AB Calculus Implicit Differentiation Homework Name: ____________________

1. Find dy/dx.

a) $y = x^{\frac{9}{4}}$

$\frac{dy}{dx} = \frac{9}{4}x^{\frac{5}{4}}$ or $\frac{9\sqrt[4]{x^5}}{4}$

b) $y = \sqrt[3]{x}$ $x^{\frac{1}{3}}$

$\frac{dy}{dx} = \frac{1}{3}(x)^{-\frac{2}{3}} \rightarrow \frac{1}{3x^{2/3}}$ or $\boxed{\frac{1}{3\sqrt[3]{x^2}}}$

c) $y = (2x+5)^{-\frac{1}{2}}$

$\frac{dy}{dx} = -\frac{1}{2}(2x+5)^{-\frac{3}{2}}(2)$ $\frac{-1(2)}{2(2x+5)^{3/2}}$ or $\frac{-1}{(\sqrt{2x+5})^3}$

d) $y = x\sqrt{x^2+1}$ $\sqrt{x^2+1} + \frac{x \cdot 2x}{2\sqrt{x^2+1}}$

$\boxed{\frac{dy}{dx} = \sqrt{x^2+1} + \frac{x^2}{\sqrt{x^2+1}}}$

e) $x^2y + xy^2 = 6$

$2xy + x^2\frac{dy}{dx} + y^2 + x2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2xy - y^2}{x^2+2xy}$

f) $y^2 = \frac{x-1}{x+1}$

$2y\frac{dy}{dx} = \frac{x+1-(x-1)}{(x+1)^2} \rightarrow \frac{dy}{dx} = \frac{2}{(x+1)^2 2y}$

$\boxed{= \frac{1}{y(x+1)^2}}$

g) $y = \sqrt{1-\sqrt{x}}$

O: $\sqrt{x} \rightarrow \frac{1}{2\sqrt{x}}$

I: $1-\sqrt{x} \rightarrow -\frac{1}{2\sqrt{x}}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{1-\sqrt{x}}} \cdot \frac{-1}{2\sqrt{x}} = \boxed{\frac{-1}{4\sqrt{x}(\sqrt{1-\sqrt{x}})}}$

h) $y = 3(\csc x)^{\frac{3}{2}}$

$\frac{dy}{dx} = \frac{9}{2}(\csc x)^{\frac{1}{2}}(-\csc x \cot x)$

$\boxed{= -\frac{9}{2}\csc^{\frac{3}{2}}x \cot x}$

i) $x = \tan y$

$\frac{dx}{dx} = 1$, so $1 = \sec^2 y \frac{dy}{dx}$

$\frac{1}{\sec^2 y} = \frac{dy}{dx} \rightarrow \boxed{\frac{dy}{dx} = \cos^2 y}$

j) $x + \tan(xy) = 0$

$1 + \sec^2(xy)(x\frac{dy}{dx} + y) = 0$

$x\sec^2(xy)\frac{dy}{dx} + y\sec^2(xy) = -1$

$\boxed{\frac{dy}{dx} = \frac{-1-y\sec^2(xy)}{x\sec^2(xy)}}$

k. $x^2 + y^2 = 1$

$2x + 2y\frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{-2x}{2y} = \boxed{-\frac{x}{y}}$

l. $y^2 = x^2 + 2x$

$2y\frac{dy}{dx} = 2x + 2$

$\frac{dy}{dx} = \frac{2x+2}{2y} \rightarrow \boxed{\frac{x+1}{y}}$

2. Find the equation of the lines that are a) tangent and b) normal to the curve at the given point.

a. $x^2 + xy - y^2 = 1$ at $(2,3)$.

$2x + x\frac{dy}{dx} + y - 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2x-y}{x-2y} = \frac{-2(2)-3}{2-2(3)} = \frac{-4-3}{2-6}$

$= \frac{-7}{-4} = \frac{7}{4}$

T: $y-3 = \frac{7}{4}(x-2)$

N: $y-3 = -\frac{4}{7}(x-2)$

b. $x^2y^2 = 9$ at $(-1,3)$.

$$2xy^2 + x^2 2y\frac{dy}{dx} = 0$$

$$2(-1)(9) + (-1)^2(6)\frac{dy}{dx} = 0$$

$$-18 + 6\frac{dy}{dx} = 0$$

$$6\frac{dy}{dx} = 18$$

$$\frac{dy}{dx} = 3$$

T: $y - 3 = 3(x+1)$

N: $y - 3 = -\frac{1}{3}(x+1)$

c. $6x^2 + 3xy + 2y^2 + 17y - 6 = 0$ at $(-2, 1)$.

$$12x + 3x\frac{dy}{dx} + 3y + 4y\frac{dy}{dx} + 17\frac{dy}{dx} = 0$$

$$-24 - 6\frac{dy}{dx} + 3 + 4\frac{dy}{dx} + 17\frac{dy}{dx} = 0$$

$$15\frac{dy}{dx} = 21$$

$$\frac{dy}{dx} = \frac{21}{15} = \frac{7}{5}$$

T: $y - 1 = \frac{7}{5}(x+2)$

N: $y - 1 = \frac{-5}{7}(x+2)$

d. $2xy + \pi\sin y = 0$ at $(0, \pi)$.

$$2x\frac{dy}{dx} + 2y + \pi\cos y\frac{dy}{dx} = 0$$

$$0 + 2\pi + \pi\underbrace{\cos\pi}_{-1}\frac{dy}{dx} = 0$$

$$2\pi - \pi\frac{dy}{dx} = 0$$

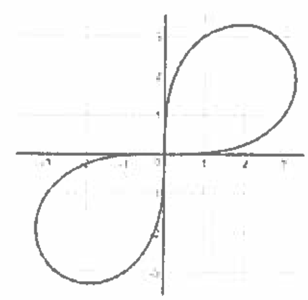
$$-\pi\frac{dy}{dx} = -2\pi$$

$$\frac{dy}{dx} = 2$$

T: $y - \pi = 2(x-0)$

N: $y - \pi = -\frac{1}{2}(x-0)$

3. Determine the slope of the graph of $3(x^2 + y^2)^2 = 100xy$ at the point $(3,1)$.



$$6(x^2+y^2)(2x + 2y\frac{dy}{dx}) = 100x\frac{dy}{dx} + 100y$$

$$6(9+1)(6 + 2\frac{dy}{dx}) = 300\frac{dy}{dx} + 100$$

$$60(6 + 2\frac{dy}{dx}) = 300\frac{dy}{dx} + 100$$

$$360 + 120\frac{dy}{dx} = 300\frac{dy}{dx} + 100$$

$$260 = 180\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{260}{180} = \frac{26}{18} = \boxed{\frac{13}{9}}$$ ✓