AP Calculus   Fall 2017 Final Exam Review Part 1   Name Key

Calculator OK.

I. Multiple Choice

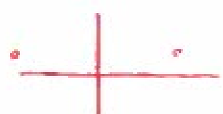
B 1. The slope of the line tangent to the graph of $y = \ln(x^2)$ at $x = e^2$ is

(A) $\frac{1}{e^2}$ (B) $\frac{2}{e^2}$ (C) $\frac{4}{e^2}$ (D) $\frac{1}{e^4}$ (E) $\frac{4}{e^4}$

$\frac{dy}{dx} = \frac{1}{x^2}(2x) = \frac{2x}{x^2} = \frac{2}{x} \rightarrow \frac{2}{e^2}$

A 2. With respect to the y-axis, point $(-4, 1)$ is symmetric to

A $(4, 1)$ B $(-4, -1)$ C $(4, -1)$ D $(1, -4)$ E $(1, 4)$



D 3. Determine the differential dy given that $y = x\cot(x)$

(A) $-\csc^2 x$ (B) $-x\csc^2 x\,dx$ (C) $-x\csc^2 x + \cot x$

(D) $(-x\csc^2 x + \cot x)dx$ (E) None of these

$\frac{dy}{dx} = x(-\csc^2 x) + 1 \cdot \cot x$

C 4. Determine the equation of the tangent line to the graph of $y = f(x)$ at the point where $x = -3$ if $f(-3) = 2$ and $f'(-3) = 5$.

(A) $y = -3$ (B) $y = 5x + 2$ (C) $y = 5x + 17$ (D) $y = 2x + 5$

(E) It can not be determined from this information.

$y - 2 = 5(x + 3)$

$y = 5x + 15 + 2$

C 5. The derivative of $y = 6x^2$ is

(A) x (B) 12   (C) $12x$ (D) 0   (E) None of these

A 6. The second derivative of $y = 8x$ is

(A) 0   (B) $8x$ (C) x (D) 8   (E) None of these

B 7. A ball is dropped from a height of 1 meter. It always bounces to one-half its previous height. The ball will bounce infinitely but it will travel a finite distance. What is the distance? $1+\frac{1}{2}+\frac{1}{2}+\frac{1}{4}+\frac{1}{4}+\frac{1}{8}+\frac{1}{8}+\ldots$

(A) 4 m (B) 3 m (C) $2\frac{31}{32}$ m (D) 2 m (E) It can not be determined

$S=\frac{1}{1-\frac{1}{2}}=2$ $S=\frac{\frac{1}{2}}{1-\frac{1}{2}}=1$ $2+1=3$

D 8. $\frac{d}{dx}\left(Sin^{-1}(2x)\right)=$ $\frac{1}{\sqrt{1-(2x)^2}}(2)$

(A) $\frac{-1}{2\sqrt{1-4x^2}}$ (B) $\frac{-2}{\sqrt{4x^2-1}}$ (C) $\frac{1}{2\sqrt{1-4x^2}}$

(D) $\frac{2}{\sqrt{1-4x^2}}$ (E) $\frac{2}{\sqrt{4x^2-1}}$

A 9. If $y=e^{nx}$, where n is a constant, then $\frac{d^n y}{dx^n}=$

(A) $n^n e^{nx}$ (B) $n!\,e^{nx}$ (C) ne^{nx} (D) $n^n e^{nx-1}$ (E) $n!\,e^x$

C 10. Water flows at 8 cubic feet per minute into a cylinder with radius 4 feet. How fast is the water level rising?

(A) 2 ft/min (B) $\frac{1}{\pi}$ ft/min (C) $\frac{1}{2\pi}$ ft/min (D) 2π ft/min

(E) None of the above

$V=\pi r^2 h$

$V=16\pi h$ $8=16\pi\frac{dh}{dt}$

$\frac{dV}{dt}=16\pi\frac{dh}{dt}$ $\frac{dh}{dt}=\frac{8}{16\pi}$

A 11. The value of $\frac{d^2y}{dx^2}$ in the equation $y^2+y=x$ at the point (2, 1) is:

(A) $\frac{-2}{27}$ (B) $\frac{1}{3}$ (C) $\frac{1}{5}$ (D) $\frac{-2}{125}$ (E) $\frac{1}{2}$

$2yy'+y'=1$ $\rightarrow y'=\frac{1}{2y+1}$ or $y'=(2y+1)^{-1}$

$y'(2y+1)=1$ $y''=-1(2y+1)^{-2}(2y')$

$y''|_{(2,1)}=-1(2\cdot1+1)^{-2}(2)\left(\frac{1}{2\cdot1+1}\right)$ $-1\left(\frac{1}{9}\right)\frac{2}{3}$

B 12. If the graph of $y = ax^3 + 4x^2 + cx + d$ has a point of inflection at $(1,0)$, then the value of a is:

(A) 2 (B) $-\frac{4}{3}$ (C) $\frac{1}{2}$ (D) $\frac{8}{3}$ (E) None of these

$y' = 3ax^2 + 8x + c$

$y'' = 6ax + 8$

$0 = 6ax + 8$

$\frac{-8}{6a} = \frac{6ax}{6a}$

$1 = \frac{-8}{6a}$

$6a = -8$

$a = -\frac{4}{3}$

E 13. The equation of the normal line to the curve $y = x^4 + 3x^3 + 2$ at the point where $x = 0$ is

$y' = 4x^3 + 9x^2$ $y' = 0$ So Normal is vertical

(A) $y = x$ (B) $y = 13x$ (C) $y = 0$ (D) $y = x + 2$ (E) $x = 0$

A 14. If $y = \sin u$, $u = 3w$, and $w = e^{2x}$, then $\frac{dy}{dx} =$

$\frac{dy}{du} \cdot \frac{du}{dw} \cdot \frac{dw}{dx} = \frac{dy}{dx}$

(A) $6e^{2x}\cos(3e^{2x})$ (B) $3\cos(e^{2x})$ (C) $e^{2\cos(3e^{2x})}$ (D) $-6\sin(6e^{2x})$

(E) $6x\cos(e^{2x})$

$\cos u \cdot 3 \cdot 2e^{2x}$

$\cos 3e^{2x} \cdot 3 \cdot 2e^{2x}$

D 15. $\frac{d}{dx}(\text{Arccos } 3x) =$

(A) $\frac{3}{\sqrt{1-x^2}}$ (B) $\frac{-1}{3\sin 3x}$ (C) $\frac{3}{\sqrt{1-3x^2}}$ (D) $\frac{-3}{\sqrt{1-9x^2}}$

(E) $\frac{3}{\sqrt{9x^2-1}}$

$\frac{-1}{\sqrt{1-(3x)^2}} \cdot 3$

A 16. If $f(x) = 2e^x + e^{2x}$, then $f'''(0) =$

(A) 10 (B) 8 (C) 6 (D) 4 (E) 3

$f'(x) = 2e^x + 2e^{2x}$

$f''(x) = 2e^x + 4e^{2x}$

$f'''(x) = 2e^x + 8e^{2x}$

$f'''(0) = 2e^0 + 8e^0 = 10$

B

17. Determine the absolute maximum value and the absolute minimum value of the function $f(x)=2x^3+3x^2-12x$ over the interval $[-3,3]$.

(A) Absolute Maximum value is 20; Absolute Minimum value is -7

(B) Absolute Maximum value is 45; Absolute Minimum value is -7

(C) Absolute Maximum value is 3; Absolute Minimum value is 1

(D) Absolute Maximum value is -2; Absolute Minimum value is 1

(E) Absolute Maximum value is 45; Absolute Minimum value is 9

$f(-3)=9$

$f(-2)=20$

$f(1)=-7$

$f(3)=45$

$f'(x)=6x^2+6x-12$

$=6(x^2+x-2)$

$=6(x+2)(x-1)$

min max min max

$[\;+\;|\;-\;|\;+\;]$

$-3 \quad -2 \quad 1 \quad 3$

B

18. Given L feet of fencing, what is the maximum number of square feet that can be enclosed if the fencing is used to make three sides of a rectangular pen, using an existing wall as the fourth side?

(A) $\frac{L^2}{4}$ (B) $\frac{L^2}{8}$ (C) $\frac{L^2}{9}$ (D) $\frac{L^2}{16}$ (E) $\frac{2L^2}{9}$

$L=2x+y$

$y=L-2x$

if $x=\frac{L}{4}$

$y=L-\frac{2L}{4}$

$A=xy$

$A=x(L-2x)$

$A=Lx-2x^2$

$\frac{dA}{dx}=L-4x$

$0=L-4x$

$-L=-4x$

$x=\frac{L}{4}$

max

$+\;|\;-$

$\frac{L}{4}$

$A=\left(\frac{L}{4}\right)\left(\frac{L}{2}\right)=\frac{L^2}{8}$

C

19. If $y=\ln(\sin x)$, then $\frac{dy}{dx}=$

(A) $\frac{1}{\sin x}$ (B) $\ln(\sin x)$ (C) $\cot x$ (D) $\frac{1}{x}\sin x+\ln(\cos x)$

(E) $(\cos x)\ln(\sin x)$

$\frac{dy}{dx}=\frac{1}{\sin x}\cdot\cos x=\cot x$

A

20. A function f is defined by $f(x)=\begin{cases}\frac{x^2-4}{x-2} & \text{if } x\neq 2\\ k & \text{if } x=2\end{cases}$

If f is continuous at $x=2$, what is the value of k?

(A) 4 (B) -2 (C) 2 (D) 0 (E) $\frac{1}{2}$

$\frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}}$

$x+2=k$ if $x=2$

$2+2=k$

$k=4$

II. Free Response

21. Prove the following derivative formula:

Given: $y = e^x$

Prove: $\frac{dy}{dx} = e^x$

$\ln y = \ln e^x$

$\ln y = x$

$\frac{d}{dx} \ln y = 1$

$\frac{1}{y} \cdot \frac{dy}{dx} = 1$

$\frac{dy}{dx} = 1 \cdot \frac{y}{1}$

$= y$

$\frac{dy}{dx} = e^x$ ✓ :)

22. a. $\lim_{x\to 0} \frac{e^x - 1}{x(1+x)}$ $\frac{0}{0}$ $x + x^2$

L'Hos. $\frac{e^x}{1+2x} \to \frac{1}{1} = 1$

b. $\lim_{x\to 0} \frac{1-\cos x}{3x}$ $\frac{0}{0}$

L'Hos

$\frac{\sin x}{3} \to \frac{0}{3} \to 0$

c. $\lim_{x\to\infty} \frac{2x^4 - x^2 - 2}{3x^4 - 1}$

$\frac{2}{3}$

d. $\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}}$ $\frac{\infty}{\infty}$

L'Hos. $\frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}}$ $\frac{1}{x} \cdot \frac{2\sqrt{x}}{1} \to \frac{2\sqrt{x}}{x}$ or $\frac{2}{\sqrt{x}}$ bottom heavy

$\to 0$

e. $\lim_{x\to \frac{\pi}{2}} \frac{\tan 5x}{\tan x}$ $\frac{\frac{\sin 5x}{\cos 5x}}{\cancel{\frac{\sin x}{\cos x}}}$ $\frac{\sin 5x}{\cos 5x} \cdot \frac{\cos x}{\sin x}$ $\to 0$ so $\frac{0}{0}$

L'H $\frac{5\cos 5x \cos x + \sin 5x(-\sin x)}{-5\sin 5x \sin x + \cos 5x \cos x}$

$\frac{-1}{-5} = \frac{1}{5}$

f. $\lim_{x\to 0} \frac{2x}{\cos^2 x - 1}$ $\frac{0}{0}$ L'H

$\frac{2}{-2\cos x \sin x}$

$= \frac{-1}{\cos x \sin x}$

$= \frac{-1}{0}$ DNE

g. $\lim_{x\to 1} \frac{\sqrt{x} - \sqrt[4]{x}}{x-1}$ $\frac{0}{0}$

L'H $\frac{\frac{1}{2\sqrt{x}} - \frac{1}{4\sqrt[4]{x^3}}}{1} \to \frac{1}{2} - \frac{1}{4}$

$= \frac{1}{4}$

h. $\lim_{x\to 0} \frac{\tan \pi x}{e^x - 1} \to \frac{0}{0}$

L'H

$\frac{\pi \sec^2(\pi x)}{e^x}$

$\to \frac{\pi}{1} = \pi$

i. $\lim_{x\to\infty} \ln\left(\frac{x^2-1}{x^2+1}\right)^3$

$3 \ln\left(\frac{x^2-1}{x^2+1}\right)$

same coeff. so $\to \frac{1}{1} = 1$

$3 \ln 1 \to 3 \cdot 0 = 0$

23. Write out the complete definition for continuity.

$\lim_{x\to a^-} = \lim_{x\to a^+} = \lim_{x\to a} = f(a)$ $\therefore$ $f(x)$ is continuous at $x = a$

therefore

$-\frac{x}{y}$ 24. Determine $\frac{dy}{dx}$, given that $x^2 + y^2 = 16$

$2x + 2y\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} = -2x$ $\quad \frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$

$\frac{-2x-y}{x}$ 25. Determine $\frac{dy}{dx}$, given that $xy + x^2 = 1607$

$x\frac{dy}{dx} + y + 2x = 0$

$x\frac{dy}{dx} = -2x - y$

$\frac{dy}{dx} = \frac{-2x-y}{x}$

$80x(4x^2-5)^9$ 26. Determine $\frac{dy}{dx}$, given that $y = (4x^2 - 5)^{10}$

$\frac{dy}{dx} = 10(4x^2-5)^9(8x)$

________ 27. Determine $\frac{dy}{dx}$, given that $y = \frac{x^2}{\cos(x)}$

$\frac{dy}{dx} = \frac{(\cos x)(2x) - (x^2)(-\sin x)}{(\cos x)^2}$

________ 28. Determine $\frac{dy}{dx}$, given that $x = 3t + 1$ and $y = t^2 + t$

$\frac{dx}{dt} = 3 \quad \frac{dy}{dt} = 2t + 1$

$\frac{\frac{dy}{dt} \cdot \frac{dt}{dx}}{\frac{\cancel{dx}}{\cancel{dt}}}$

$2t + 1 \cdot \left(\frac{1}{3}\right) = \frac{2t+1}{3}$

$x \geq 0$

$x < -4$ 29. Determine the domain of $y = \sqrt{\dfrac{x}{x+4}}$

$\dfrac{x}{x+4} > 0$

$x \to 0$

$x \to -4$

$\frac{-}{-} = +$ ✓ $\frac{-}{+} = -$ X $\frac{+}{+} = +$ ✓

-4 0

______ 30. Determine the equation of the tangent to the curve $\sin(y) = \cos(x)$ at the point $\left(\dfrac{\pi}{2}, 0\right)$.

$\cos y \cdot \dfrac{dy}{dx} = -\sin x$

$\cos 0 \cdot \dfrac{dy}{dx} = -\sin \dfrac{\pi}{2}$

$\boxed{y - 0 = -1\left(x - \frac{\pi}{2}\right)}$

$1 \cdot \dfrac{dy}{dx} = -1$ $\dfrac{dy}{dx} = -1$

1,024 ft 31. A particle projected vertically upward with an initial velocity of 256 ft/sec reaches an elevation $s = 256t - 16t^2$ at the end of t seconds. How high does the particle rise?

$s' = 256 - 32t$

$0 = 256 - 32t$

$32t = 256$

$t = 8$

$s(8) = 256(8) - 16(8)^2$

9 32. For $x \neq 4$, the function $h(x) = \dfrac{x^2 + x - 20}{x - 4}$. What value should be assigned to $h(4)$ to make $h(x)$ continuous at $x = 4$?

$\dfrac{\cancel{(x-4)}(x+5)}{\cancel{x-4}}$ $4 + 5 = 9$

$\frac{d}{dx} f(f(x)) = f'(f(x)) \cdot f'(x)$
↳ not needed this time though

± 3 ______ 33. Determine the value of x if $f(x) = x^2$ and $f'(f(x)) + f(f'(x)) = 54$.

$f'(x) = 2x$

$f'(x^2) + f(2x) = 54$

$2(x^2) + (2x)^2 = 54$

$2x^2 + 4x^2 = 54$

$6x^2 = 54$

$x^2 = 9$

$x = \pm 3$

$-(2^{50})\sin 2x$ ______ 34. If $y = \sin(2x)$, determine the 50^{th} derivative of y with respect to x.

$y' = 2\cos 2x$

$y'' = -4\sin 2x$

$y''' = -8\cos 2x$

$y'''' = 16\sin 2x$

$\frac{50}{4} = 12 \text{ R}2$

a = -1 ______
b = 2 ______
c = 5 ______

35. The curve $y = ax^2 + bx + c$ passes through the points $(2, 5)$ and $(-2, -3)$. The value of y is greatest when $x = 1$. Determine the values of a, b, and c.

$y' = 2ax + b$

$0 = 2ax + b$

$0 = 2a + b$

$0 = 2a + 2$

$a = -1$

$5 = 4a + 2b + c$

$-3 = 4a - 2b + c$

$c = 5 - 4a - 2b$

$c = -3 - 4a + 2b$

$5 - 4a - 2b = -3 - 4a + 2b$

$5 + 3 = 4b$

$8 = 4b$ $\quad b = 2$

$5 = -4 + 4 + c$

$c = 5$

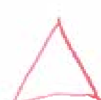
2.089 ______ 36. Solve for y *(to three decimal places)*: $8^y = 77$

$y \ln 8 = \ln 77$

$y = \frac{\ln 77}{\ln 8}$

$\frac{dh}{dt} = \frac{3}{8\pi}$ ft/min ______ 37. Grain pouring from a chute at the rate of $6\ ft^3/\text{min}$ forms a conical pile whose altitude is always twice its radius. How fast is the altitude of the pile changing when the radius is 4 feet?

$\frac{dr}{dt} = \frac{3}{16\pi}$ ft/min



$h = 2r$

$\frac{dh}{dt} = 2\frac{dr}{dt}$

$\frac{dV}{dt} = 6 ft^3/min$

$V = \frac{1}{3}\pi r^2 h$ $\quad V = \frac{1}{3}\pi r^2(2r)$

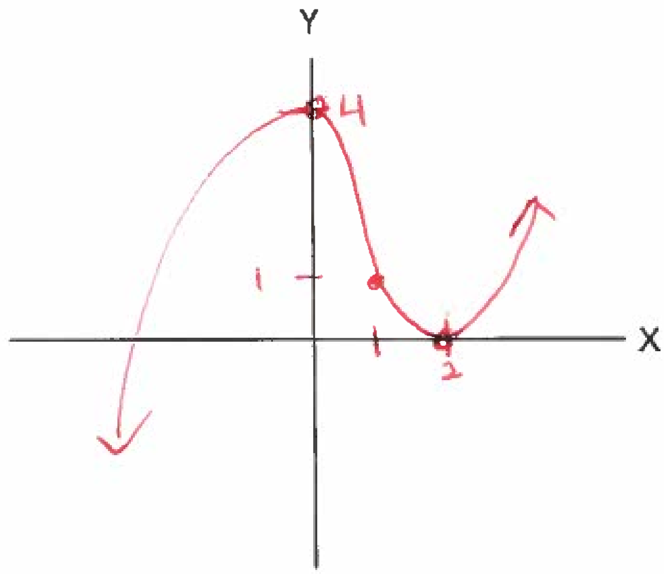
$V = \frac{2}{3}\pi r^3$ $\quad \frac{dV}{dt} = 2\pi r^2 \frac{dr}{dt}$ $\quad 6 = 2\pi(4)^2 \frac{dr}{dt}$

38. Sketch a curve which satisfies the following conditions:

$\frac{dy}{dx} > 0$ on $(-\infty, 0)$ and $(2, +\infty)$ inc inc $\qquad \frac{dy}{dx} < 0$ on $(0, 2)$ dec

$\frac{d^2y}{dx^2} > 0$ on $(1, +\infty)$ CCU $\qquad \frac{d^2y}{dx^2} < 0$ on $(-\infty, 1)$ CCD

$f(0) = 4 \qquad f(2) = 0 \qquad f(1) = 1$



$-3/4$ ______ 39. Given $f(1) = 2$, $f'(1) = 4$, and $g(x) = (f(x))^{-3}$

Determine $\frac{d}{dx}(g(x))|_{x=1}$

$\rightarrow -3(f(x))^{-4} \cdot f'(x)$

$\rightarrow -3(f(1))^{-4} \cdot f'(-1)$

$\rightarrow -3(2)^{-4} \cdot (4) \rightarrow \frac{-3 \cdot 4}{16} \rightarrow \frac{-3}{4}$

$\frac{80}{27}$ ______ 40. Use differentials to determine the value of $\sqrt[3]{26}$. $\sqrt[3]{27} = 3$

$\sqrt[3]{x}$

$f(x) = \sqrt[3]{x} \qquad f'(x) = \frac{1}{3}x^{-2/3}$

$f(27) = 3 \qquad f'(27) = \frac{1}{27} \qquad y - 3 = \frac{1}{27}(x - 27) \qquad y = \frac{1}{27}(26 - 27) + 3$

$-\frac{1}{27} + \frac{81}{27} = 80/27$