

$$a) f(x) = e^x \cos x \quad 0 \leftrightarrow \pi$$

$$\frac{f(\pi) - f(0)}{\pi - 0} = \frac{(e^\pi \cos \pi) - (e^0 \cos 0)}{\pi}$$

$$= \frac{-e^\pi - 1}{\pi}$$

$$b) f'(x) = e^x \cos x + e^x (-\sin x)$$

$$f'\left(\frac{3\pi}{2}\right) = e^{\frac{3\pi}{2}} \cos \frac{3\pi}{2} + e^{\frac{3\pi}{2}} \left(-\sin \frac{3\pi}{2}\right)$$

$$= e^{3\pi/2}$$

$$c) f'(x) = 0 \quad 0 = e^x (\cos x - \sin x)$$

$$\cos x - \sin x = 0 \quad \text{at } x = \frac{\pi}{4}$$

| x | f(x) |
|----------|----------------------------------|
| 0 | 1 |
| $\pi/4$ | $e^{\pi/4} (\sqrt{2}/2)$ |
| $5\pi/4$ | $e^{5\pi/4} \cdot (-\sqrt{2}/2)$ |
| 2π | $e^{2\pi} \cdot 1$ |

$$\text{abs min} = \left(e^{5\pi/4} \right) \left(\frac{-\sqrt{2}}{2} \right) \quad x = \frac{5\pi}{4}$$

$$d) \lim_{x \rightarrow \pi/2} \frac{f(x)}{g(x)}$$

$$\lim_{x \rightarrow \pi/2} f(x) = 0 \quad \text{by } \lim_{x \rightarrow \pi/2} \frac{f(x)}{g(x)} = \lim_{x \rightarrow \pi/2} \frac{f'(x)}{g'(x)}$$

L'Hospital's

$$\lim_{x \rightarrow \pi/2} g(x) = 0$$

$$= \frac{-e^{\pi/2}}{2}$$

a) slope field

b) $\frac{dy}{dx} = \frac{1}{3}x(y-2)^2$ $\frac{dy}{dx} \Big|_{(1,0)} = \frac{1}{3}(1)(0-2)^2 = \frac{4}{3}$

Find tangent when $x=1$

TL $y-0 = \frac{4}{3}(x-1)$

y -value = 0

$$f(7) \approx \frac{4}{3}(7-1)$$

c) $\frac{dy}{dx} = \frac{1}{3}x(y-2)^2$

$$\int \frac{dy}{(y-2)^2} = \int \frac{1}{3}x \, dx$$

$$\int (y-2)^{-2} dy = \int \frac{1}{3}x \, dx$$

$$-1(y-2)^{-1} = \frac{1}{3} \frac{1}{2}x^2 + C$$

$$-1(0-2)^{-1} = \frac{1}{6} + C$$

$$\frac{+1}{+2} = \frac{1}{6} + C$$

$$C = \frac{3}{3} \frac{1}{2} - \frac{1}{6} = \frac{3-1}{6} = \frac{2}{6} = \frac{1}{3}$$

$$-1(y-2)^{-1} = \frac{1}{6}x^2 + \frac{1}{3}$$

$$(y-2) \frac{-1}{y-2} = \left(\frac{1}{6}x^2 + \frac{1}{3}\right)(y-2)$$

$$\frac{-1}{\frac{1}{6}x^2 + \frac{1}{3}} = \frac{\left(\frac{1}{6}x^2 + \frac{1}{3}\right)(y-2)}{\frac{1}{6}x^2 + \frac{1}{3}}$$

$$y-2 = \frac{-1}{\frac{1}{6}x^2 + \frac{1}{3}}$$

$$y = \frac{-1}{\frac{1}{6}x^2 + \frac{1}{3}} + 2$$

a) slope field

b) $\frac{dy}{dx} = \frac{x(y-1)}{4}$

Tangent line

$$y-3 = \frac{1}{2}(x-1)$$

$$f(1.4) \approx \frac{1}{2}(1.4-1) + 3$$

Tangent line at (1,3)

$$\frac{dy}{dx} \Big|_{(1,3)} = \frac{1}{2}$$

$$c) \quad \frac{dy}{dx} = \frac{x(y-1)}{4}$$

$$\int \frac{dy}{y-1} = \int \frac{x}{4} dx$$

$$\ln|y-1| = \frac{1}{4} \frac{x^2}{2} + C$$

$$\ln 2 = \frac{1}{8} + C$$

$$C = \ln 2 - \frac{1}{8}$$

$$e^{\ln|y-1|} = e^{\frac{1}{8}x^2 + \ln 2 - \frac{1}{8}}$$
$$y-1 = e^{\frac{1}{8}x^2 + \ln 2 - \frac{1}{8}}$$
$$y = e^{\frac{1}{8}x^2 + \ln 2 - \frac{1}{8}} + 1$$
$$y = 2e^{\frac{1}{8}x^2 - \frac{1}{8}} + 1$$

$$f(x) = (x^2 - 2x - 1)e^x$$

$$a) \quad \lim_{x \rightarrow \infty} f(x) = \infty$$

$$\lim_{x \rightarrow -\infty} f(x) = 0$$

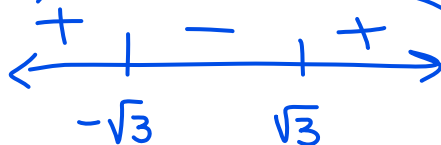
b.) $f(x)$ INCREASING?

$$f'(x) = e^x(2x-2) + e^x(x^2-2x-1)$$

$$0 = e^x(2x-2+x^2-2x-1)$$

$$0 = e^x(x^2-3)$$

$$x = \pm\sqrt{3}$$



$f(x)$ is increasing
 $(-\infty, -\sqrt{3})$ $(\sqrt{3}, \infty)$

$f'(x) > 0$

c) concavity (down)

$$f'(x) = e^x(x^2-3)$$

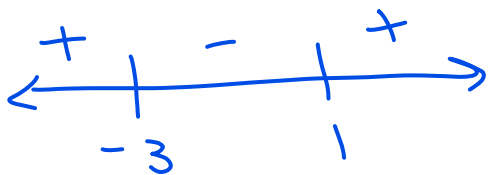
$$f''(x) = e^x(x^2-3) + e^x(2x)$$

$$0 = e^x(x^2-3+2x)$$

$$0 = e^x(x^2+2x-3)$$

$$0 = e^x(x-1)(x+3)$$

$$x = 1, -3$$



Concave Down

$(-3, 1)$ $f''(x) < 0$

