AB Calculus Integrals as Net Change Practice **Name:** ______________________________

1. a) Integrating velocity gives displacement (change in position).

   b) Integrating the absolute value of velocity (speed) gives distance traveled.

   c) New Position = old position + displacement.

   $f(a) + \int_a^b f'(x)\,dx = f(b)$

2. (Calculator) A particle starts at $x = 0$ and moves along the x-axis so that its velocity at time t is given by

$$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right)$$

a) Find the acceleration of the particle at time $t = 2$.

$a(t) = v'(t)$ $\quad a(2) = 1.588$

b) Find all times t in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.

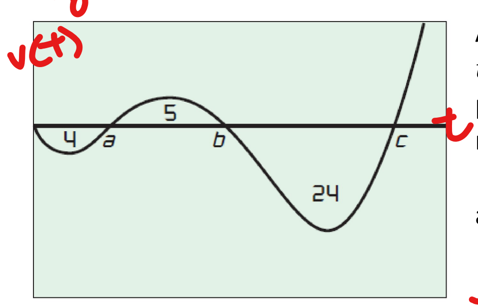
$v(t) = 0$ $\quad t = 2.507$ $\quad v(t)$ changes sign $-$ to $+$

c) Find the total distance traveled by the particle from time $t = 0$ to time $t = 3$.

$\int_0^3 |v(t)|\,dt = 4.334$

d) During the time interval $0 \le t \le 3$, what is the greatest distance between the particle and the origin? Show the work that leads to your answer.

$\int_0^{2.507} v(t)\,dt = -3.265$ $\quad$ greatest distance $= 3.265$

3. 

A particle moves along the x-axis (units in cm). Its initial position at $t = 0$ seconds is $x(0) = 15$. The figure shows the graph of the particle's velocity $v(t)$. The numbers are the areas of the enclosed regions.

a) What is the particle's displacement between $t = 0$, and $t = c$?

$\int_0^c v(t)\,dt = -4 + 5 - 24 = -23$ cm

b) What is the total distance traveled by the particle in the same time period as part a?

$\int_0^c |v(t)|\,dt = 4 + 5 + 24 = 33$ cm

c) Give the positions of the particle at times a, b, and c.

$x(a) = x(0) + \int_0^a v(t)\,dt \Rightarrow x(a) = 15 + (-4) = 11$ / $x(b) = 16$ / $x(c) = -8$

d) Approximately where does the particle achieve its greatest positive acceleration on the interval $[0, b]$?

at "a"

e) Approximately where does the particle achieve its greatest positive acceleration on the interval $[0, c]$?

at "c"

4. The rate at which people enter an amusement park on a given day is modeled by the function $E(t)$ defined by

$$E(t) = \frac{15600}{t^2 - 24t + 160}$$

The rate at which people leave the same amusement park is modeled by the function $L(t)$ defined by

$$L(t) = \frac{9890}{t^2 - 38t + 370}$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$. There are no people in the park.

a) How many people have entered the park by 5:00 pm ($t = 17$)? Round your answer to the nearest whole number.

$\int_9^{17} E(t)\,dt = 6004$ people

b) The price of admission to the park is \$15 until 5:00 pm ($t = 17$). After 5:00 pm, the price of admission to the park is \$11. How much money is collected by the park on the given day? Round to the nearest dollar.

$\int_{17}^{23} E(t)\,dt = 1271$ $15 \times 6004 + 11 \times 1271$
$= \$104,041$

c) Let $H(t) = \int_9^t \big(E(x) - L(x)\big)dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$ and explain the meaning of $H(17)$ and $H'(17)$ in the context of the park.

$H'(t) = E(t) - L(t)$
$H'(17) = E(17) - L(17)$
$= -380\ 281$

$H(17) = 3,725 \rightarrow$ the # of people in the park at 5pm

$H'(17) = -380\ 281$ means the number of people in the park is decreasing (at a rate of) 380 281 people/hr at 5pm

d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum? Show the work that leads to your answer.

$H'(t) = 0$
$0 = E(t) - L(t)$
$E(t) = L(t)$

at $t = 15.795$