AB Calculus Finding Limits Algebraically Homework Name: Key

1. When evaluating limits, what does it mean if direct substitution produces the result $\frac{7}{0}$?

either DNE, ∞, or $-\infty$

2. When evaluating limits, what does it mean if direct substitution produces the result $\frac{0}{0}$?

indeterminate → probably a hole

3. What are the options we discussed for dealing with the result $\frac{0}{0}$?

rationalizing (conjugate), factor, simplify (common denominator)

* 1st step is to plug in value and if $\frac{0}{0}$, manipulate algebraically. Always show the initial step where you get $\frac{0}{0}$.

4. Evaluate the following limits without a calculator. Show all work.

a) $\lim\limits_{x\to 3} \frac{x^2-x-6}{x-3}$ $\quad \frac{\cancel{(x-3)}(x+2)}{\cancel{x-3}}$

$3+2 = 5$

e) $\lim\limits_{x\to 0} \frac{\frac{1}{2+x}-\frac{1}{2}}{x} \cdot \frac{2+x}{2+x}$ $\quad \frac{2-2-x}{2x(2+x)} \to \frac{-x}{2x(2+x)}$

$\to \frac{-1}{2(2+x)} \to -\frac{1}{4}$

b) $\lim\limits_{x\to 0} \frac{\sqrt{2x+1}-1}{x} \cdot \frac{\sqrt{2x+1}+1}{\sqrt{2x+1}+1}$

$\frac{2x+1-1}{x(\sqrt{2x+1}+1)} \to \frac{2x}{x(\sqrt{2x+1}+1)}$

$\to \frac{2}{\sqrt{2x+1}+1} \to \frac{2}{\sqrt{1}+1} = \frac{2}{1+1} = \frac{2}{2} = 1$

f) $\lim\limits_{x\to 4} \frac{\sqrt{x+5}-3}{x-4} \cdot \frac{\sqrt{x+5}+3}{\sqrt{x+5}+3}$

$\frac{x+5-9}{x-4(\sqrt{x+5}+3)} \to \frac{x-4}{(x-4)(\sqrt{x+5}+3)} \to \frac{1}{\sqrt{x+5}+3}$

$\to \frac{1}{3+3} = \frac{1}{6}$

c) $\lim\limits_{x\to 1} \frac{x-1}{x^2-1}$ $\quad \frac{x-1}{(x-1)(x+1)} \to \frac{1}{x+1}$

$= \frac{1}{2}$

g) $\lim\limits_{x\to 0} \frac{(4+x)^2-16}{x} \to \frac{16+8x+x^2-16}{x}$

$\frac{8x+x^2}{x} \to 8+x \to 8$

d) $\lim\limits_{t\to 2} \frac{t^2-3t+2}{t^2-4} \to \frac{\cancel{(t-2)}(t-1)}{\cancel{(t-2)}(t+2)}$

$\to \frac{1}{4} \to \frac{1}{4}$

h) $\lim\limits_{x\to 0} \frac{(3+x)^3-8}{x}$

~~$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$~~

~~$27+27x+9x^2+x^3-8$~~

$\frac{(3+0)^3-8}{0}$

$\frac{27-8}{0} \to \frac{19}{0}$

$.0000001 \to \infty$

$-.0000001 \to -\infty$

DNE

One of the limits you should commit to memory is $\lim_{x \to 0} \frac{\sin x}{x} = $ __1__. This limit only works when the denominator matches the inside of the sine function. If they do not match, you cannot change the inside of the sine function without a trig identity. However, you can algebraically manipulate the non-sine part to get it to match the inside of the sine function.

5. Evaluate each of the following limits.

a) $\lim_{x \to 0} \frac{\sin x}{5x}$ $\rightarrow \frac{1}{5} \cdot \frac{\sin x}{x} \rightarrow \frac{1}{5} \cdot 1$ $= \frac{1}{5}$

b) $\lim_{x \to 0} \frac{\sin 4x}{x}$ $\cdot \frac{4}{4} \rightarrow \frac{4}{1} \cdot \frac{\sin 4x}{4x}$ 1 $= 4$

c) $\lim_{x \to 0} \frac{\sin x}{2x^2 - x}$ $\frac{\sin x}{x(2x-1)} \rightarrow \frac{\sin x}{x} \cdot \frac{1}{2x-1}$ $= \frac{1}{-1} = -1$

d) $\lim_{x \to 0} \frac{x + \sin x}{x}$ $\frac{x}{x} + \frac{\sin x}{x} = 2$ ↳ except if $x=0$

6. Evaluate $\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$ $\rightarrow \frac{x^2 + 2xh + h^2 - x^2}{h} \rightarrow \frac{2xh + h^2}{h} \rightarrow 2x + h \rightarrow 2x$

Hint: h is approaching 0, not x. Your final answer will contain an the variable x.

7. Use the following definition of $g(x)$ to evaluate each limit.

$$g(x) = \begin{cases} 2 - x, & x \leq 1 \\ \frac{x}{2} + 1, & x > 1 \end{cases}$$

a) $\lim_{x \to 1^-} g(x) =$ $2 - 1 = 1$

b) $\lim_{x \to 1^+} g(x) =$ $\frac{1}{2} + 1 = \frac{3}{2}$

c) $\lim_{x \to 1} g(x) =$ DNE

d) $g(1) =$ $2 - 1 = 1$

8. Evaluate the following using the graph below.

a) $\lim_{x \to -1^-} f(x) =$ 2

b) $\lim_{x \to -1^+} f(x) =$ 4

c) $\lim_{x \to -1} f(x) =$ DNE

d) $\lim_{x \to 2} f(x) =$ ~~2~~ 1

e) $\lim_{x \to 1} f(x) =$ 2

f) $f(1) =$ 4