AB Calculus Implicit Differentiation Day 2 Homework Name: Key

1. Find dy/dx. If you use a table for the chain rule, try to find the derivative of these without it.

a) $y = \frac{1}{(2x+1)^2}$ → $(2x+1)^{-2}$

$\frac{dy}{dx} = \frac{-2}{(2x+1)^3} \cdot 2$

b) $y = \sqrt[3]{\sin x}$

$\sqrt[3]{x} \to \frac{1}{3\sqrt[3]{x^2}}$

$\sin x \to \cos x$

$\frac{dy}{dx} = \frac{1}{3\sqrt[3]{\sin^2 x}} \cdot \cos x$

c) $y = \cos(\tan x)$

$\cos x \to -\sin x$

$\tan x \to \sec^2 x$

$\frac{dy}{dx} = -\sin(\tan x) \cdot \sec^2 x$

d) $y = \csc(7x^2 + 1)$

$\csc x \to -\csc x \cot x$

$7x^2 + 1 \to 14x$

$\frac{dy}{dx} = -\csc(7x^2+1)\cot(7x^2+1) \cdot 14x$

2. Find dy/dx.

a) $6x^3 + y^4 = 6x$

$18x^2 + 4y^3 y' = 6$

$y' = \frac{6 - 18x^2}{4y^3} \to \frac{3 - 9x^2}{2y^3}$

b) $x^2 + y^2 - 4x + 7y = 15$

$2x + 2y \cdot y' - 4 + 7y' = 0$

$y' = \frac{-2x + 4}{2y + 7}$

c) $xy - x - 3y - 4 = 0$

$xy' + y - 1 - 3y' = 0$

$y' = \frac{1 - y}{x - 3}$

d) $\sin^2 y + \cos^2 y = y + 2$

$(\sin y)^2 + (\cos y)^2 = y + 2$

$2\sin y \cos y \cdot y' - 2\cos y \sin y \cdot y' = y'$

$0 = y'$

3. Find the slope of the curve at the given point.

a. $\sqrt{xy} = 1$ at $\left(2, \frac{1}{2}\right)$.

$\frac{1}{2\sqrt{xy}} \cdot (xy' + y) = 0$

$\frac{1}{2\sqrt{2 \cdot \frac{1}{2}}}\left(2y' + \frac{1}{2}\right) = 0$

$\frac{1}{2} \cdot 2y' + \frac{1}{4} = 0$

$2y' = \frac{-1/4}{1/2}$

$2y' = -\frac{1}{2}$

$y' = -\frac{1}{4}$

b. $x^3 + 5x^2 y + 2y^2 = 4y + 11$ at $(1, 2)$.

$(5x^2)y$

$3x^2 + 10xy + 5x^2 y' + 4y \cdot y' = 4y'$

$3 + 20 + 5y' + 8y' = 4y'$

$23 = -9y'$

$y' = \frac{-23}{9}$

4. Find the equation of the tangent and normal lines of the curves at the given point.

a. $x^2 + y^2 = 25$ at the point $(4, 3)$.

$2x + 2y \cdot y' = 0$

$8 + 6y' = 0$

$y' = \frac{-8}{6} = \frac{-4}{3}$

T: $y - 3 = -\frac{4}{3}(x-4)$

N: $y - 3 = \frac{3}{4}(x-4)$

b. $y + \sin y + x^2 = 9$ at $(3, 0)$.

$y' + \cos y \cdot y' + 2x = 0$

$y' + \cos 0 \cdot y' + 6 = 0$

$y' + y' = -6$

$2y' = -6$

$y' = -3$

T: $y - 0 = -3(x-3)$

N: $y - 0 = \frac{1}{3}(x-3)$

5. Find $\frac{d^2y}{dx^2}$ in terms of x and y if $xy + y^2 = 4$.

$xy' + y + 2yy' = 0$

$y' = \frac{-y}{x+2y}$

$y'' = \frac{(x+2y)(-1)y' - (-y)(1+2y')}{(x+2y)^2}$

$y'' = \frac{-(x+2y)\left(\frac{-y}{x+2y}\right) + y\left(1 + 2\left(\frac{-y}{x+2y}\right)\right)}{(x+2y)^2}$

$\frac{y + y + \frac{(-2)y^2}{x+2y}}{(x+2y)^2}$

$\frac{(x+2y)2y - 2y^2}{(x+2y)^3}$

6. Find the points where the graph of $x^2 + 3y^2 = 7$ has a horizontal tangent line.

$2x + 6y \cdot y' = 0$

$y' = \frac{-2x}{6y} = \frac{-x}{3y} = 0$ if $x = 0$

$0 + 3y^2 = 7$

$y^2 = \frac{7}{3}$

$\left(0, \pm\sqrt{\frac{7}{3}}\right)$

7. Find the points where the graph of $x^2 + 3y^2 = 7$ has a vertical tangent line.

$2x + 6y \cdot y' = 0$

$y' = \frac{-x}{3y}$ is und. if $3y = 0 \rightarrow y = 0$

$x^2 + 0 = 7$

$(\pm\sqrt{7}, 0)$