The twice-differentiable function f is defined for all real numbers and satisfies the following conditions:

$$f(0) = 2,\ f'(0) = -4, \text{ and } f''(0) = 3.$$

(a) The function g is given by $g(x) = e^{ax} + f(x)$ for all real numbers, where a is a constant. Find $g'(0)$ and $g''(0)$ in terms of a. Show the work that leads to your answers.

(b) The function h is given by $h(x) = \cos(kx) f(x)$ for all real numbers, where k is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x = 0$.

(a) $g'(x) = ae^{ax} + f'(x)$

$g'(0) = ae^{a(0)} + f'(0)$

$= a - 4$

$g''(x) = a^2e^{ax} + f''(x)$

$g''(0) = a^2 + f''(0)$

$= a^2 + 3$

(b) $\boxed{h'(x) = -\sin(kx)k \cdot f(x) + \cos(kx) \cdot f'(x)}$

tangentline of h at $x = 0$

$\boxed{h'(x),\ x,\ h(x)}$

$h(0) = \cos(k\,0)\,f(0)$

$= 1\,2$

$= 2$

$h'(0) = 0 + 1 f'(0)$

$= -4$

$\boxed{y - 2 = -4(x - 0)}$

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table above.

(a) Find the average acceleration of rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$.

(c) Rocket B is launched upward with an acceleration of $a(t) = \frac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

(a) $\frac{v(80)-v(0)}{80-0} = \frac{49-5}{80-0}$ ft/s/s

(b) The displacement of the rocket in feet from $t=10$ to $t=70$

$\frac{70-10}{3} = \frac{60}{3} = 20$ $\quad \int_{10}^{70} v(t)dt \approx 20(22+35+44)$ feet

(c) V_B? when a_B $\quad \int \frac{3}{\sqrt{t+1}} dt = \int 3(t+1)^{-\frac{1}{2}} dt$

$V_B(t) = 2 \cdot 3(t+1)^{\frac{1}{2}} + c$

$2 = 6(1)^{\frac{1}{2}} + c$

$c = -4$

$V_B(t) = 6(t+1)^{\frac{1}{2}} - 4$

$V_B(80) = 6(80+1)^{\frac{1}{2}} - 4$

$= 54 - 4 = 50$

$50 > V_A(80) = 49$ so Rocket B is faster at $t=80$.