$$2\left(x\frac{dy}{dx}+1\,y\right)+2y\frac{dy}{dx}=0$$

$$2x\frac{dy}{dx}+2y+2y\frac{dy}{dx}=0$$

$$2x\frac{dy}{dx}+2y\frac{dy}{dx}=-2y$$

Consider the curve given by the equation $2xy+y^2=8$ for $y>0$.

(a) Show that $\frac{dy}{dx}=\frac{-y}{x+y}$.

$$\frac{dy}{dx}(2x+2y)=-2y$$

$$\frac{dy}{dx}=\frac{-2y}{2x+2y}=\frac{-y}{x+y}$$

(b) Write an equation for the line tangent to the curve at the point $(1,2)$.

$x=1$ $y=2$ $\left.\frac{dy}{dx}\right|_{(1,2)}=\frac{-2}{1+2}=-\frac{2}{3}$

$$y-2=-\frac{2}{3}(x-1)$$

(c) Evaluate $\frac{d^2y}{dx^2}$ at the point $(1,2)$.

$$\frac{d}{dx}\frac{dy}{dx}=\frac{d}{dx}\left(\frac{-y}{x+y}\right)\rightarrow\frac{d^2y}{dx^2}=\frac{(x+y)\left(-\frac{dy}{dx}\right)-(-y)\left(1+\frac{dy}{dx}\right)}{(x+y)^2}$$

$$\left.\frac{d^2y}{dx^2}\right|_{1,2}=\frac{(1+2)\left(\frac{2}{3}\right)-(-2)\left(1-\frac{2}{3}\right)}{(1+2)^2}$$

$$=\frac{2+\frac{2}{3}}{9}=\frac{\frac{8}{3}}{9}=\frac{1}{9}=\frac{8}{27}$$

(d) The points $(1,2)$ and $\left(\frac{7}{2},1\right)$ are on the curve. Find the value of $\left(y^{-1}\right)'(1)$.

$$\left(y^{-1}\right)'(1)=\frac{1}{y'\left(\frac{7}{2}\right)}$$

$y'\left(\frac{7}{2}\right)$ implies $\left.\frac{dy}{dx}\right|_{\left(\frac{7}{2},1\right)}$

$$\left.\frac{dy}{dx}\right|_{\left(\frac{7}{2},1\right)}=\frac{-1}{\frac{7}{2}+1}=\frac{-1}{\frac{9}{2}}=-\frac{2}{9}$$

Final answer $=\frac{1}{-2/9}=\boxed{-\frac{9}{2}}$

## Unit 3 Progress Check: FRQ Part B

Please respond on separate paper, following directions from your teacher.

**2.** NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

| $x$ | $-3$ | $2$ | $3$ | $8$ |
|---|---|---|---|---|
| $f(x)$ | $-9$ | $4$ | $2$ | $6$ |
| $f'(x)$ | $-\frac{7}{2}$ | $\frac{3}{2}$ | $-\frac{2}{5}$ | $\frac{1}{3}$ |

$g'(x) = \dfrac{f(x^3)\ \frac{1}{x} - \ln x\ f'(x^3)\ 3x^2}{[f(x^3)]^2}$

$g'(2) = \dfrac{f(2^3)\cdot\frac{1}{2} - \ln 2\ f'(2^3)\ 3(2)^2}{[f(2^3)]^2} = \dfrac{f(8)\ \frac{1}{2} - \ln 2\ f'(8)\ 12}{[f(8)]^2}$

(a) Let $g$ be the function defined by $g(x) = \frac{\ln x}{f(x^3)}$. Find $g'(2)$.

$= \dfrac{6\ \frac{1}{2} - \ln 2\ \frac{1}{3}\cdot 12}{6^2}$

$= \dfrac{3 - 4\ln 2}{36}$

(b) Let $h$ be the function defined by $h(x) = f(f(-3x))$. Find $h'(-1)$.

$h'(x) = f'(f(-3x))\ f'(-3x)\ -3$

$h'(-1) = f'(f(3))\ f'(3)\ -3$

$= f'(2)\ -\frac{2}{5}\ -3 = \frac{3}{2}\ -\frac{2}{5}\ -\frac{3}{1} = \frac{18}{10} = \frac{9}{5}$

## Unit 3 Progress Check: FRQ Part B

Please respond on separate paper, following directions from your teacher.

(c) Let $k$ be the function defined by $k(x) = f(x) \cdot \arctan\left(\frac{x}{3}\right)$. Find $k'(-3)$.

$$k'(x) = f(x) \frac{1}{1+\left(\frac{x}{3}\right)^2} \frac{1}{3} + f'(x) \arctan\left(\frac{x}{3}\right)$$

$$k'(-3) = f(-3) \frac{1}{1+\left(\frac{-3}{3}\right)^2} \frac{1}{3} + f'(-3) \arctan\left(\frac{-3}{3}\right)$$

$$= -9 \frac{1}{2} \cdot \frac{1}{3} + -\frac{7}{2} -\frac{\pi}{4}$$

$$= -\frac{9}{6} + \frac{7\pi}{8}$$