Precalculus Name_____

Trig Review Notes

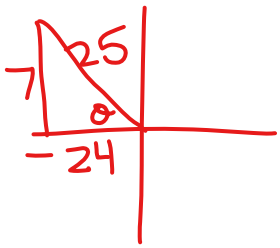
Date_____ Period____

Find the exact values of the five trigonometric ratios not given.

1) $\cot\theta = -\frac{24}{7}$ and $\sin\theta > 0$

II | I
III | IV

$\tan\theta = \frac{-7}{24}$



$\sin\theta = \frac{7}{25}$

$\csc\theta = \frac{25}{7}$

$\cos\theta = \frac{-24}{25}$

$\sec\theta = \frac{-25}{24}$

2) Two straight roads diverge at an angle of 78°. Two cars leave the intersection at 2:38 P.M., one traveling at 36 mi/hr and the other at 72 mi/hr. How far apart are the cars at 4:26 P.M.?

$\frac{D}{T} = R$

$D = RT$

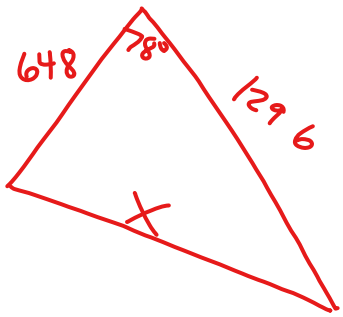
$T = \frac{D}{R}$

36 mi/hr: 4 × 36 = 144; 18 × 288; 36 × 648

72 mi/hr: 1 × 72; 18 × 576; 72 × 1296

22 min + 60 min + 26 min = 108 min

60)108 → 60, 480, 480, 0 → 18



$x^2 = (648)^2 + (1296)^2 - 2(1296)(648)\cos 78°$

$x = 132.3$ miles

Find the exact value of each expression.

3) $\cos^{-1}\left(-\frac{1}{2}\right)$ Q2 for sure

$\frac{2\pi}{3}$

4) $\cos^{-1}\frac{\sqrt{3}}{2}$ Q1

$\frac{\pi}{6}$

5) $\cot^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

$\frac{2\pi}{3}$

6) $\cot^{-1} 0$

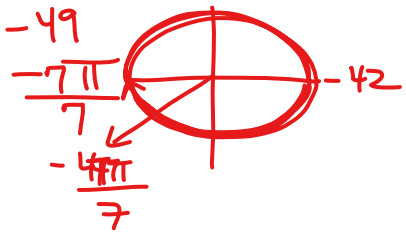
$\frac{\pi}{2}$

7) $\sec^{-1}\left(\csc -\frac{\pi}{4}\right)$

$\sec^{-1}\left(\frac{2}{-\sqrt{2}} \text{ or } -\sqrt{2}\right)$

then $\cos^{-1}\left(\frac{-\sqrt{2}}{2}\right) = \frac{3\pi}{4}$

8) $\cos^{-1}\left(\cos -\frac{47\pi}{7}\right) = \frac{5\pi}{7}$

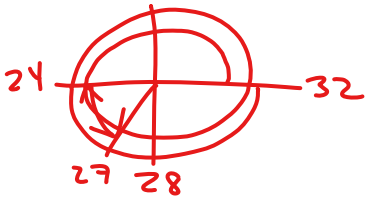


9) $\tan^{-1}\left(\tan -\frac{80\pi}{9}\right)$

$\frac{\pi}{9}$

10) $\sin^{-1}\left(\sin \frac{27\pi}{8}\right)$

$-\frac{3\pi}{8}$

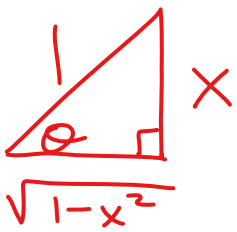


ref $< = \frac{3\pi}{8}$

Write each trigonometric expression as an algebraic expression.

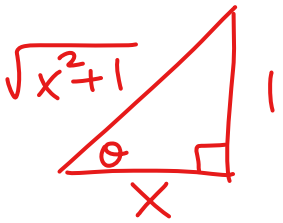
11) $\tan \sin^{-1} x$

$\frac{x}{\sqrt{1-x^2}}$



12) $\sin \cot^{-1} x$

$\frac{1}{\sqrt{x^2+1}}$

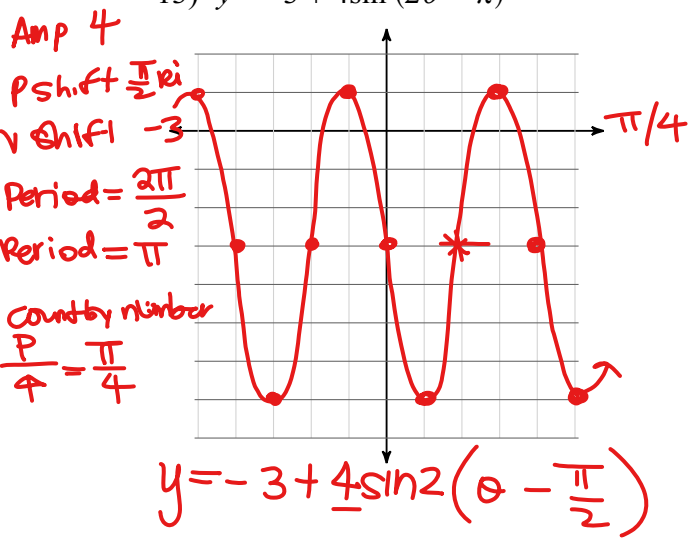


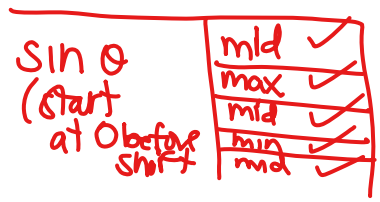
$\sin\theta = \sin^{-1} x$

$\sin\theta = x$

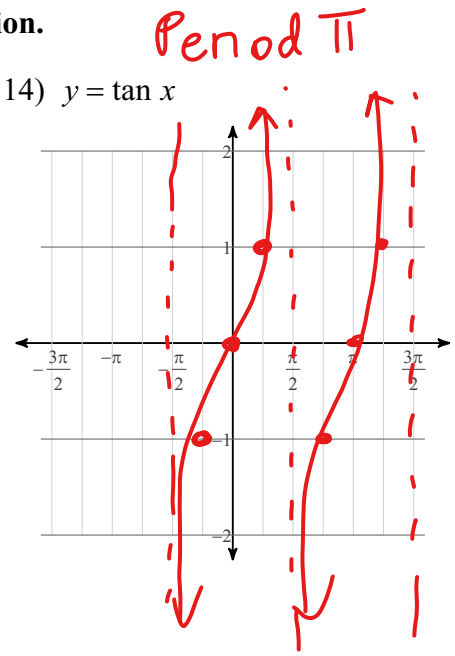
Graph the function. State the period for each function.

13) $y = -3 + 4\sin(2\theta - \pi)$





14) $y = \tan x$



Find the exact value of each trigonometric function.

15) $\sec \frac{\pi}{3}$ 2

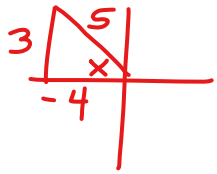
16) $\sin -\frac{17\pi}{6}$ $-\frac{1}{2}$

17) $\csc -\frac{28\pi}{3}$ $\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

18) $\cot \frac{17\pi}{3}$ $-\frac{\sqrt{3}}{3}$

Assuming $\csc x = \frac{5}{3}$ **if** $\frac{\pi}{2} < x < \pi$ **and** $\cot y = \frac{5}{12}$ **if** $\pi < y < \frac{3\pi}{2}$

19) $\sin(x + y)$

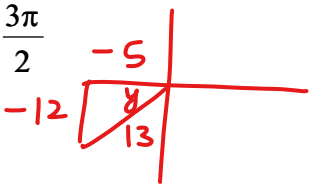


$\sin x \cos y + \sin y \cdot \cos x$

$\left(\frac{3}{5}\right)\left(\frac{-5}{13}\right) + \left(\frac{-12}{13}\right)\left(-\frac{4}{5}\right)$

$\frac{-15}{65} + \frac{48}{65} = \frac{33}{65}$

20) $\tan(x - y)$



$\frac{\tan x - \tan y}{1 + \tan x \tan y}$

$\frac{\frac{5}{5}\cdot\frac{-3}{4} - \frac{12}{5}\cdot\frac{4}{4}}{\frac{20}{20} 1 + \left(\frac{-3}{4}\right)\left(\frac{12}{5}\right)} = \frac{-15 - 48}{20 - 36}$

$= \frac{-63}{-16} = \frac{63}{16}$

21) $\sec(x - y)$

*pretend I asked for cos

$\cos(x - y) = \cos x \cos y + \sin x \sin y$

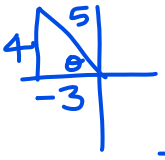
$= \left(-\frac{4}{5}\right)\left(\frac{-5}{13}\right) + \left(\frac{3}{5}\right)\left(-\frac{12}{13}\right)$

$= \frac{20}{65} - \frac{36}{65}$

$= -\frac{16}{65}$ → so . ans $\frac{65}{-16}$

Find the exact value of each. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

22) $\csc\theta = \frac{5}{4}$ where $90 \le \theta < 180$

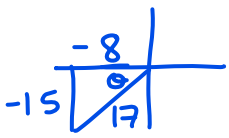


Find $\tan 2\theta = \frac{2\left(\frac{4}{3}\right)}{1-\left(\frac{-4}{3}\right)^2}$

$\frac{\frac{-8}{3}\ \frac{3}{3}}{\frac{9}{9}-\frac{16}{9}} \rightarrow \frac{+24}{+7}$

23) $\tan\theta = \frac{15}{8}$ where $180 \le \theta < 270$

Find $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= 1-2\sin^2\theta$
$= 2\cos^2\theta - 1$

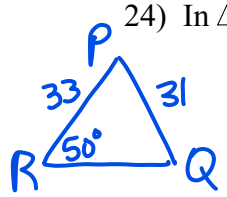


$= \left(\frac{-8}{17}\right)^2 - \left(\frac{-15}{17}\right)^2$

$= \frac{64}{289} - \frac{225}{289} = \frac{-161}{289}$

Solve each triangle. Round your answers to the nearest tenth.

24) In $\triangle RPQ$, $m\angle R = 50°$, $q = 33$ yd, $r = 31$ yd



$\frac{\sin 50°}{31} = \frac{\sin Q}{33}$

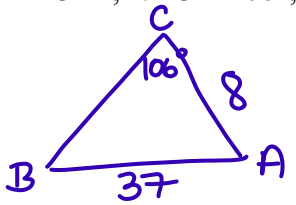
$\frac{33\sin 50°}{31} = \frac{31\sin Q}{31}$

$\sin^{-1}\left(\frac{33\sin 50°}{31}\right) = Q = 54\,63°$

Triangle 1	Triangle 2
$\angle Q = 54\,63°$	$\angle Q = 180 - 54\,63° = 125\,37°$
$\angle P = 75\,37°$	$\angle P = 4\,63°$
$p = 39.2$	$p = 3.2$

$\frac{\sin 50°}{31} = \frac{\sin 75\,37°}{p} \rightarrow \frac{p\sin 50°}{\sin 50°} = \frac{31\sin 75\,37°}{\sin 50°}$

25) In $\triangle CAB$, $m\angle C = 106°$, $b = 8$ mi, $c = 37$ mi



$\frac{\sin 106°}{37} = \frac{\sin B}{8}$

$\frac{8\sin 106°}{37} = \sin B$

$B = \sin^{-1}\left(\frac{8\sin 106°}{37}\right) = 12° = B$

$\frac{\sin 62°}{a} = \frac{\sin 106°}{37}$

$a = 34$

$\angle A = 62°$

Solve each equation for $0 \le \theta < 2\pi$. Round your answers to the nearest hundredth.

26) $-4 + 2\cos\left(3\theta + \frac{\pi}{4}\right) = -2.62$

$+4 \qquad +4$

$\frac{2\cos(3\theta + \frac{\pi}{4})}{2} = \frac{1.38}{2}$

$\cos(3\theta + \frac{\pi}{4}) = .69$

$\cos^{-1}(.69) = .8093$

$3\theta + \frac{\pi}{4} = .8093 + 2\pi n$

$3\theta + \frac{\pi}{4} = -.8093 + 2\pi n$

$\theta = \frac{(\pm .8093 - \frac{\pi}{4}) + 2\pi n}{3}$

$\theta = \{.008, .210, 1.197, 1.563, 3.657, 5.75\}$

Solve each equation for $0 \le \theta < 2\pi$.

27) $-\sqrt{3}\sin\theta = -\sin 2\theta$

$\sqrt{3}\sin\theta = 2\sin\theta\cos\theta$

$-\sqrt{3}\sin\theta \qquad -\sqrt{3}\sin\theta$

$0 = 2\sin\theta\cos\theta - \sqrt{3}\sin\theta$

$0 = \sin\theta(2\cos\theta - \sqrt{3})$

$\sin\theta = 0 \qquad \cos\theta = \frac{\sqrt{3}}{2}$

$\theta = \{0, \pi, \frac{\pi}{6}, \frac{11\pi}{6}\}$

28) $\cos 2\theta = -6\sin^2\theta + 3$

$\cos 2\theta + 6\sin^2\theta - 3 = 0$

$1 - 2\sin^2\theta + 6\sin^2\theta - 3 = 0$

$4\sin^2\theta - 2 = 0$

$\sqrt{\sin^2\theta} = \sqrt{\frac{1}{2}}$

$\sin\theta = \pm\frac{\sqrt{2}}{2}$

$\theta = \{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\}$

29) $2\sqrt{3}\cos\theta - 2\csc\theta = 3\cos\theta\csc\theta - 2\csc\theta$

$+2\csc\theta \qquad +2\csc\theta$

$2\sqrt{3}\cos\theta = 3\cos\theta\csc\theta$

$0 = 3\cos\theta\csc\theta - 2\sqrt{3}\cos\theta$

$0 = \cos\theta(3\csc\theta - 2\sqrt{3})$

$\cos\theta = 0 \qquad \csc\theta = \frac{2\sqrt{3}}{3}$ (so $\sin\theta = \frac{\sqrt{3}}{2}$)

$\theta = \{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{2\pi}{3}\}$

$\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta - 1 = -\cos^2\theta$

30) $0 = 2(-\cos^2\theta) + 3\sin\theta + \sin^2\theta$

$0 = 2[+\sin^2\theta - 1] + 3\sin\theta + \sin^2\theta$

$0 = 2\sin^2\theta + 3\sin\theta + 1$

$0 = (2\sin\theta + 1)(\sin\theta + 1)$

$\theta = \left\{ \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{3\pi}{2} \right\}$

$\sin\theta = -\frac{1}{2}$ $\quad \sin\theta = -1$

Find the area of each triangle to the nearest tenth.

31) In $\triangle ZXY$, $y = 9$ m, $x = 11.5$ m, $z = 15$ m

LOC

$15^2 = 9^2 + 11.5^2 - 2(9)(11.5)\cos Z$

$Z = 93.254°$

Per = 35.5

$S = 17.75$

Area $= \frac{1}{2}bh$

Area $= \frac{1}{2}ab\sin C$

Area $= \frac{1}{2}(9)(11.5)\sin$

$A = 51.667$

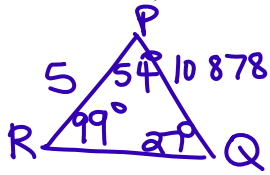
Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$\sqrt{17.75(17.75-9)(17.75-11.5)(17.75-15)}$

$\sqrt{17.75(8.75)(6.25)(2.75)}$

$A = 51.667$

32) In $\triangle RPQ$, $q = 5$ cm, $m\angle R = 99°$, $m\angle P = 54°$



$\frac{\sin 27°}{5} = \frac{\sin 99°}{r}$

$r = \frac{5\sin 99°}{\sin 27°}$

$r = 10.878$

54
+99
153

180
−153
27

$A = \frac{1}{2}(5)(10.878)\sin 54° = 22 \text{ cm}^2$

Find all solutions to each equation in radians.

33) $\frac{-2+\sqrt{2}}{2} - 2\cos\left(4\theta + \frac{3\pi}{2}\right) = -1 - 3\cos\left(4\theta + \frac{3\pi}{2}\right) + 3$

$-\frac{2}{2} + \frac{\sqrt{2}}{2} \rightarrow -1 + \frac{\sqrt{2}}{2}$

$\frac{\sqrt{2}}{2} + \cos\left(4\theta + \frac{3\pi}{2}\right) = 0$

$\cos\left(4\theta + \frac{3\pi}{2}\right) = -\frac{\sqrt{2}}{2}$

$4\theta + \frac{3\pi}{2} = \frac{3\pi}{4} + 2\pi n$ $\quad -\frac{3\pi}{2} \rightarrow -\frac{6\pi}{4}$

$\frac{4\theta}{4} = \frac{-3\pi}{4} + 2\pi n$ $\quad \div 4$

$4\theta + \frac{3\pi}{2} = \frac{5\pi}{4} + 2\pi n$ $\quad -\frac{3\pi}{2} \rightarrow -\frac{6\pi}{4}$

$\frac{4\theta}{4} = \frac{-\pi}{4} + 2\pi n$ $\quad \div 4$

$\theta = \left\{\frac{-3\pi}{16}, \frac{-\pi}{16}\right\} + \frac{\pi}{2}n$

Simplify each expression.

34) $\frac{\sin 2x}{\cos x} + \frac{\cos 2x}{\sin x}$

$\frac{2\sin x \cos x}{\sin x \cos x} + \frac{1 - 2\sin^2 x}{\sin x}$

$\frac{1}{\sin x} = \csc x$

35) $\frac{\cos x}{1 + \sin x} + \tan x$

$\frac{\cos x}{\cos x}\frac{\cos x}{(1+\sin x)} + \frac{\sin x}{\cos x}\frac{(1+\sin x)}{(1+\sin x)}$

$\frac{\cos^2 x + \sin x + \sin^2 x}{\cos x(1+\sin x)} = \frac{1+\sin x}{\cos x(1+\sin x)}$

$\sec x = \frac{1}{\cos x}$

Verify each identity. $(\cos 2x) = (1 - 2\sin^2 x)$

36) $\frac{1 - \cos 2x}{\cos^2 x} = 2\tan^2 x$

$\frac{1 - 1 + 2\sin^2 x}{\cos^2 x}$

$\frac{2\sin^2 x}{\cos^2 x} = 2\tan^2 x$

37) $\cos(R - 90) = \dfrac{1}{\csc R}$

$\cos R\cos 90 + \sin R\sin 90^\circ$

$\sin R = \dfrac{1}{\csc R}$ ✓

38) $\cos\left(D + \dfrac{\pi}{2}\right) = -\dfrac{1}{\csc D}$

$\cos D\cos\frac{\pi}{2} - \sin D\sin\frac{\pi}{2}$

$-\sin D = -\dfrac{1}{\csc D}$

39) $\cot x\tan x + \sec x = \dfrac{1 + \cos x}{\cos x}$

$1 + \sec x$

$\dfrac{\cos x}{\cos x} + \dfrac{1}{\cos x} = \dfrac{1+\cos x}{\cos x}$ ✓

40) $\dfrac{\cot x\sec x}{\sin x} = \cot^2 x + 1$

$\dfrac{\dfrac{\cancel{\cos x}}{\sin x}\dfrac{1}{\cancel{\cos x}}}{\dfrac{\sin x}{1}} \rightarrow \dfrac{1}{\sin^2 x} \rightarrow \csc^2 x = \cot^2 x + 1$

$\dfrac{\sin^2 x}{\sin^2 x} + \dfrac{\cos^2 x}{\sin^2 x} = \dfrac{1}{\sin^2 x}$

$1 + \cot^2 x = \csc^2 x$

41) Determine the period of the function

$y = -5\cot 7x - 18$

$\frac{\pi}{7}$

42) Determine the period of the function

$y = 18 + 11\cos\frac{x}{5}$

$\frac{2\pi}{1/5} = 10\pi$

43) For what value of k is the period of the function $y = -6\sin kx$ equal to 80°

$\frac{360°}{k} = 80°$

$360° = 80°k$

$\frac{360°}{80°} = k = \frac{36}{8} = \frac{9}{2} = k$

44) For what value of k is the period of the function $y = 8\tan kx$ equal to 260°

$\frac{180°}{k} = 260°$

$180° = 260°k$

$k = \frac{180°}{260°}$

$k = \frac{18}{26} = \frac{9}{13}$

45) Express as an inverse function the value of x for which $2\sqrt{1 - \cos x} - 1 = 0$

$(2\sqrt{1-\cos x} = 1)^2$

$4(1-\cos x) = 1$

$1 - \cos x = \frac{1}{4}$

$-\cos x = -\frac{3}{4}$

$\cos x = \frac{3}{4}$

$x = \cos^{-1}\left(\frac{3}{4}\right)$

Find the exact value of each.

46) $\tan 105°$

$\tan(60° + 45°)$ or $\tan\left(\frac{210°}{2}\right)$

$= \frac{\tan 60° + \tan 45°}{1 - \tan 60°\tan 45°}$

$= \frac{\sqrt{3} + 1}{1 - \sqrt{3}(1)} = \frac{\sqrt{3}+1}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}}$

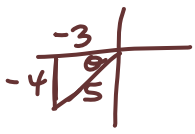
$= \frac{\sqrt{3}+3+1+\sqrt{3}}{1-3}$

$= \frac{2\sqrt{3}+4}{-2} = -\sqrt{3} - 2$

47) $\tan\theta = \frac{4}{3}$ where $540 \le \theta < 630$

Find $\sin\frac{\theta}{2}$

$\frac{-360 \quad -360}{180° \le \theta \le 270°}$



$\cos 2\theta = 1 - 2\sin^2\theta$

$\cos\theta = 1 - 2\sin^2\left(\frac{\theta}{2}\right)$

$\cos\theta - 1 = -2\sin^2\left(\frac{\theta}{2}\right)$

$\frac{1-\cos\theta}{2} = \sin^2\left(\frac{\theta}{2}\right)$

$\frac{1-\left(\frac{-3}{5}\right)}{2} \rightarrow \frac{\frac{5}{5}+\frac{3}{5}}{2} = \frac{\frac{8}{5}}{2} = \frac{8}{10}$

$= \frac{4}{5} = \sin^2\left(\frac{\theta}{2}\right) \rightarrow \sin\left(\frac{\theta}{2}\right) = \frac{2}{\sqrt{5}}$

Precalculus Name_____

Trig Review Notes

Date_____ Period____

Find the exact values of the five trigonometric ratios not given.

1) $\cot\theta = -\frac{24}{7}$ and $\sin\theta > 0$

$\sin\theta = \frac{7}{25}$, $\cos\theta = -\frac{24}{25}$, $\tan\theta = -\frac{7}{24}$

$\csc\theta = \frac{25}{7}$, $\sec\theta = -\frac{25}{24}$

2) Two straight roads diverge at an angle of 78°. Two cars leave the intersection at 2:38 P.M., one traveling at 36 mi/hr and the other at 72 mi/hr. How far apart are the cars at 4:26 P.M.?

132.3 mi

Find the exact value of each expression.

3) $\cos^{-1} -\frac{1}{2}$

$\frac{2\pi}{3}$

4) $\cos^{-1} \frac{\sqrt{3}}{2}$

$\frac{\pi}{6}$

5) $\cot^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

$\frac{2\pi}{3}$

6) $\cot^{-1} 0$

$\frac{\pi}{2}$

7) $\sec^{-1}\left(\csc -\frac{\pi}{4}\right)$

$\frac{3\pi}{4}$

8) $\cos^{-1}\left(\cos -\frac{47\pi}{7}\right)$

$\frac{5\pi}{7}$

9) $\tan^{-1}\left(\tan -\frac{80\pi}{9}\right)$

$\frac{\pi}{9}$

10) $\sin^{-1}\left(\sin \frac{27\pi}{8}\right)$

$-\frac{3\pi}{8}$

Write each trigonometric expression as an algebraic expression.

11) $\tan \sin^{-1} x$

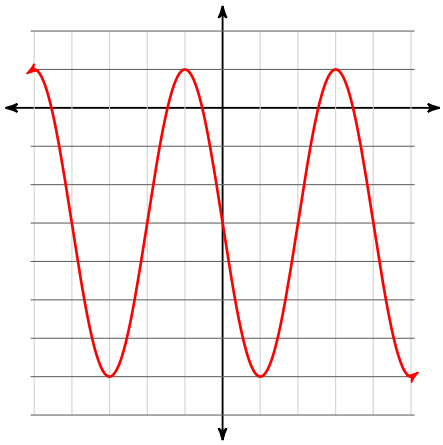
$\frac{x}{\sqrt{1-x^2}}$

12) $\sin \cot^{-1} x$

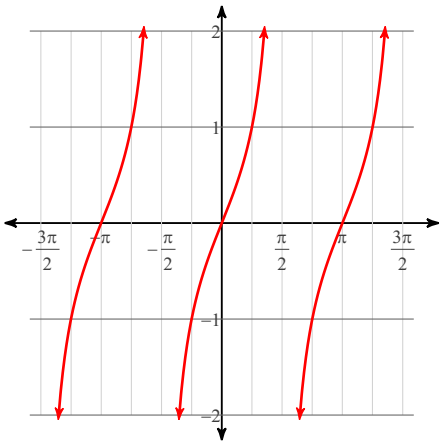
$\frac{1}{\sqrt{x^2+1}}$

Graph the function. State the period for each function.

13) $y = -3 + 4\sin(2\theta - \pi)$



14) $y = \tan x$



Find the exact value of each trigonometric function.

15) $\sec \frac{\pi}{3}$

2

16) $\sin -\frac{17\pi}{6}$

$-\frac{1}{2}$

17) $\csc -\frac{28\pi}{3}$ $\frac{2\sqrt{3}}{3}$

18) $\cot \frac{17\pi}{3}$

$-\frac{\sqrt{3}}{3}$

Assuming $\csc x = \frac{5}{3}$ **if** $\frac{\pi}{2} < x < \pi$ **and** $\cot y = \frac{5}{12}$ **if** $\pi < y < \frac{3\pi}{2}$

19) $\sin(x + y)$

$\frac{33}{65}$

20) $\tan(x - y)$ $\frac{63}{16}$

21) $\sec(x - y)$

$-\frac{65}{16}$

Find the exact value of each.

22) $\csc\theta = \frac{5}{4}$ where $90 \le \theta < 180$

Find $\tan 2\theta$

$\frac{24}{7}$

23) $\tan\theta = \frac{15}{8}$ where $180 \le \theta < 270$

Find $\cos 2\theta$

$-\frac{161}{289}$

Solve each triangle. Round your answers to the nearest tenth.

24) In $\triangle RPQ$, $m\angle R = 50°$, $q = 33$ yd, $r = 31$ yd

$m\angle P = 75.4°$, $m\angle Q = 54.6°$, $p = 39.2$ yd
Or $m\angle P = 4.6°$, $m\angle Q = 125.4°$, $p = 3.2$ yd

25) In $\triangle CAB$, $m\angle C = 106°$, $b = 8$ mi, $c = 37$ mi

$m\angle A = 62°$, $m\angle B = 12°$, $a = 34$ mi

Solve each equation for $0 \le \theta < 2\pi$. Round your answers to the nearest hundredth.

26) $-4 + 2\cos\left(3\theta + \frac{\pi}{4}\right) = -2.62$

$\{0.01, 1.56, 2.1, 3.66, 4.2, 5.75\}$

Solve each equation for $0 \le \theta < 2\pi$.

27) $-\sqrt{3}\sin\theta = -\sin 2\theta$

$\left\{0, \frac{\pi}{6}, \pi, \frac{11\pi}{6}\right\}$

28) $\cos 2\theta = -6\sin^2\theta + 3$

$\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

29) $2\sqrt{3}\cos\theta - 2\csc\theta = 3\cos\theta\csc\theta - 2\csc\theta$

$\left\{\frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{2}\right\}$

30) $0 = 2 - \cos^2\theta + 3\sin\theta + \sin^2\theta$

$\left\{\frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\right\}$

Find the area of each triangle to the nearest tenth.

31) In $\triangle ZXY$, $y = 9$ m, $x = 11.5$ m, $z = 15$ m

51.7 m²

32) In $\triangle RPQ$, $q = 5$ cm, $m\angle R = 99°$, $m\angle P = 54°$

22 cm²

Find all solutions to each equation in radians.

33) $\frac{-2+\sqrt{2}}{2} - 2\cos\left(4\theta + \frac{3\pi}{2}\right) = -1 - 3\cos\left(4\theta + \frac{3\pi}{2}\right)$

$\left\{-\frac{3\pi}{16} + \frac{\pi n}{2}, -\frac{\pi}{16} + \frac{\pi n}{2}\right\}$

Simplify each expression.

34) $\frac{\sin 2x}{\cos x} + \frac{\cos 2x}{\sin x}$

$\csc x$

35) $\frac{\cos x}{1+\sin x} + \tan x$

$\sec x$

Verify each identity.

36) $\frac{1-\cos 2x}{\cos^2 x} = 2\tan^2 x$

$\frac{1-\cos 2x}{\cos^2 x}$ Use $\cos^2 x = \frac{1+\cos 2x}{2}$

$\frac{2(1-\cos 2x)}{1+\cos 2x}$ Use $\tan^2 x = \frac{1-\cos 2x}{1+\cos 2x}$

$2\tan^2 x$ ■

37) $\cos(R - 90) = \dfrac{1}{\csc R}$

$\sin R = \dfrac{1}{\csc R}$

38) $\cos\left(D + \dfrac{\pi}{2}\right) = -\dfrac{1}{\csc D}$

$-\sin D = -\dfrac{1}{\csc D}$

39) $\cot x \tan x + \sec x = \dfrac{1 + \cos x}{\cos x}$

$\cot x \tan x + \sec x$ Decompose into sine and cosine

$\dfrac{\cos x}{\sin x} \cdot \dfrac{\sin x}{\cos x} + \dfrac{1}{\cos x}$ Simplify

$\dfrac{1 + \cos x}{\cos x}$ ■

40) $\dfrac{\cot x \sec x}{\sin x} = \cot^2 x + 1$

$\dfrac{\cot x \sec x}{\sin x}$ Decompose into sine and cosine

$\dfrac{\dfrac{\cos x}{\sin x} \cdot \dfrac{1}{\cos x}}{\sin x}$ Simplify

$\dfrac{1}{\sin^2 x}$ Use $\csc x = \dfrac{1}{\sin x}$

$\csc^2 x$ Use $\cot^2 x + 1 = \csc^2 x$

$\cot^2 x + 1$ ■

41) Determine the period of the function

$y = -5\cot 7x - 18$

$\frac{\pi}{7}$

42) Determine the period of the function

$y = 18 + 11\cos\frac{x}{5}$

10π

43) For what value of k is the period of the function $y = -6\sin kx$ equal to 80°

$\frac{9}{2}$

44) For what value of k is the period of the function $y = 8\tan kx$ equal to 260°

$\frac{9}{13}$

45) Express as an inverse function the value of x for which $2\sqrt{1 - \cos x} - 1 = 0$

$\cos^{-1}\frac{3}{4}$

Find the exact value of each.

46) tan 105°

$-2 - \sqrt{3}$

47) $\tan\theta = \frac{4}{3}$ where $540 \le \theta < 630$

Find $\sin\frac{\theta}{2}$

$-\frac{2\sqrt{5}}{5}$