdot f'(6) - f(6) \cdot g'(6)}{(g(6))^2} = \frac{3 \cdot 1 - 2 \cdot 2}{3^2} = \frac{3 - 4}{9} = -\frac{1}{9}$$

$$5.) 2 \cdot f(3) \cdot f'(3) = 2(3)(-1) = -6$$

$$6.) f'(g(3)) \cdot g'(3) = f'(4) \cdot (-1) = (-1) \cdot (-1) = 1$$

Evaluate each limit. Evaluate the trig value. Don't simplify your answer for the radical problem.

$$12) \lim_{h \rightarrow 0} \frac{\sin\left(\frac{\pi}{3} + h\right) - \sin\frac{\pi}{3}}{h}$$

Limit Def. of Derivative

$$f(x) = \sin x$$

$$f'(x) = \cos x$$

$$x = \frac{\pi}{3}$$

$$f'\left(\frac{\pi}{3}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$$

$$13) \lim_{h \rightarrow 0} \frac{\sqrt[3]{2+h} - \sqrt[3]{2}}{h}$$

$$f(x) = \sqrt[3]{x} \rightarrow x^{\frac{1}{3}}$$

$$f'(x) = \frac{1}{3} x^{-\frac{2}{3}}$$

$$= \frac{1}{3\sqrt[3]{x^2}}$$

$$x = 2$$

$$f'(2) = \frac{1}{3\sqrt[3]{2^2}} \text{ or } \frac{1}{3\sqrt[3]{4}}$$

