AP Calculus Fall 2017 Final Exam Review Part 2 Name Key

NO Calculator

I. Multiple Choice

A 1. If $f(x) = e^x$, which of the following is an asymptote to the graph of f?

(A) $y = 0$ (B) $x = 0$ (C) $y = -x$ (D) $y = 1$ (E) $y = x$

D 2. If $\log_a(2^a) = \frac{a}{4}$, then $a =$

(A) 2 (B) 4 (C) 8 (D) 16 (E) 32

$a \log_a 2 = \frac{a}{4}$ so $\log_a 2 = \frac{1}{4}$

$(a^{\frac{1}{4}} = 2)^4$

$a = 16$

E 3. The area of a circle is given by $A = \pi r^2$. Assuming that the radius is changing, the formula for the instantaneous rate of change of A with respect to r is:

(A) π (B) 0 (C) 2π (D) πr^2 (E) $2\pi r$

$\frac{dA}{dr} = 2\pi r$

E 4. If $f(x) = 3x^3 - 7x + 9$, then $f''(x) = \frac{d^2y}{dx^2} =$

(A) $9x - 7$ (B) $27x^2 - 7$ (C) $9x^2 - 7$ (D) $9x^2$ (E) $18x$

$9x^2 - 7$

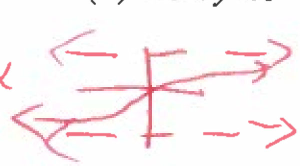
$18x$

B 5. Let $f(x) = \cos(Tan^{-1}x)$. What is the range of $f(x)$?

(A) $\frac{-\pi}{2} < y < \frac{\pi}{2}$ (B) $0 < y \le 1$ → never actually gets to $\pm\frac{\pi}{2}$ (C) $0 \le y \le 1$

(D) $-1 \le y \le 1$ (E) $-1 < y < 1$

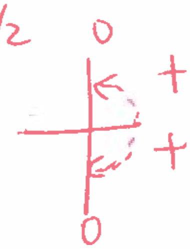
$\tan^{-1} x$ ↳



$\pi/2$

$-\pi/2$

$\cos(-\frac{\pi}{2} \leftrightarrow \frac{\pi}{2}) \rightarrow$



B 6. Evaluate $\lim_{h \to 0} \dfrac{\cos\left(\frac{\pi}{2}+h\right) - \cos\left(\frac{\pi}{2}\right)}{h}$

$\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

(A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) ∞

$f(x) = \cos x \quad f'(x) = -\sin x$

E 7. $\log\left(\dfrac{x^2}{3y}\right)$ is equivalent to

$x = \frac{\pi}{2} \quad f'(\frac{\pi}{2}) = -\sin\frac{\pi}{2} = -1$

(A) $\log(2x) - \log(3y)$ (B) $2\log(x) - 3\log(y)$ (C) $2\log(x) - \log(3) + \log(y)$
(D) $\log(x) + \log(2) - \log(3y)$ (E) $2\log(x) - \log(3) - \log(y)$

$\log x^2 - \log 3y \rightarrow 2\log x - \log 3 - \log y$

C 8. Evaluate $\log_3\left(\dfrac{1}{27}\right) = x$ $\quad 3^x = \frac{1}{27} \quad 3^x = 3^{-3}$

(A) $\dfrac{-1}{3}$ (B) $\dfrac{1}{3}$ (C) -3 (D) 3 (E) 9 $\quad x = -3$

E 9. The set of all points (e^t, t) where t is a real number is the graph of:

(A) $y = \dfrac{1}{e^x}$ (B) $y = (e)^{\frac{1}{x}}$ (C) $y = x(e)^{\frac{1}{x}}$ (D) $y = \dfrac{1}{\ln x}$ (E) $y = \ln x$

$t = \ln e^t$

$y = \ln x$

D 10. If $y = -x^2 + 4x + 25$, What is the maximum value for y?

(A) 25 (B) -16 (C) 28 (D) 29 (E) 18

$\dfrac{dy}{dx} = -2x + 4 \quad 0 = -2x + 4$

$x = 2$

max

$+ \quad -$

2

$y = -(2)^2 + 4(2) + 25 = -4 + 8 + 25$

A 11. Which of the following is a point of discontinuity for $f(x)=\dfrac{x^2-4}{x^2+2x-3}$?

(A) -3 (B) 2 (C) 0 (D) -1 (E) -2

$(x+3)(x-1)$

A 12. $\lim_{x\to 0}\left(\left(\frac{\sin x}{x}\right)\left(\frac{x+1}{x-1}\right)\right)=$

$\lim_{x\to 0}\frac{\sin x}{x}\Rightarrow 1$ then $\frac{0+1}{0-1}\to -1$

(A) -1 (B) 1 (C) 0 (D) π (E) $+\infty$

$1\cdot -1$
$=-1$

C 13. The graph of $y=2x^3+5x^2-6x+7$ has a point of inflection at x =

(A) 0 (B) $\frac{-3}{5}$ (C) $\frac{-5}{6}$ (D) $\frac{2}{5}$ (E) None of these

$y'=6x^2+10x-6$

$y''=12x+10$

$x=\frac{-10}{12}=\frac{-5}{6}$

$-$ | $+$

$-5/6$

D 14. Determine the exact value of $Sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\frac{\pi}{3}$ radians or $60°$

(A) 0 (B) $\frac{\pi}{6}$ radians (C) $\frac{1}{2}$ (D) 60 degrees (E) None of these

C 15. If $f(x)=\ln x$, then the inverse function $f^{-1}(x)=$

(A) $\frac{1}{x}$ (B) $\frac{1}{\ln x}$ (C) e^x (D) e^{-x} (E) x

$y=\ln x$

$x=\ln y$

$e^x=e^{\ln y}$

$e^x=y$

Inc CCD

B 16. If $f'(x) > 0$ and $f''(x) < 0$ over the same interval, which of the following statements is true for $f(x)$ over that interval?

(A) $f(x)$ is increasing and concave up

(B) $f(x)$ is increasing and concave down

(C) $f(x)$ is decreasing and concave up

(D) $f(x)$ is decreasing and concave down

(E) None of the statements are true

A 17. Given a function *f*, *how many* of the following statements are true?

(i) If $f''(a) < 0$, then the graph of $y = f(x)$ is concave upward at $x = a$.

(ii) If $f'(a) < 0$, then the graph of $y = f(x)$ is concave downward at $x = a$.

C.N. (iii) If $f'(a) = 0$ and $f''(a) > 0$ (ccu), then $f(a)$ is a relative maximum.

(iv) If $f'(a) = 0$ and $f''(a) = 0$, then $f'''(a) = 0$.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

C 18. A square piece of tin has 10 inches on a side. An open box is formed by cutting out equal square pieces from the four corners and then bending up the sides. Determine the length of the side of the squares that will result in the maximum volume of the box.

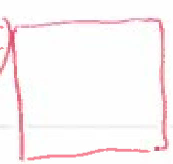
↳ cut

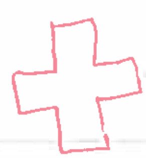
(A) 1 inch (B) $\frac{3}{5}$ inches (C) $\frac{5}{3}$ inches (D) 5 inches

(E) None of the above

(squares) cut $= \frac{5}{3}$

Side of tin $= 10 - 2(\frac{5}{3}) = \frac{30}{3} - \frac{10}{3} = \frac{20}{3}$





$V = (10-2x)(10-2x)(x)$

$V = (100 - 40x + 4x^2)(x)$

$V = 100x - 40x^2 + 4x^3$

$\frac{dV}{dx} = 100 - 80x + 12x^2$

$\frac{dV}{dx} = 4(25 - 20x + 3x^2)$

$0 = 4(3x-5)(x-5)$

+ max − min +

$\frac{5}{3}$ 5

E 19. If $f(x) = x^3 - 3x^2 - 2x + 5$ and $g(x) = 2$

Then $g(f(x)) =$

(A) $2x^3 - 6x^2 - 4x + 10$ (B) $2x^2 - 6x + 1$

(C) -6 (D) -3 (E) 2

$g(f(x)) = 2$

C 20. Which of the following statements is true?

(A) $\log(A - B) = \log\left(\frac{A}{B}\right)$

(B) $\log\left(\frac{A}{B}\right) = \frac{\log A}{\log B}$

(C) $\log\left(\frac{A}{B}\right) = \log(A) - \log(B)$

(D) $\log(A - B) = \log(A) - \log(B)$

II. Free Response

21. Prove the following derivative formula:

Given: $y = \sec(x)$

Prove: $\frac{dy}{dx} =$ sec ~~x~~ tan x

$y = \frac{1}{\cos x}$

$y = (\cos x)^{-1}$

$\frac{dy}{dx} = -1(\cos x)^{-2}(-\sin x)$

$= \frac{+\sin x}{\cos^2 x} \rightarrow \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} \rightarrow \sec x \tan x$ ✓

22. Write out the complete definition of the derivative.

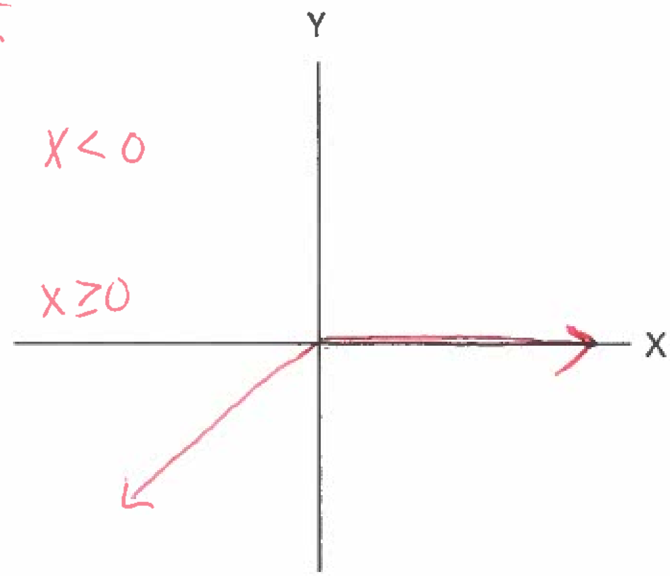
$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ or $\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$;

Slope of the tangent line ; Instantaneous rate of change

23. Graph the function $y = \frac{x - |x|}{2}$ on the axes below:

$y = \frac{x}{2} - \frac{|x|}{2}$

$y = \begin{cases} x & x < 0 \\ 0 & x \geq 0 \end{cases}$



$\frac{x}{2} - \frac{x}{2} = 0$

$\frac{x}{2} - \frac{-x}{2} = \frac{2x}{2} = x$

24. Find the length and width of a rectangle that has an area of 72 square feet and whose perimeter is a minimum.

a. 36 ft x 2 ft **b.** 18 ft x 4 ft

c. 9 ft x 8 ft **d.** $36\sqrt{2}$ x $\sqrt{2}$

e. $6\sqrt{2}$ x $6\sqrt{2}$

$A = 72 = lw$ $\quad w = \frac{72}{L}$

$P = 2L + 2w$

$P = 2L + \frac{2 \cdot 72}{L} \rightarrow 144L^{-1}$

$\frac{dP}{dL} = 2 + \frac{-144}{L^2}$

$0 = \frac{2L^2 - 144}{L^2}$ $\quad L^2 \rightarrow 0$ not significant here

$2L^2 - 144 = 0$

$L^2 = 72$

$L = \pm\sqrt{72}$ ($+$ → not important)

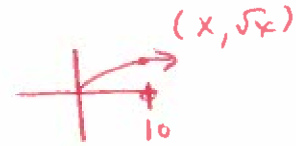
min

$-$ $+$

0 $\quad \sqrt{72}$

$L = \sqrt{72}$

$w = \frac{72}{\sqrt{72}} = \sqrt{72}$



b. Find the point on the graph of the function $f(x) = \sqrt{x}$ that is closest to the point $(10, 0)$.

a. $\left(\frac{19}{2}, \sqrt{\frac{21}{2}}\right)$ b. $\left(\frac{19}{2}, \sqrt{\frac{19}{2}}\right)$

c. $\left(\frac{21}{2}, \sqrt{\frac{21}{2}}\right)$ d. $\left(\sqrt{\frac{19}{2}}, \frac{19}{2}\right)$

e. $\left(\sqrt{\frac{21}{2}}, \frac{19}{2}\right)$

$d = \sqrt{(x-10)^2 + (\sqrt{x} - 0)^2}$

$d = \sqrt{x^2 - 20x + 100 + x}$

$\frac{dd}{dx} = \frac{1}{2\sqrt{x^2 - 19x + 100}} \cdot (2x - 19)$

↳ $b^2 - 4ac < 0$

$0 = 2x - 19$

$x = \frac{19}{2}$

$-$ $+$ $\frac{19}{2}$

25. Given $f(x) = x^3 + 2x + 1$, find $\left(f^{-1}\right)'(1)$.

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$

(C) 0 (D) 4

(E) None of these

↓ input of inverse = output of function

$1 = x^3 + 2x + 1$

$0 = x^3 + 2x$

$0 = x(x^2 + 2)$

↓ ↳ never 0

$x = 0$

$(f^{-1})'(1) = \frac{1}{f'(0)}$

$f' = 3x^2 + 2$

$f'(0) = 2$

-2 26. Evaluate the limit: $\lim_{x \to \infty} \left(\frac{2x^2}{2006 - x^2} \right)$

2 27. Evaluate the limit: $\lim_{x \to 3} \frac{2x^2 + x - 9}{x + 3}$

$\frac{2(3)^2 + 3 - 9}{3 + 3} = \frac{18 - 6}{6} = 3 - 1 = 2$

$\frac{1}{3}$ 28. Evaluate the limit: $\lim_{x \to 0} \frac{\sin(x)}{3x}$

$\frac{\sin(0)}{3 \cdot 0} \to \frac{0}{0}$

L'H. $\lim_{x \to 0} \frac{\cos x}{3} \to \frac{1}{3}$

8 ______ 29. Evaluate the limit: $\lim_{x\to 4} \frac{x^2-16}{x-4}$

$\frac{(\cancel{x-4})(x+4)}{\cancel{x-4}}$ $4+4 = 8$

$-2\sin 2x$ ______ 30. Evaluate the limit: $\lim_{h\to 0} \frac{\cos(2(x+h))-\cos 2x}{h}$

$f(x)=\cos(2x)$ $f'(x)=-2\sin(2x)$

$f(x+h)=\cos(2(x+h))$

______ 31. Determine the differential dy, given that $y=x^4-5x^2+2006$.

$\frac{dy}{dx}=4x^3-10x$

$dy=(4x^3-10x)dx$ ✓

7 ______ 32. Determine the derivative of $\ln(e^{7x})$.

$\frac{1}{e^{7x}}\cdot e^{7x}\cdot 7 = 7$

$y=5^x$ ______ 33. Determine the inverse function of $y=\log_5 x$.

$x=\log_5 y$ $5^x=y$

______ 34. Determine $\frac{d}{dx}(x^{\cos x})$ $y=x^{\cos x}$

$\ln y = \ln x^{\cos x}$

$\ln y = \cos x \ln x$

$\frac{1}{y}\cdot\frac{dy}{dx} = \cos x\cdot\frac{1}{x} - \sin x \ln x$

$\frac{dy}{dx} = \left(\frac{\cos x}{x} - \sin x\cdot\ln x\right)(x^{\cos x})$ ✓

$3x(x-2)$

$+ \quad - \quad +$ (0, 2)

$6(x-1)$

$- \quad +$ (1)

35-36. Given the equation $y = x^3 - 3x^2 + 4$, determine the following:

First Derivative: $y' = 3x^2 - 6x$

Increasing on: $(-\infty, 0)$ $(2, \infty)$

Decreasing on: $(0, 2)$

Relative Maximum at: $(0, 4)$

Relative Minimum at: $(2, 0)$

Second Derivative: $y'' = 6x - 6$

Concave Up on: $(1, \infty)$

Concave Down on: $(-\infty, 1)$

Point of Inflection at: $(1, 2)$

$x^2(x^2-4) \rightarrow x^2(x-2)(x+2) \rightarrow 0, 2, -2$

$f'(x) = 4x^3 - 8x$

$0 = 4x(x^2 - 2)$

$0 \qquad \pm\sqrt{2}$

$- \quad + \quad - \quad +$ ($-\sqrt{2}$, 0, $\sqrt{2}$)

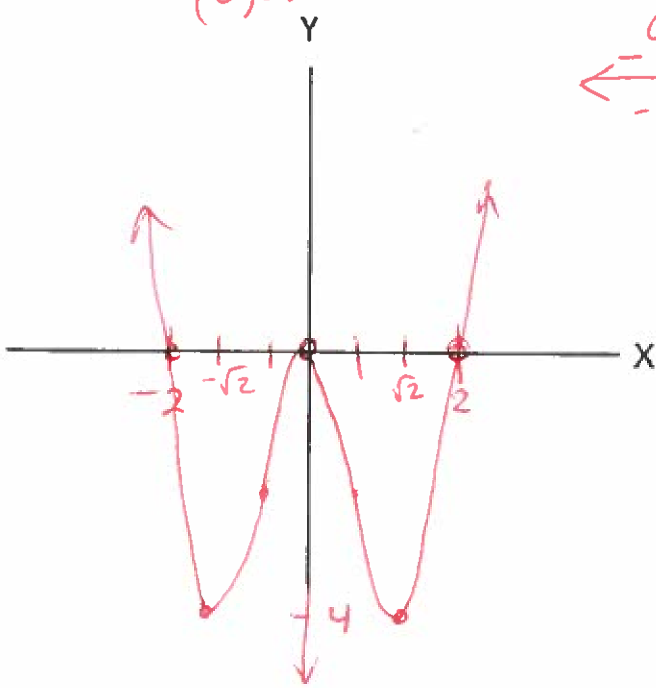
37. Sketch the graph of the curve $f(x) = x^4 - 4x^2$

$(0,0)$

$\left(\sqrt{\frac{2}{3}}\right)^4 - 4\left(\sqrt{\frac{2}{3}}\right)^2$

$\frac{4}{9} - 4\left(\frac{2}{3}\right)$

$\frac{4}{9} - \frac{24}{9} = \frac{-20}{9}$



$f(\sqrt{2}) = 4 - 4(2)$
$= -4$

$f''(x) = 12x^2 - 8$

$0 = 4(3x^2 - 2)$

$x^2 = \frac{2}{3}$

$x = \pm\sqrt{\frac{2}{3}}$

$+ \quad - \quad +$ ($-\sqrt{\frac{2}{3}}$, $\sqrt{\frac{2}{3}}$)

P.O.I.

$y = 6 - x^2 \qquad x = 6 - y^2$

38. Find the inverse of $f(x) = 6 - x^2$, $x \geq 0$ and specify its domain.

(A) $f^{-1}(x) = 6 - y^2$; domain $[0,6]$ (B) $f^{-1}(x) = \pm\sqrt{6 - x^2}$; domain $(-\infty, 6]$

(C) $f^{-1}(x) = \sqrt{6 - x}$; domain $(-\infty, 6]$ (D) $f^{-1}(x) = \sqrt{6 - x^2}$; domain $(-\infty, 6]$

(E) $f^{-1}(x) = \sqrt{6 - x}$; domain $(-\infty - 6) \cup (-6, 6)$

$y^2 = 6 - x \qquad y = \sqrt{6 - x}$

39. If $f(x) = \ln x$ and $g(x)$ is the inverse of $f(x)$, find $g'(1)$. input of inverse output of function

(A) 1 (B) 0

(C) $\frac{1}{e}$ (D) e

(E) does not exist

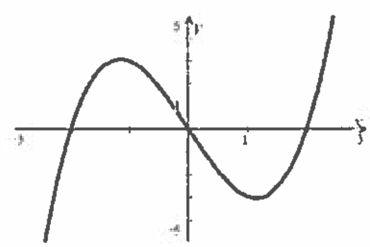
$1 = \ln x \qquad f'(x) = \frac{1}{x}$

$e^1 = x$

$f'(e) = \frac{1}{e}$

$g'(1) = \frac{1}{f'(e)} = \frac{1}{1/e} = e$

40. The graph of a twice-differentiable function f is shown in the figure to the right. Arrange these expressions from largest to smallest.



$0 + 0 = 0 \qquad +(+) + = +$

I. $f(-2) + f(2)$ II. $f'(-2) + f'(2)$ III. $f''(-2) - f''(2)$

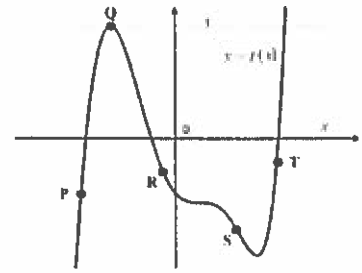
$-(-) + = -$

(A) I, II, III (B) I, III, II

(C) II, III, I (D) II, I, III

(E) III, II, I

41. The graph of a function $y = f(x)$ is given to the right. $\frac{dy}{dx} < 0$ and $\frac{d^2y}{dx^2} = 0$ at only one point. Which is it?



(A) P

(B) Q

(C) R

(D) S

(E) T

P.O.I. and decreasing

42. A news report states that the national price of gas is decreasing but not as fast as it was last week. If P represents the price of gas at the time of the report, which of the following statements are true?

i. $P > 0$ ii. $P < 0$ iii. $\frac{dP}{dt} > 0$ iv. $\frac{dP}{dt} < 0$ v. $\frac{d^2P}{dt^2} > 0$ vi. $\frac{d^2P}{dt^2} < 0$

(A) i, iii, v

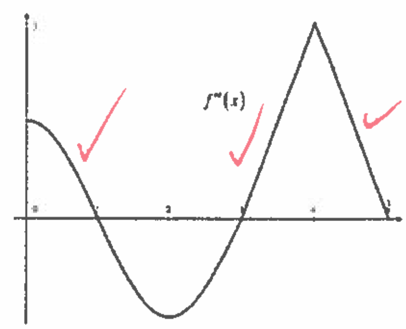
(B) i, iv, vi

(C) i, iii, vi

(D) i, iv, v

(E) ii, iii, v

43. The graph of $y = f''(x)$, the second derivative of f, on the open interval $(0,5)$, is shown to the right. On which of the following intervals is $f'(x)$ increasing?



A function incr. when it's deriv. is >0. so if $f''(x) > 0$, $f'(x)$ inc

(A) (3,4) only

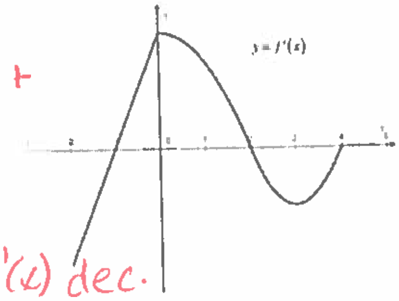
(B) (2,4) only

(C) (1,3) only

(D) (0,1) and (3,4)

(E) (0,1) and (3,5)

44. The graph of $y = f'(x)$ is shown in the figure to the right.
How many of the following statements are ***false***?



✓ i. f has a relative minimum at $x = -1$. $f'(x)$ $-$ to $+$
✓ ii. f is increasing on $(-1, 2)$ $(f'(x) > 0)$
✗ iii. f has a local maximum at $x = 0$. $f'(x)$. $+$ to $-$
✓ iv. f is differentiable at $x = 0$.
✓ v. f is concave down on $(2, 3)$ $f''(x) < 0$ so $f'(x)$ dec.

(A) None
(B) One
(C) Two
(D) Three
(E) Four

Evaluate each limit.

45) $\lim_{x \to 2} \dfrac{\sqrt{x+2} - 2}{x - 2} \cdot \dfrac{\sqrt{x+2} + 2}{\sqrt{x+2} + 2}$

$\dfrac{x+2-4}{(\cancel{x-2})(\sqrt{x+2}+2)} \to \cancel{x-2} \to \dfrac{1}{\sqrt{2+2}+2} \to \frac{1}{4}$

46) $\lim_{x \to \infty} \dfrac{x}{\ln x}$ $\frac{\infty}{\infty}$ L'H $\to \dfrac{1}{1/x} \to x$

∞

47) $\lim_{x \to 0} \dfrac{x^2}{1 - \cos(x)}$

$\dfrac{0}{1-1} \Rightarrow \dfrac{0}{0}$ L'H $\to \dfrac{2x}{\sin x} \to \dfrac{0}{0} \to$ L'H

$\dfrac{2}{\cos x} \to \dfrac{2}{1} = 2$

48) $\lim_{x \to 0^+} x^2 \ln x$ $\boxed{x^{-2}}$

$\dfrac{\ln x}{\frac{1}{x^2}} \to \dfrac{-\infty}{\infty}$

L'H $\Rightarrow \dfrac{\frac{1}{x}}{\frac{-2}{x^3}} \to \dfrac{1}{x} \cdot \dfrac{x^3}{-2} \Rightarrow \dfrac{x^2}{-2} \to 0$

0

49) $\lim_{x \to \infty} \left(\dfrac{3x^2}{x - 1} - \dfrac{3x^2}{x + 1} \right)$

$\dfrac{3x^2(x+1) - 3x^2(x-1)}{(x-1)(x+1)}$

$3x^3 + 3x^2 - 3x^3 + 3x^2 \to$

$\dfrac{6x^2}{x^2 - 1} \to 6$

EBM

50) $\lim_{x \to 0} (x + 1)^{\frac{1}{x}}$

$\frac{1}{x} \ln(x+1)$

$\to \dfrac{\ln(x+1)}{x} \to \dfrac{0}{0}$ L'H.

$\dfrac{\frac{1}{x+1}}{1} \to \frac{1}{1} \to 1$ undo ln

e^1