Key

AP Calculus
5.4 Worksheet Day 1

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

For questions 1 – 10, use the Fundamental Theorem of Calculus (Evaluation Part) to evaluate each definite integral. Use your memory of derivative rules and/or the chart from your notes. You should start making a list of all the rules on ONE page!

1. $\int_1^4 \left(x^3 + \frac{5}{\sqrt{x}}\right)dx$ $\frac{x^4}{4} + 10x^{\frac{1}{2}}\Big|_1^4$

$5x^{-\frac{1}{2}}$

$\left(\frac{4^4}{4} + 10\,4^{\frac{1}{2}}\right) - \left(\frac{1^4}{4} + 10\,1^{\frac{1}{2}}\right)$

$= (64 + 20) - \left(\frac{1}{4} + 10\right) = \frac{295}{4}$

2. $\int_3^5 \frac{dx}{x}$ $\int_3^5 \frac{1}{x}\,dx \to \ln|x|\Big|_3^5$

$\ln 5 - \ln 3$

$= \ln\left(\frac{5}{3}\right)$

3. $\int_{1/2}^{\sqrt{2}/2} \frac{1}{\sqrt{1-x^2}}dx$ $\sin^{-1}x\Big|_{1/2}^{\frac{\sqrt{3}}{2}}$

$\sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}\frac{1}{2} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$

4. $\int_{-1}^{\sqrt{3}} \frac{1}{1+x^2}dx$ $= \tan^{-1}x\Big|_{-1}^{\sqrt{3}}$

$\tan^{-1}\sqrt{3} - \tan^{-1}(-1)$

$\frac{\pi}{3} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{3} + \frac{\pi}{4} = \frac{7\pi}{12}$

5. $\int_0^2 5^x\,dx$ $= \frac{5^x}{\ln 5}\Big|_0^2 = \frac{1}{\ln 5}(5^2 - 5^0)$

$= \frac{24}{\ln 5}$

6. $\int_{-5}^{12} 7x\,dx$ $= \frac{7x^2}{2}\Big|_{-5}^{12}$

$= \frac{7}{2}\left(12^2 - (-5)^2\right) = \frac{7}{2}(144 - 25) = \frac{7}{2}(119)$

$= 833/2$

7. $\int_{-2}^{5} 6\,dx$ $= 6x\Big|_{-2}^{5}$

$6(5 - (-2)) = 6 \cdot 7 = 42$

8. $\int_{\pi/2}^{\pi} 5\sin(x)dx$

$= -5\cos x\Big|_{\frac{\pi}{2}}^{\pi}$

$= -5\left(\cos\pi - \cos\frac{\pi}{2}\right) = -5(-1 - 0)$

$= 5$

9. $\int_0^{\pi/4} \sec^2(x)dx$

$= \tan x\Big|_0^{\pi/4}$

$\tan\frac{\pi}{4} - \tan 0 = 1 - 0 = 1$

10. $\int_{-1}^{3} e^x\,dx$ $= e^x\Big|_{-1}^{3}$

$e^3 - e^{-1} = e^3 - \frac{1}{e}$

If you would like more practice with the FTOC (Evaluation part)? ... page 303 #27 – 40 (ask to borrow a book)

For questions 11 and 12, setup and evaluate an expression involving definite integrals in order to find the total AREA of the region between the curve and the x-axis. [No Calculator!]

$3(x-1)(x+1)$

11. $y = 3x^2 - 3$ on the interval $-2 \le x \le 2$

$-2\int_0^1 3x^2 - 3\,dx \quad -2\left[1^3 - 3(1) - (0^3 - 3(0))\right]$

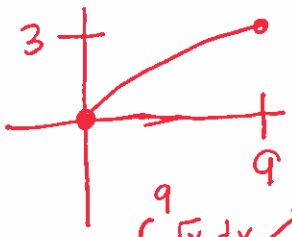
$2\int_1^2 3x^2 - 3\,dx \quad +2\left[2^3 - 3(2) - (1^3 - 3(1))\right]$

$= -2(-2) + 2(8 - 6 - 1 + 3)$

$= 4 + 8 = 12$

$= -2\left[x^3 - 3x\right]_0^1 + 2\left[x^3 - 3x\right]_1^2$

12. $y = \sqrt{x}$ on the interval $0 \le x \le 9$



$\int_0^9 \sqrt{x}\,dx \quad x^{\frac{1}{2}}$

$\frac{2}{3}x^{\frac{3}{2}}\Big|_0^9$

$= \frac{2}{3}\left(9^{\frac{3}{2}} - 0^{\frac{3}{2}}\right)$

$= \frac{2}{3}\left(3^{2\cdot\frac{3}{2}}\right)$

$= \frac{2}{3}\cdot 27 = 18$

For questions 13 – 16, find the average value of the function on the specified interval <u>without a calculator</u>.

13. $g(x) = 9 - 3x^2$ on the interval $[0, 4]$

$\frac{1}{4-0}\int_0^4 (9 - 3x^2)\,dx \quad \frac{1}{4}\left[9x - x^3\right]_0^4$

$\frac{1}{4}\left(9(4) - 4^3 - (9\cdot 0 - 0^3)\right) = -7$

14. $h(x) = \csc(x)\cot(x)$ on the interval $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

$\frac{1}{\frac{\pi}{2} - \frac{\pi}{4}}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x \csc x\,dx$

$= \frac{-\csc x\Big|_{\pi/4}^{\pi/2}}{\pi/4} \quad \frac{-1}{\frac{\pi}{4}}\left(\csc\frac{\pi}{2} - \csc\frac{\pi}{4}\right)$

$= -\frac{4}{\pi}\left(1 - \sqrt{2}\right)$

15. $y = \begin{cases} 5x & \text{if } 0 \le x \le 2 \\ 12 - x & \text{if } 2 < x \le 12 \end{cases}$

$\frac{1}{12}\int_0^2 5x\,dx + \frac{1}{12}\int_2^{12} 12 - x\,dx$

$\frac{1}{12}\left(\frac{5x^2}{2}\Big|_0^2 + 12x - \frac{x^2}{2}\Big|_2^{12}\right) \quad \frac{1}{12}\left[10 + 72 - 20\right]$

$\frac{1}{12}\left[\frac{5}{2}(4) - 0 + \left(12(12) - \frac{12^2}{2}\right) - \left(12(2) - \frac{2^2}{2}\right)\right] \quad = \frac{1}{12}(62) = \frac{31}{6}$

16. $f(x) = \sec^2 x$ on the interval $\left[0, \frac{\pi}{4}\right]$

$\frac{1}{\frac{\pi}{4} - 0}\int_0^{\pi/4} \sec^2 x\,dx$

$\frac{4}{\pi}\left(\tan x\Big|_0^{\pi/4}\right) = \frac{4}{\pi}\left(\tan\frac{\pi}{4} - \tan 0\right)$

$= \frac{4}{\pi}(1 - 0) = \frac{4}{\pi}$

17. Including start-up costs, it costs a printer \$50 to print 24 copies of a newsletter, after which the marginal cost (in dollars per copy) at x copies is given by $C'(x) = \frac{2}{\sqrt{x}}$. Find the total cost of printing 2500 newsletters.

$50 + \int_{25}^{2500} \frac{2}{\sqrt{x}}\,dx$

$50 + \int_{25}^{2500} 2x^{-\frac{1}{2}}\,dx$

$50 + 4x^{\frac{1}{2}}\Big|_{25}^{2500}$

$50 + 4\left(2500^{\frac{1}{2}} - 25^{\frac{1}{2}}\right)$

$50 + 4(50 - 5)$

$50 + 4(45)$

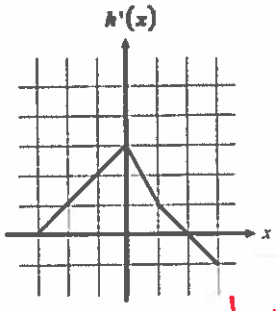
$50 + 180 = \$230$

18. If you know $\int_{-7}^{9} f'(x)dx = 15$, and you know $f(-7) = 4$, what does $f(9) = ?$

$\int_{-7}^{9} f'(x)dx = f(9) - f(-7)$

$15 = f(9) - 4 \qquad f(9) = 19$

19. The graph of $h'(x)$ is given below. If $h(-2) = 6$, what does $h(3) = ?$

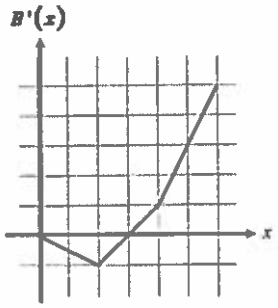


$\int_{-2}^{3} h'(x)dx = h(3) - h(-2)$

$6 = h(3) - 6 \qquad h(3) = 12$

$\int_{-2}^{0} h'(x)dx = 4 \quad \int_{0}^{1} h'(x)dx = 2 \quad \int_{1}^{3} h'(x)dx = 0 \quad$ so $\int_{-2}^{3} h'(x)dx = 4 + 2 + 0 = 6$

20. The graph of $B'(x)$ is given below. If you know that $B(0) = 5$, what does $B(5) = ?$



$\int_{0}^{5} B'(x)dx = B(5) - B(0)$

$1 = B(5) - 5$

$B(5) = 6$

$\int_{0}^{2} B'(x)dx = -1$

$\int_{2}^{4} B'(x)dx = 0$

$\int_{4}^{5} B'(x)dx = 2$

so $\int_{0}^{5} B'(x)dx = -1 + 0 + 2 = 1$

ABCALC FTOC and Integration Methods Day 1 Homework Name: Key

1. Evaluate each integral.

a) $\int_{\frac{1}{2}}^{3}\left(2-\frac{1}{x}\right)dx$

$2x-\ln|x|\Big]_{\frac{1}{2}}^{3}$

$(2(3)-\ln 3)-(2(\tfrac{1}{2})-\ln\tfrac{1}{2})$

$6-\ln 3-1+\ln\tfrac{1}{2}$

$5+\ln\frac{1}{6}$ or $5-\ln 6$

b) $\int_{0}^{1}(x^2+-\sqrt{x})dx$ $\quad -x^{\frac{1}{2}}$

$\frac{x^3}{3}-\frac{2}{3}x^{\frac{3}{2}}\Big|_0^1$

$\left(\frac{1^3}{3}-\frac{2}{3}1^{\frac{3}{2}}\right)-\left(\frac{0^3}{3}-\frac{2}{3}0^{\frac{3}{2}}\right)=-\frac{1}{3}$

c) $\int_{1}^{32}\left(x^{-\frac{6}{5}}\right)dx$

$-5x^{-\frac{1}{6}}\Big|_1^{32}$

$-5(32^{-\frac{1}{5}}-1^{-\frac{1}{5}})$

$=-5\left(\frac{1}{2}-1\right)=-5\left(-\frac{1}{2}\right)=\frac{5}{2}$

d) $\int_{0}^{\pi}\sin x\,dx$

$-\cos x\Big|_0^{\pi}$

$-\cos\pi+\cos 0$

$-(-1)+1$

$=2$

e) $\int_{0}^{\frac{\pi}{3}}2\sec^2\theta\,d\theta$

$2\tan\theta\Big|_0^{\frac{\pi}{3}}$

$2\left(\tan\frac{\pi}{3}-\tan 0\right)$

$2(\sqrt{3}-0)=2\sqrt{3}$

f) $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\csc x\cot x\,dx$

$-\csc x\Big|_{\pi/4}^{3\pi/4}$

$-\csc\frac{3\pi}{4}+\csc\frac{\pi}{4}$

$-\sqrt{2}+\sqrt{2}=0$

g) $\int_{-1}^{1}(r+1)^2\,dr$

$\int_{-1}^{1}r^2+2r+1\,dr$

$\frac{r^3}{3}+r^2+r\Big|_{-1}^{1}$

$\left(\frac{1^3}{3}+1^2+1\right)-\left(\frac{-1^3}{3}+(-1)^2-1\right)$ $\quad \left(\frac{1}{3}+2\right)-\left(-\frac{1}{3}\right)$

$=2\frac{2}{3}=\frac{8}{3}$

h) $\int_{0}^{1}3e^x\,dx$

$=3e^x\Big|_0^1$

$=3e^1-3e^0$

$=3e-3$

i) $\int_{1}^{4}\left(x^2-\frac{1}{\sqrt{x}}\right)dx$

$x^2-1x^{-\frac{1}{2}}$

$\frac{x^3}{3}-2x^{\frac{1}{2}}\Big]_1^4$

$\left(\frac{4^3}{3}-2(4)^{\frac{1}{2}}\right)-\left(\frac{1^3}{3}-2(1)^{\frac{1}{2}}\right)$

$\frac{64}{3}-4-\frac{1}{3}+2=21-2=19$

j) $\int_{\frac{1}{2}}^{1}\left(\frac{1}{\sqrt{1-x^2}}\right)dx$

$=\sin^{-1}x\Big|_{1/2}^{1}$

$\sin^{-1}1-\sin^{-1}\frac{1}{2}$

$\frac{\pi}{2}-\frac{\pi}{6}=\frac{\pi}{3}$