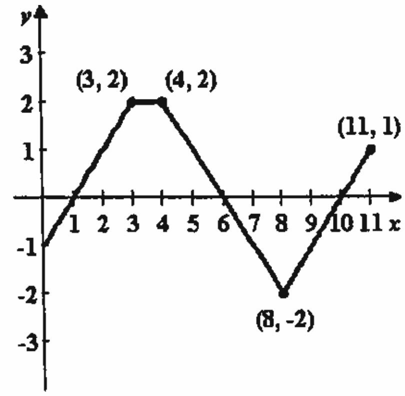
AP Calculus Name Key

1. The graph of f shown to the right consists of line segments.

   Evaluate $\int_0^8 f(x)\,dx$ using geometric formulas. $-.5 + 6 - 2$ [illegible]



   (A) 4.5 (B) 6.5

   (C) 3.5 (D) 2.5

   (E) 5.5

2. Given $\int_0^2 f(x)\,dx = -1$, $\int_1^2 f(x)\,dx = \frac{3}{2}$, $\int_2^5 f(x)\,dx = \frac{5}{2}$. What is the value of $\int_5^0 3f(x)\,dx$?

   (A) $-\frac{3}{2}$ (B) $\frac{3}{2}$ (C) $-\frac{9}{2}$

   (D) $\frac{9}{2}$ (E) -9

$\int_0^1 + \int_1^2 + \int_2^5 = \int_0^5$

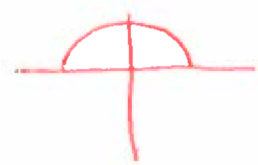
$\int_0^1 + \int_1^2 = \int_0^2$

$-2.5 + 1.5 = -1$

$-2.5 + 1.5 + 2.5 = 1.5$

$3(-1.5) = -4.5$

3. Find the area depicted by the definite integral $\int_{-3}^{3} \sqrt{9-x^2}\,dx$.



   (A) 9π (B) $\frac{9\pi}{4}$

   (C) $\frac{9}{2}$ (D) $\frac{9\pi}{2}$

   (E) $\frac{9}{8}\pi$

$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2}\pi(3)^2$

$= \frac{9\pi}{2}$

4. A car travels in a straight line for ~~one~~ several hours. Its velocity, v, in miles per hour is shown in the table. For each problem, approximate the distance the car traveled using the given method and number of rectangles/trapezoids, n.

Time (~~min~~ hr)	0	2	5	7	11
Velocity (mph)	1	5	8	9	11

a.) Left Rectangles, $n=4$

$2(1)+3(5)+2(8)+4(9)$

$2+15+16+36$

$17+52$ $\boxed{69 \text{ miles}}$

d) 79 miles

b.) Right Rectangles, $n=4$

$2(5)+3(8)+2(9)+4(11)$

$10+24+18+44$

$34+62$ 96 miles

c.) Trapezoids, $n=4$

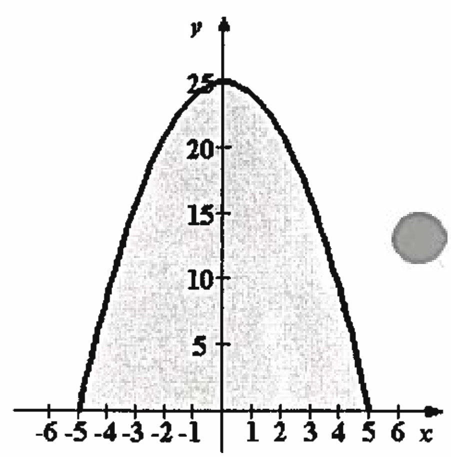
$\frac{1}{2}(2)(1+5)+\frac{1}{2}(3)(5+8)+\frac{1}{2}(2)(8+9)+\frac{1}{2}(4)(9+11)$ $\boxed{\text{or } \frac{165}{2}}$

$6+\frac{39}{2}+17+40 = 25.5+57 = \boxed{82.5 \text{ miles}}$

5. The graph of the function $f(x)=25-x^2$ is shown to the right. Which of the following definite integrals yields the area of the shaded region?

a. $\int_0^5 (25-x^2)\,dx$ **b.** $\int_{-25}^{25} (25-x^2)\,dx$ **c.** $\int_0^{25} (25-x^2)\,dx$

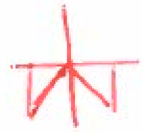
d. $2\int_0^5 (25-x^2)\,dx$ **e.** $2\int_{-5}^5 (25-x^2)\,dx$



6. Find the area depicted by the definite integral $\int_{-5}^{5}(5-|x|)\,dx$.

a. 36 b. 25 c. 75 d. 22 e. 129.5

$\int_{-5}^{5} 5\,dx$



$-\frac{25}{2} - \frac{25}{2} = -25$

$5x\Big|_{-5}^{5} = 25+25$
$= 50$

$50 - 25$

7. Given $\int_{2}^{3} x\,dx = \frac{5}{2}$. What is the value of $\int_{3}^{2} 12x\,dx$?

a. −12 b. −30 c. −76 d. 30 e. −195

$\int_{3}^{2} = -\frac{5}{2} \cdot 12 = -30$

8.

x	−4	−3	−2	−1
$f(x)$	0.75	−1.5	−2.25	−1.5
$f'(x)$	−3	−1.5	0	1.5

The table above gives values of a function f and its derivative at selected values of x. If f' is continuous on the interval $[-4, -1]$, what is the value of $\int_{-4}^{-1} f'(x)\,dx$? $f(-1) - f(-4)$

(A) −4.5 (B) −2.25 (C) 0 (D) 2.25 (E) 4.5

$-1.5 - .75 = -2.25$

9. Evaluate $\int_{0}^{1} x\sqrt{1-x^2}\,dx$

a. $\frac{1}{6}$ b. $\frac{1}{3}$ c. 1 d. $\frac{2}{3}$ e. $-\frac{1}{3}$

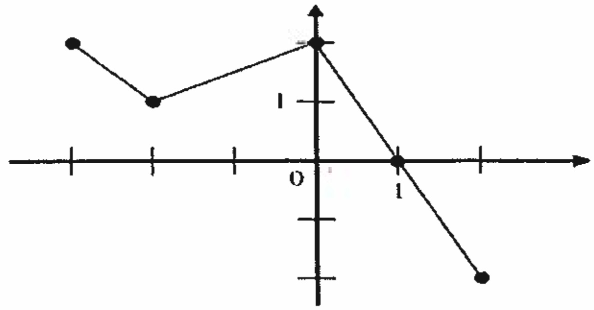
$u = 1 - x^2$ $du = -2x\,dx$ $\frac{-1}{2}\int_{1}^{0} \sqrt{u}\,du$

$u = 1 - 1 = 0$ $\frac{du}{-2} = x\,dx$ $-\frac{1}{2} \cdot \frac{2}{3} u^{\frac{3}{2}} \Big|_{1}^{0}$

$u = 1 - 0 = 1$

$-\frac{1}{3}(0 - 1) = \frac{1}{3}$

10.



Graph of f

The graph of the piecewise linear function f is shown in the figure above. If $g(x)=\int_{-2}^{x} f(t)\,dt$, which of the following values is greatest? $g'(x)=f(x)$

(A) $g(-3)$ $-$
(B) $g(-2)$ 0
(C) $g(0)$ $+$
(D) $g(1)$ big +
(E) $g(2)$ less +

11. Evaluate $\int \frac{\cos x}{\sin^3 x}\,dx$ $u=\sin x$ $du=\cos x\,dx$

a. $-\frac{(\cos x)^{-2}}{2}+C$

b. $\frac{(\sin x)^{-3}}{2}+C$

c. $-\frac{(\cos x)^{-2}}{3}+C$

d. $-\frac{(\sin x)^{-2}}{3}+C$

e. $-\frac{(\sin x)^{-2}}{2}+C$

$\int \frac{1}{u^3}\,du$ $\frac{u^{-2}}{-2}+C$ $\frac{(\sin x)^{-2}}{-2}+C$

12. Evaluate $\int_2^4 (4t+1)\,dt$

a. 8   b. 52   c. 46   d. 26   e. 2

$\frac{4t^2}{2}+t\Big|_2^4$ $\left(\frac{4(4)^2}{2}+4\right)-\left(2(2)^2+2\right)$

$36-10$

13. Evaluate $\int_0^1 \sqrt{x}\,dx$

a. $\frac{3}{2}$ b. $\frac{5}{4}$ c. $\frac{7}{6}$ d. $\frac{9}{5}$ e. $\frac{2}{3}$

$\frac{2}{3}x^{\frac{3}{2}}\Big|_0^1 = \frac{2}{3}(1-0)$

14. Evaluate $\int_0^{6\pi}(5\sin x+2\cos x)\,dx$

a. 1 b. 0 c. 2 d. −1 e. −2

$-5\cos x+2\sin x\Big|_0^{6\pi}$

$-5\cos 6\pi+2\sin 6\pi$ $\quad -5+0$

$+5\cos 0-2\sin 0$ $\quad +5-0=0$

15. Find the average value of the function $f(x)=30-6x^2$ over the interval $-2\le x\le 2$.

a. 46 b. 38 c. 22 d. 14 e. 6

$\dfrac{\int_{-2}^{2}(30-6x^2)\,dx}{4}=\dfrac{30x-2x^3\Big|_{-2}^{2}}{4}$ $\quad \dfrac{(60-16)-(-60+16)}{4}$

$\dfrac{120-32}{4}=$ $\quad 30-8=22$

16. Find $\dfrac{d}{dx}\left[\int_0^{x^2}\dfrac{1}{\sqrt{1+t^3}}\,dt\right]$. $\quad \dfrac{1}{\sqrt{1+x^6}}(2x)$

a. $\dfrac{1}{\sqrt{1+x^3}}$ b. $\dfrac{2x}{\sqrt{1+x^6}}$ c. $\dfrac{2x}{\sqrt{1+x^5}}$ d. $\dfrac{x^2}{\sqrt{1+x^6}}$ e. none of the above

17. Evaluate $\int_1^3\dfrac{1}{\sqrt{4x+1}}\,dx$. $\quad u=4x+1$, $du=4\,dx$ $\quad u=13$, $u=5$ $\quad \frac{1}{4}\int_5^{13}u^{-\frac{1}{2}}\,du$

a. $\dfrac{\sqrt{13}-\sqrt{5}}{4}$ b. $\dfrac{\sqrt{13}+\sqrt{5}}{4}$ c. $\dfrac{\sqrt{13}+\sqrt{5}}{2}$ d. $\dfrac{\sqrt{13}-\sqrt{5}}{2}$ e. $\dfrac{\sqrt{5}-\sqrt{13}}{2}$

$\frac{1}{4}\cdot 2\sqrt{u}\Big|_5^{13}$

18. Evaluate the integral $\int(2x^4+4x^3-2x)\,dx$. $\quad \frac{2}{5}x^5+x^4-x^2+C$

a. $8x^3+12x^2-2+C$ b. $2x^5+5x^4-5x^2+C$ c. $\frac{2}{5}x^5+x^4-x^2$ d. $\frac{2}{5}x^5+x^4-x^2+C$ e. none of these

$x^{1.5-.5}$

19. Evaluate the integral $\int \frac{7+\sqrt{x^3}}{\sqrt{x}}dx$.

a. $\frac{7}{2}\sqrt{x}+\frac{3}{2}x^2+C$ b. $14\sqrt{x}+\frac{3}{2}x^2+C$ c. $-\frac{7}{2}x^{-3/2}+3+C$ d. $\frac{7}{2}x^{-3/2}+\frac{3}{2}x^2+C$ **e. none of these**

$\int 7x^{-\frac{1}{2}}+x$

$14x^{\frac{1}{2}}+\frac{x^2}{2}+C$

20. Evaluate the integral $\int \frac{\sec^3\theta \tan\theta}{1+\tan^2\theta}d\theta$. *Don't forget Pythagorean Identities from Trig*

$\sin^2+\cos^2=1$ $\tan^2+1=\sec^2$

a. $\frac{1}{4}\sec^4\theta+C$ b. $\frac{1}{2}\sec^2\theta+C$ c. $\frac{1}{4}\sec^2\theta\tan^2\theta+C$ **d.** $\sec\theta+C$ e. none of these

$\int \frac{\sec^3\theta \tan\theta}{\sec^2\theta}=\int \sec\theta\tan\theta=\sec\theta+C$

21. Find $y=f(x)$ if $f''(x)=x+2$, $f'(0)=3$, $f(0)=-1$.

a. $y=\frac{1}{6}x^3+x^2+3x-1$ b. $y=\frac{x^3}{6}+2x^2+C$ c. $y=x^3+6x^2+18x-6$

d. $y=\frac{1}{6}x^3+x^2+\frac{21}{2}x+\frac{61}{6}+C$ e. none of these

$\int f''=f'=\frac{x^2}{2}+2x+C \rightarrow C=3$

$\int f'=f=\frac{x^3}{6}+x^2+3x+d$ $d\rightarrow -1$

22. Use $a(t)=-32$ ft/sec^2 as the acceleration due to gravity. A ball is thrown vertically upward from the ground with an initial velocity of 16 feet per second. How high will the ball go?

a. 32 feet b. 16 feet c. 2 feet **d. 4 feet** e. none of these

$\int a(t)dt=v(t)=-32t+C \rightarrow C=16$

$v(t)=-32t+16=0$ $t=1/2$

$s(t)=-16t^2+16t+C$

$-16(\frac{1}{2})^2+16(\frac{1}{2})=-4+8=4$

23.

$\int \frac{x}{x^2-4}\,dx =$

$u = x^2 - 4$

$\frac{du}{2} = x\,dx$

(A) $\frac{-1}{4\left(x^2-4\right)^2}+C$

(B) $\frac{1}{2\left(x^2-4\right)}+C$

$\frac{1}{2}\int \frac{1}{u}\,du$

(C) $\frac{1}{2}\ln\left|x^2-4\right|+C$

(D) $2\ln\left|x^2-4\right|+C$

$\frac{1}{2}\ln|u| + c$

(E) $\frac{1}{2}\arctan\left(\frac{x}{2}\right)+C$

$\frac{1}{2}\ln|x^2-4| + c$

24.

If $\int_{-5}^{2} f(x)\,dx = -17$ and $\int_{5}^{2} f(x)\,dx = -4$, what is the value of $\int_{-5}^{5} f(x)\,dx$?

(A) -21 (B) -13 (C) 0 (D) 13 (E) 21

$\int_{-5}^{2} + \int_{2}^{5} = \int_{-5}^{5}$ $-17 + 4 = -13$

25.

If $G(x)$ is an antiderivative for $f(x)$ and $G(2) = -7$, then $G(4) =$

(A) $f'(4)$

$G(x) = \int f(x)$

(B) $-7 + f'(4)$

$\int_2^4 f(x) = G(4) - G(2)$

(C) $\int_2^4 f(t)\,dt$

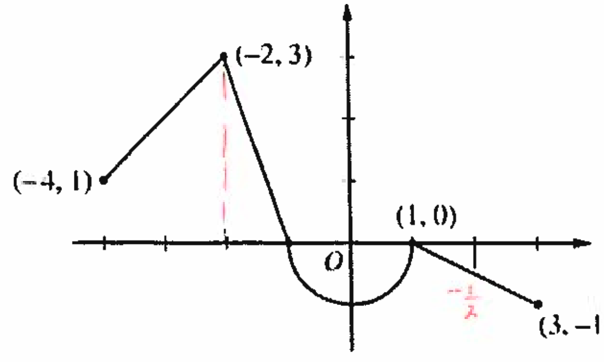
$\int_2^4 f(x) = G(4) + 7$

(D) $\int_2^4 (-7 + f(t))\,dt$

$\int_2^4 f(x) - 7 = G(4)$

(E) $-7 + \int_2^4 f(t)\,dt$

Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt$.



Graph of f

(a) Find the values of $g(2)$ and $g(-2)$.

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

$g'(x) = f(x)$

a) $g(2) = \int_1^2 f(t)\,dt = \frac{1}{2}(1)\left(-\frac{1}{2}\right) = -\frac{1}{4}$

$g(-2) = \int_{+1}^{-2} f(t)\,dt = +\frac{1}{2}(\pi)(1)^2 - \frac{1}{2}(3)(1)$

$= \frac{\pi}{2} - \frac{3}{2}$

b.) $g'(-3) = f(-3) = 2$

$g''(-3) = f'(-3) = 1$

c.) $g'(x) = 0 = f(x)$

$x = -1$
rel max
f + to −

$x = 1$
neither
f − to −

d) $g''(x)$ changes signs
$f'(x)$ changes

$x = -2, 0, 1$
f' changes signs