a) $f(x) = e^x \cos x \quad 0 \leftrightarrow \pi$

$$\frac{f(\pi) - f(0)}{\pi - 0} = \frac{(e^{\pi} \cos \pi) - (e^{0} \cos 0)}{\pi}$$

$$= \frac{-e^{\pi} - 1}{\pi}$$

b) $f'(x) = e^x \cos x + e^x(-\sin x)$

$$f'\left(\frac{3\pi}{2}\right) = e^{\frac{3\pi}{2}} \cos \frac{3\pi}{2} + e^{\frac{3\pi}{2}}\left(-\sin \frac{3\pi}{2}\right)$$

$$= e^{3\pi/2}$$

c.) $f'(x) = 0 \quad 0 = e^x(\cos x - \sin x)$

$\cos x - \sin x = 0$ at $x = \frac{\pi}{4}$

$x = \frac{5\pi}{4}$

x	$f(x)$
0	1
$\pi/4$	$e^{\pi/4}(\sqrt{2}/2)$
$5\pi/4$	$e^{5\pi/4} \cdot (-\sqrt{2}/2)$
2π	$e^{2\pi}$ 1

$$\text{abs min} = \left(e^{5\pi/4}\right)\left(-\frac{\sqrt{2}}{2}\right)$$

d.) $\lim\limits_{x \to \pi/2} \frac{f(x)}{g(x)}$

$\lim\limits_{x \to \frac{\pi}{2}} f(x) = 0$

$\lim\limits_{x \to \frac{\pi}{2}} g(x) = 0$

by L'Hospitals $\lim\limits_{x \to \frac{\pi}{2}} \frac{f(x)}{g(x)} = \lim\limits_{x \to \frac{\pi}{2}} \frac{f'(x)}{g'(x)}$

$$= \frac{-e^{\pi/2}}{2}$$

a) slope field

b.) $\frac{dy}{dx} = \frac{1}{3}x(y-2)^2$ $\left.\frac{dy}{dx}\right|_{(1,0)} = \frac{1}{3}(1)(0-2)^2 = \frac{4}{3}$

Find tangent when $x = 1$ TL $y - 0 = \frac{4}{3}(x-1)$

y-value = 0

$f(7) \approx \frac{4}{3}(7-1)$

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c.) $\frac{dy}{dx} = \frac{1}{3}x(y-2)^2$

$\int \frac{dy}{(y-2)^2} = \int \frac{1}{3}x \, dx$

$\int (y-2)^{-2} dy = \int \frac{1}{3} x \, dx$

$-1(y-2)^{-1} = \frac{1}{3}\frac{1}{2}x^2 + C$

$-1(0-2)^{-1} = \frac{1}{6} + C$

$\frac{+1}{+2} = \frac{1}{6} + C$

$C = \frac{3}{3}\frac{1}{2} - \frac{1}{6} = \frac{3-1}{6} = \frac{2}{6} = \frac{1}{3}$

$-1(y-2)^{-1} = \frac{1}{6}x^2 + \frac{1}{3}$

$(y-2)\ \frac{-1}{y-2} = \left(\frac{1}{6}x^2 + \frac{1}{3}\right)(y-2)$

$\frac{-1}{\frac{1}{6}x^2 + \frac{1}{3}} = \frac{\left(\frac{1}{6}x^2 + \frac{1}{3}\right)(y-2)}{\frac{1}{6}x^2 + \frac{1}{3}}$

$y - 2 = \frac{-1}{\frac{1}{6}x^2 + \frac{1}{3}}$

$y = \frac{-1}{\frac{1}{6}x^2 + \frac{1}{3}} + 2$

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a) slope field

b) $\frac{dy}{dx} = \frac{x(y-1)}{4}$

Tangent Line

$y - 3 = \frac{1}{2}(x-1)$

$f(1.4) \approx \frac{1}{2}(1.4-1) + 3$

Tangent Line at (1,3) $\left.\frac{dy}{dx}\right|_{(1,3)} = \frac{1}{2}$

c) $\frac{dy}{dx} = \frac{x(y-1)}{4}$

$$\int \frac{dy}{y-1} = \int \frac{x}{4}\, dx$$

$$\ln|y-1| = \frac{1}{4}\frac{x^2}{2} + C$$

$$\ln 2 = \frac{1}{8} + C$$

$$C = \ln 2 - \frac{1}{8}$$

$$e^{\ln|y-1|} = e^{\frac{1}{8}x^2 + \ln 2 - \frac{1}{8}}$$

$$y-1 = e^{\frac{1}{8}x^2 + \ln 2 - \frac{1}{8}}$$

$$y = e^{\frac{1}{8}x^2 + \ln 2 - \frac{1}{8}} + 1$$

$$y = 2e^{\frac{1}{8}x^2 - \frac{1}{8}} + 1$$

$$f(x) = (x^2 - 2x - 1)e^x$$

(a)

$$\lim_{x \to \infty} f(x) = \infty$$

$$\lim_{x \to -\infty} f(x) = 0$$

b.) f(x) INCREASING?

$$f'(x) = e^x(2x-2) + e^x(x^2-2x-1)$$

$$0 = e^x(2x-2+x^2-2x-1)$$

$$0 = e^x(x^2-3)$$

$$x = \pm\sqrt{3}$$

+ | − | +

$-\sqrt{3}$ $\sqrt{3}$

f(x) is increasing
$(-\infty, -\sqrt{3})$ $(\sqrt{3}, \infty)$
$f'(x) > 0$

ⓒ concavity (down)

$$f'(x) = e^x(x^2-3)$$

$$f''(x) = e^x(x^2-3) + e^x(2x)$$

$$0 = e^x(x^2-3+2x)$$

$$0 = e^x(x^2+2x-3)$$

$$0 = e^x(x-1)(x+3)$$

$$x = 1, -3$$

+ | − | +

−3   1

Concave Down
$(-3, 1)$ $f''(x) < 0$