Integrals 1 Test Review

1. What is the average value of y for the part of the curve $y = 3x - x^2$, which is the *first quadrant*?

(A) −6

(B) −2

(C) $\frac{3}{2}$ ✓

(D) $\frac{9}{4}$

(E) $\frac{9}{2}$

---

2. If the function *f* given by $f(x) = x^3$ has an average value of 9 on the closed interval [0, *k*], then *k* =

(A) 3

(B) $3^{\frac{1}{2}}$

(C) $18^{\frac{1}{3}}$

(D) $36^{\frac{1}{4}}$

(E) $36^{\frac{1}{3}}$ ✓

---

3. The average (mean) value of $\sqrt{x}$ over the interval $0 \leq x \leq 2$ is

Integrals 1 Test Review

(A) $\frac{1}{3}\sqrt{2}$

(B) $\frac{1}{2}\sqrt{2}$

(C) $\frac{2}{3}\sqrt{2}$ ✔

(D) 1

(E) $\frac{4}{3}\sqrt{2}$

---

4. The average value of 1/x on the closed interval [1,3] is

(A) 12

(B) 23

(C) ln2/2

(D) ln3/2 ✔

(E) ln3

---

5. $\frac{d}{dx}\left(\int_0^{x^3} \ln\left(t^2+1\right)\,dt\right) =$

Integrals 1 Test Review

(A) $\frac{2x^3}{x^6+1}$

(B) $\frac{3x^2}{x^6+1}$

(C) $\ln\ (x^6+1)$

(D) $2x^3\ \ln\ (x^6+1)$

(E) $3x^2\ \ln\ (x^6+1)$ ✓

---

6. For all $x > 1$, if $f(x) = \int_t^x \frac{1}{t}dt$, then $f'(x)$ =

(A) 1

(B) $\frac{1}{x}$ ✓

(C) ln x − 1

(D) ln x

(E) e^x

---

7. Let g be a function with first derivative given by $g'\ (x) = \int_0^x e^{-t^2}\ dt$. Which of the following must be true on the interval $0 < x < 2$?

Integrals 1 Test Review

(A) *g* is increasing, and the graph of *g* is concave up. ✔

(B) *g* is increasing, and the graph of *g* is concave down.

(C) *g* is decreasing, and the graph of *g* is concave up.

(D) *g* is decreasing, and the graph of *g* is concave down.

(E) *g* is decreasing, and the graph of *g* has a point of inflection on $0 < x < 2$.

---

8. $\frac{d}{dx}\left(\int_0^{x^2} \sin\left(t^3\right)\, dt\right) =$

(A) $-\cos\left(x^6\right)$

(B) $\sin\left(x^3\right)$

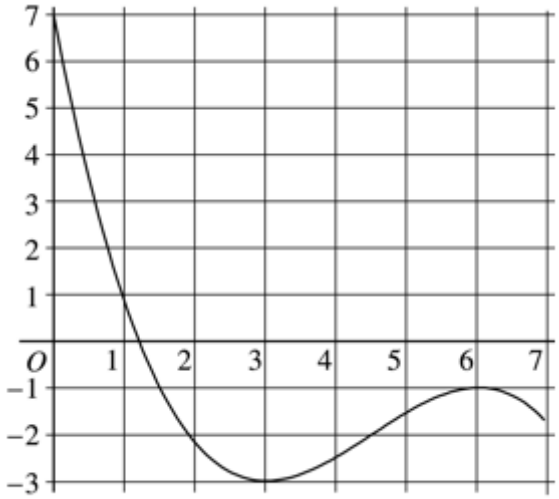
(C) $\sin\left(x^6\right)$

(D) $2x\sin\left(x^3\right)$

(E) $2x\sin\left(x^6\right)$ ✔

Integrals 1 Test Review

9.



Graph of f

The graph of the function f shown in the figure above has horizontal tangents at $x = 3$ and $x = 6$. If $g(x) = \int_0^{2x} f(t)\,dt$, what is the value of $g'(3)$?

(A) 0

(B) -1

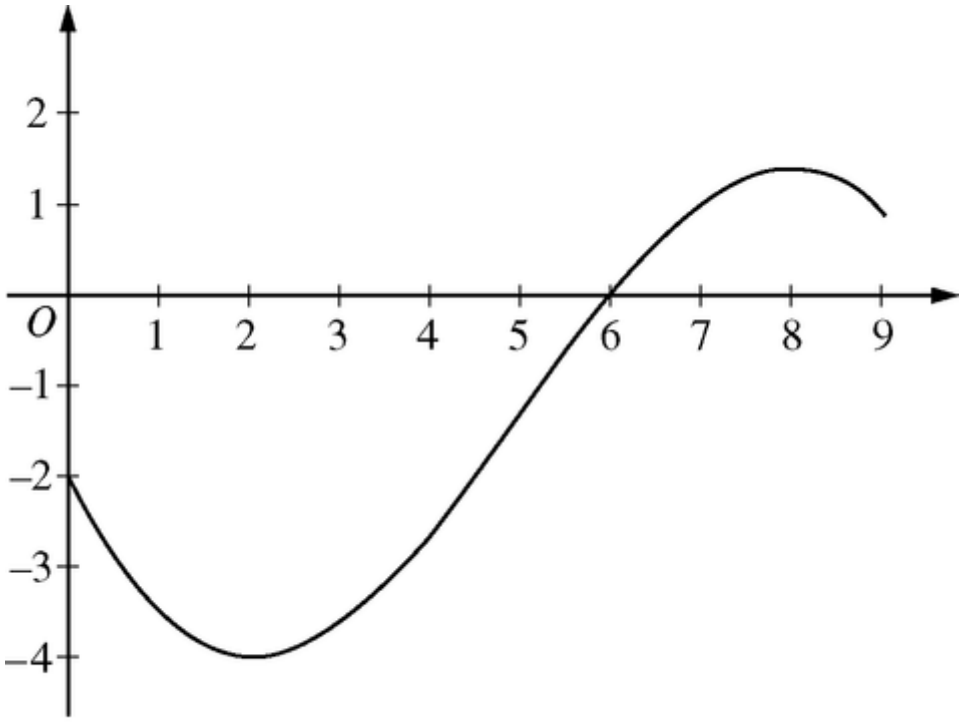
(C) -2 ✓

(D) -3

(E) -6

Integrals 1 Test Review

10.



Graph of f

The graph of a differentiable function f is shown above. If $h(x) = \int_0^x f(t)\ dt$, which of the following is true?

(A) $h(6) < h'(6) < h''(6)$ ✓

(B) $h(6) < h''(6) < h'(6)$

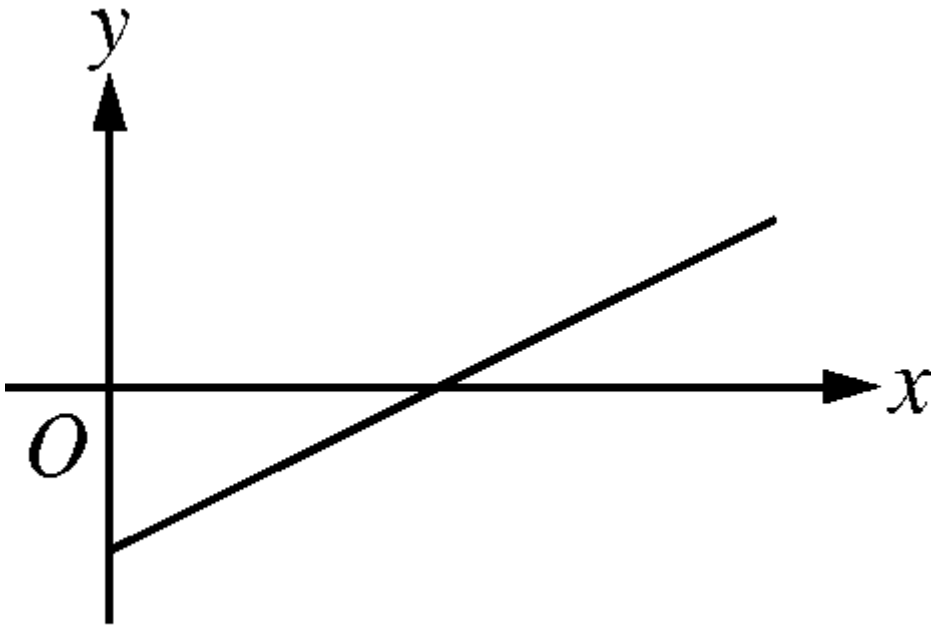
(C) $h'(6) < h(6) < h''(6)$

(D) $h''(6) < h(6) < h'(6)$

(E) $h''(6) < h'(6) < h(6)$

Integrals 1 Test Review

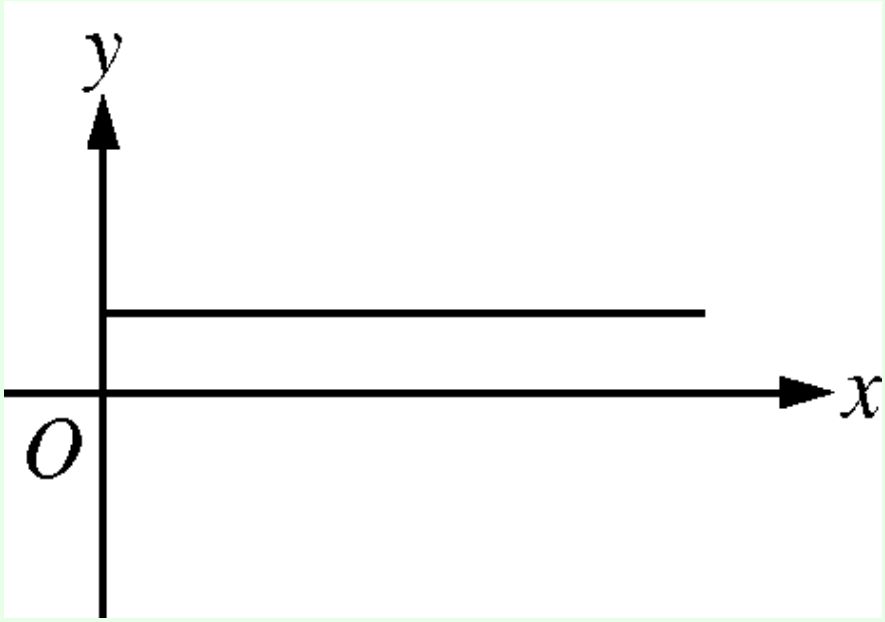
11.



Graph of f

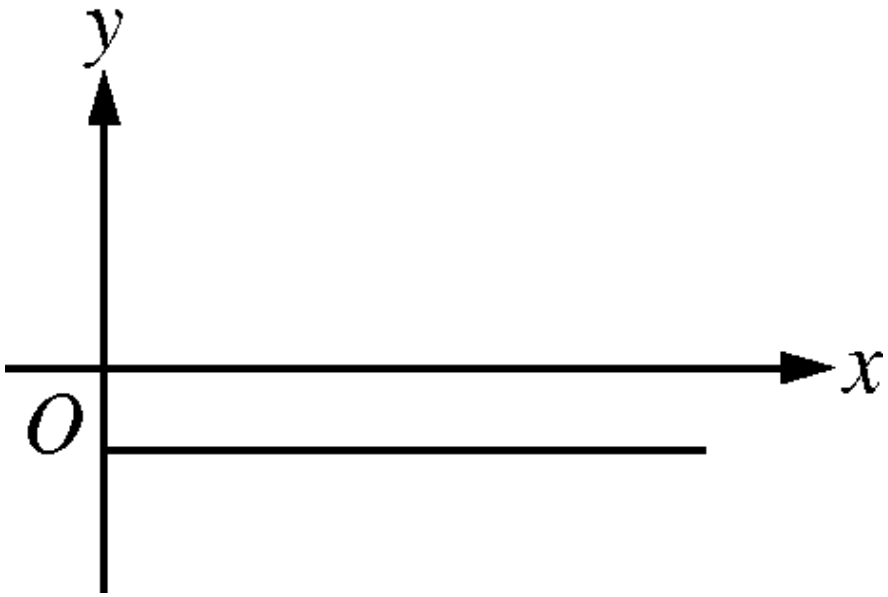
The figure above shows the graph of *f*. If $f\left(x\right)=\int_{2}^{x}g\left(t\right)\ dt$, which of the following could be the graph of $y=g\left(x\right)$?

A

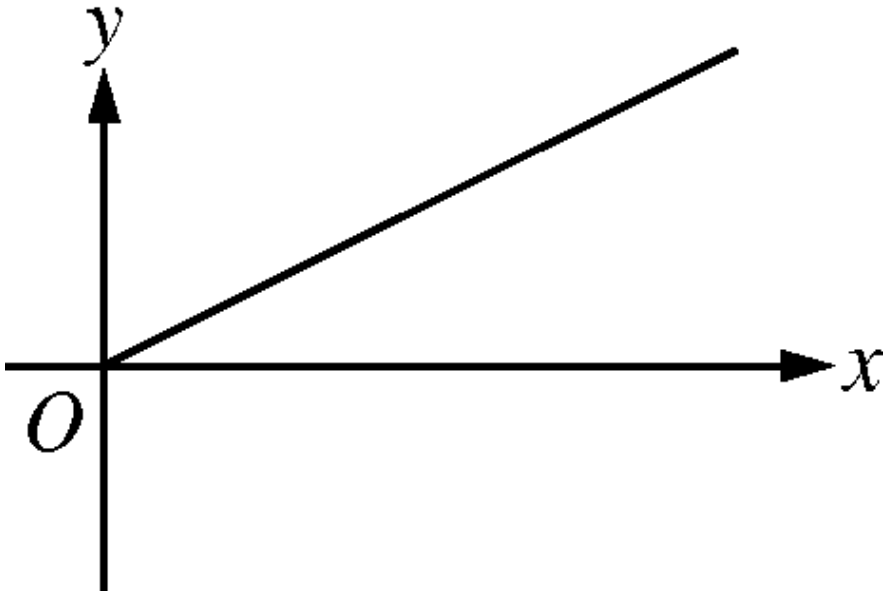


Integrals 1 Test Review

B

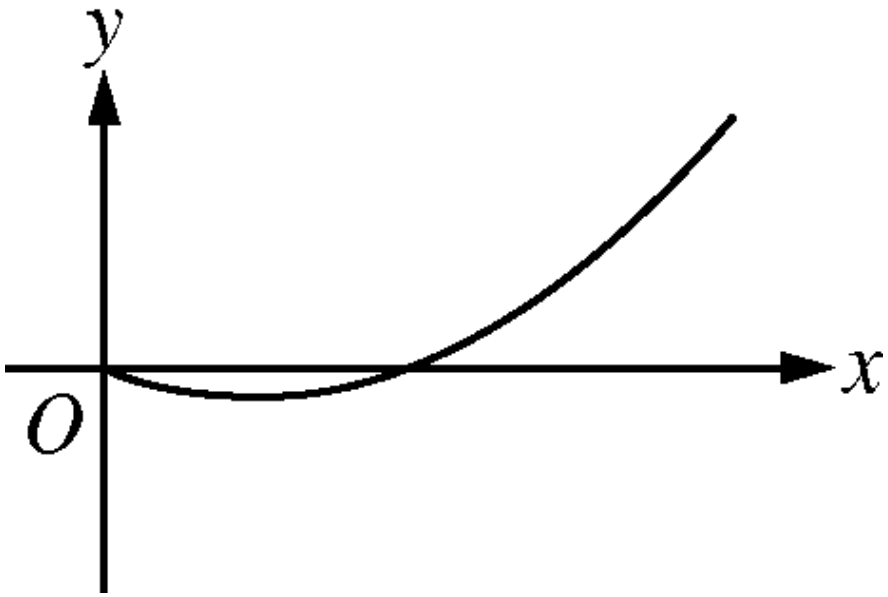


C

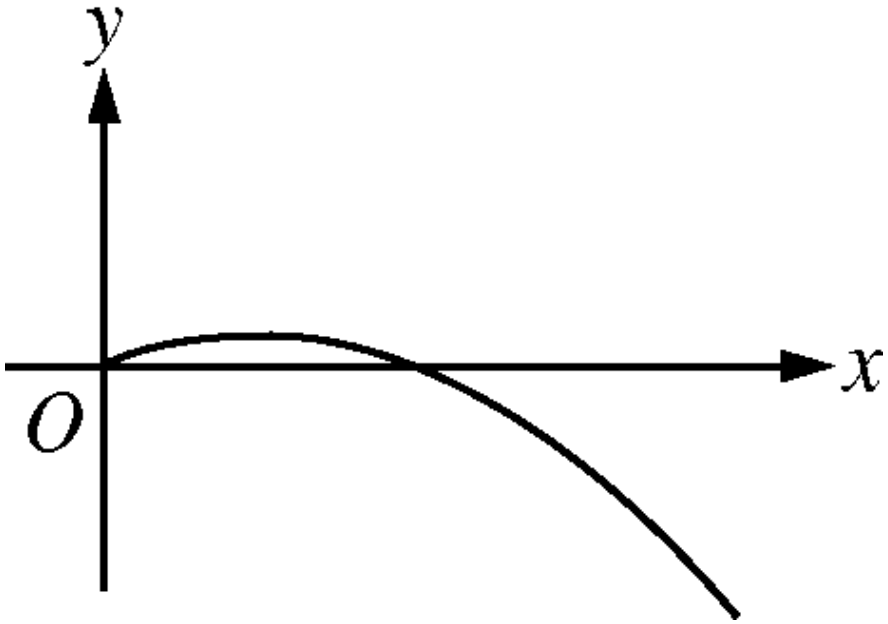


Integrals 1 Test Review

D



E



12. $\int_1^4 |x-3|dx =$

Integrals 1 Test Review

(A) $-\frac{3}{2}$

(B) $\frac{3}{2}$

(C) $\frac{5}{2}$ ✓

(D) $\frac{9}{2}$

(E) 5

---

13. If $\int_1^{10} f(x)dx = 4$ and $\int_{10}^{3} f(x)dx = 7$, then $\int_1^3 f(x)dx =$

(A) −3

(B) 0

(C) 3

(D) 10

(E) 11 ✓

---

14. The function *f* is defined by $f(x) = \begin{cases} 2 & \text{for } x < 3 \\ x - 1 & \text{for } x \geq 3. \end{cases}$ What is the value of $\int_1^5 f(x)\ dx$?

Integrals 1 Test Review

(A) 2

(B) 6

(C) 8

(D) 10 ✔

(E) 12

---

15. Given $f(x) = \begin{cases} x+1 & \text{for } x < 0 \\ \cos \pi & \text{for } x \geq 0 \end{cases}$ $\int_{-1}^{1} f(x)dx =$

(A) $\frac{1}{2} + \frac{1}{\pi}$

(B) $-\frac{1}{2}$

(C) $\frac{1}{2} - \frac{1}{\pi}$

(D) $\frac{1}{2}$ ✔

(E) $-\frac{1}{2} + \pi$

---

16. If f is a linear function and $0<a<b$, then $\int_{a}^{b} f''(x)dx =$

Integrals 1 Test Review

(A) 0 ✓

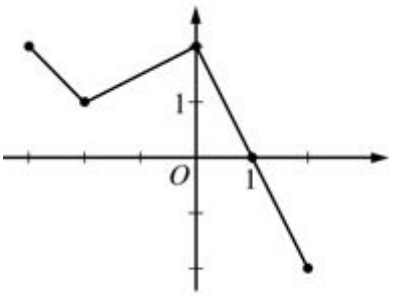
(B) 1

(C) $\frac{ab}{2}$

(D) b-a

(E) $\frac{b^2-a^2}{2}$

17.



Graph of f

The graph of the piecewise linear function f is shown in the figure above. If $g(x) = \int_{-2}^{x} f(t)\ dt$, which of the following values is greatest?

(A) $g(-3)$

(B) g(−2)

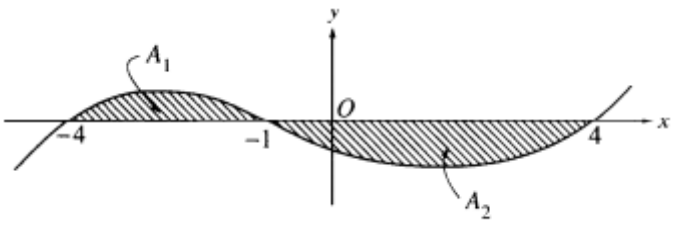
(C) g(0)

(D) g(1) ✓

(E) g(2)

Integrals 1 Test Review

18.



The graph of *y=f(x)* is shown in the figure above. If A_1 and A_2 are positive numbers that represent the areas of the shaded regions, then in terms of A_1 and A_2,

$$\int_{-4}^{4} f(x)dx - 2\int_{-1}^{4} f(x)dx =$$

(A) A_1

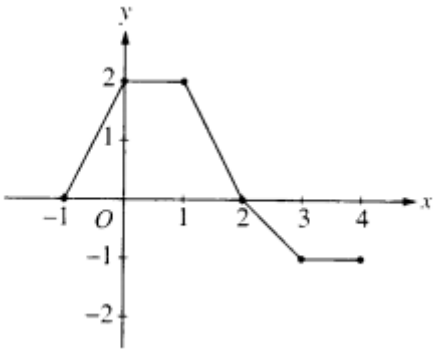
(B) A_1-A_2

(C) $2A_1$-A_2

(D) A_1+A_2 ✔

(E) A_1+$2A_2$

19.



The graph of a piecewise-linear function *f*, for $-1 \le x \le 4$, is shown above. What is the value of $\int_{-1}^{4} f(x)dx$?

Integrals 1 Test Review

(A) 1

(B) 2.5 ✓

(C) 4

(D) 5.5

(E) 8

---

20. If $\int_0^k (2kx - x^2)dx = 18$, then $k =$

(A) −9

(B) −3

(C) 3 ✓

(D) 9

(E) 18

---

21. $\int_0^1 (3x-2)^2 dx =$

Integrals 1 Test Review

(A) $-\frac{7}{3}$

(B) $-\frac{7}{9}$

(C) $\frac{1}{9}$

(D) 1 ✓

(E) 3

---

22. $\int_0^{\frac{\pi}{4}} \sin x \, dx =$

(A) $-\frac{\sqrt{2}}{2}$

(B) $\frac{\sqrt{2}}{2}$

(C) $-\frac{\sqrt{2}}{2} - 1$

(D) $-\frac{\sqrt{2}}{2} + 1$ ✓

(E) $\frac{\sqrt{2}}{2} - 1$

---

23. $\int_1^2 \frac{x-4}{x^2} dx$

Integrals 1 Test Review

(A) $-\frac{1}{2}$

(B) ln2-2 ✓

(C) ln2

(D) 2

(E) ln2+2

---

24. $\int_0^1 \sqrt{x}(x+1)dx =$

(A) 0

(B) 1

(C) $\frac{16}{15}$ ✓

(D) $\frac{7}{5}$

(E) 2

---

25. What are all values of *k* for which $\int_{-3}^{k} x^2 dx = 0$?

Integrals 1 Test Review

(A) -3 ✓

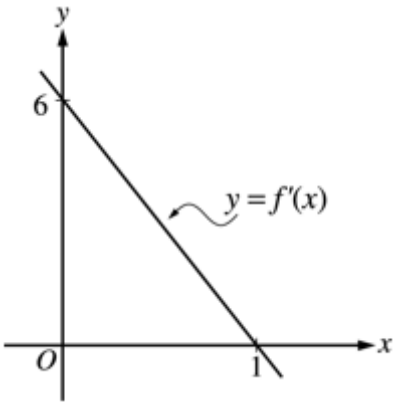
(B) 0

(C) 3

(D) -3 and 3

(E) -3, 0, 3

26.



The graph of f', the derivative of f, is the line shown in the figure above. If $f(0) = 5$, then $f(1) =$

(A) 0

(B) 3

(C) 6

(D) 8 ✓

(E) 11

Integrals 1 Test Review

27. $\int \sec^2 x\,dx =$

(A) $\tan x + c$ ✓

(B) $\csc^2 x + c$

(C) $\cos^2 x + c$

(D) $\frac{\sec^3 x}{3} + c$

(E) $2\sec^2 x \tan x + c$

---

28. If the second derivative of *f* is given by $f''(x) = 2x - \cos x$, which of the following could be f(*x*)?

(A) $\frac{x^3}{3} + \cos x - x + 1$ ✓

(B) $\frac{x^3}{3} - \cos x - x + 1$

(C) $x^3 + \cos x - x + 1$

(D) $x^2 - sinx + 1$

(E) $x^2 + \sin x + 1$

---

29. $\int_1^e \frac{x^2 + 1}{x}\,dx =$

Integrals 1 Test Review

A $\frac{e^2-1}{2}$

B $\frac{e^2+1}{2}$ ✓

C $\frac{e^2+2}{2}$

D $\frac{e^2-1}{e^2}$

E $\frac{2e^2-8e+6}{3e}$

30.

x	2	3	5	8	13
$f(x)$	6	–2	–1	3	9

The function *f* is continuous on the closed interval [2,13] and has values as shown in the table above. Using the intervals [2,3], [3,5], [5,8], and[8,13] what is the approximation of $\int_2^{13} f(x)\ dx$ obtained from a left Riemann sum?

A 6

B 14 ✓

C 28

D 32

E 50

Integrals 1 Test Review

31.

t (hours)	4	7	12	15
$R(t)$ (liters/hour)	6.5	6.2	5.9	5.6

A tank contains 50 liters of oil at time $t = 4$ hours. Oil is being pumped into the tank at a rate $R(t)$, where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are given in the table above. Using a right Riemann sum with three subintervals and data from the table, what is the approximation of the number of liters of oil that are in the tank at time $t = 15$ hours?

(A) 64.9

(B) 68.2

(C) 114.9 ✔

(D) 116.6

(E) 118.2

32.

x	2	5	10	14
$f(x)$	12	28	34	30

The function f is continuous on the closed interval [2,14] and has values as shown in the table above. Using the subintervals [2,5], [5,10], and [10,14], what is the approximation of $\int_2^{14} f(x)\,dx$ found by using a right Riemann sum?

Integrals 1 Test Review

(A) 296

(B) 312

(C) 343

(D) 374 ✔

(E) 390

---

33. If the average value of a continuous function *f* on the interval [−2, 4] is 12, what is $\int_{-2}^{4} \frac{f(x)}{8} dx$?

(A) $\frac{3}{2}$

(B) 3

(C) 9 ✔

(D) 72

---

34. Let *f* be a differentiable function such that $f(0) = -5$ and $f'(x) \leq 3$ for all *x*. Of the following, which is not a possible value for *f*(2)?

Integrals 1 Test Review

(A) -10

(B) -5

(C) 0

(D) 1

(E) 2 ✔

---

35. Which of the following is an equation for the line tangent to the graph of $y = 3 - \int_{-1}^{x} e^{-t^3}\,dt$ at the point where x = -1 ?

(A) y - 3 = -3e(x + 1)

(B) y - 3 = -e(x + 1) ✔

(C) y - 3 = 0

(D) y - 3 = -1/e(x + 1)

(E) y - 3 = 3e(x + 1)

---

36. Let *g* be the function defined by $g(x) = \int_{-1}^{x} \frac{t^3 - t^2 - 6t}{\sqrt{t^2 + 7}}\,dt$. On which of the following intervals is *g* decreasing?

Integrals 1 Test Review

(A) $x \leq -2 \, and \, 0 \leq x \leq 3$ ✓

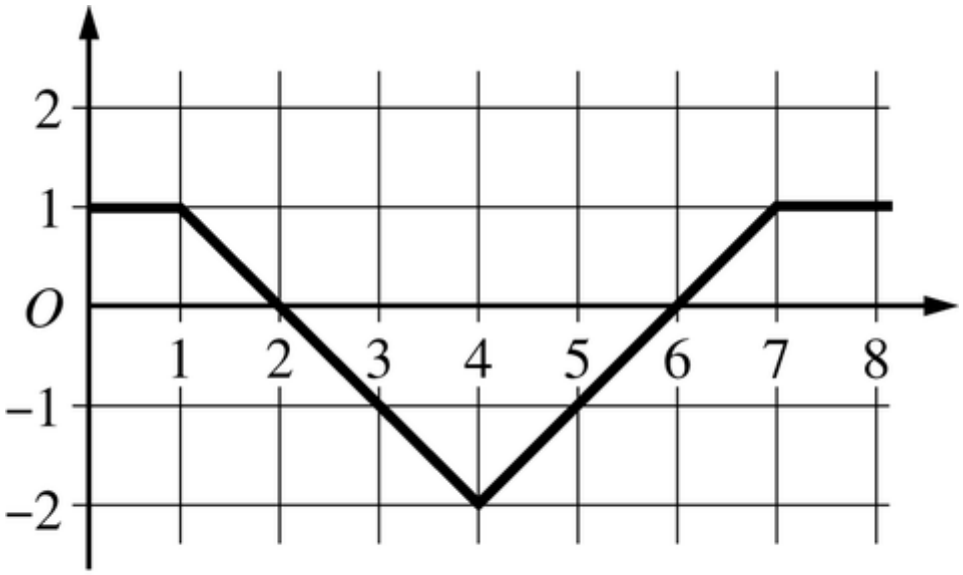
(B) $x \leq -2 \, and \, x \geq 3$

(C) $-2 \leq x \leq 0 \, and \, x \geq 3$

(D) $-2 \leq x \leq 3$

(E) $x \leq -1$

37.



Graph of f

The graph of the function *f* in the figure above consists of four line segments. Let *g* be the function defined by $g(x) = \int_0^x f(t) \, dt$. Which of the following is an equation of the line tangent to the graph of *g* at $x = 5$?

Integrals 1 Test Review

(A) $y+1=x-5$

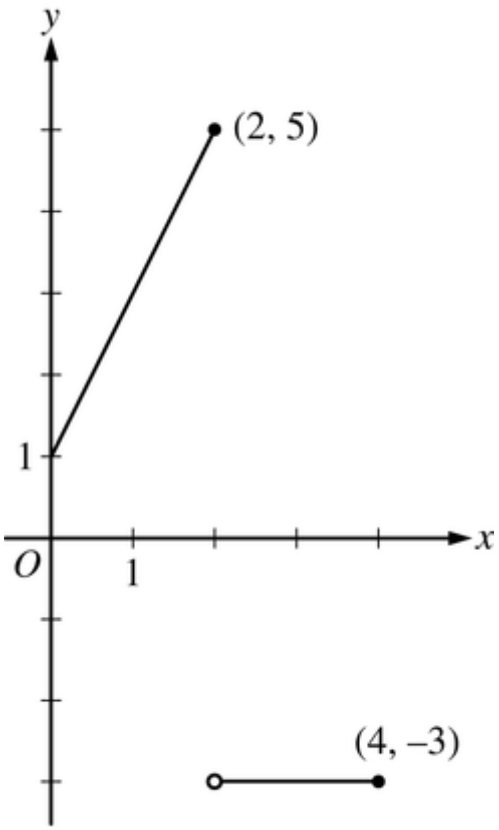
(B) $y-2=x-5$

(C) $y-2=-1(x-5)$

(D) $y+2=x-5$

(E) $y+2=-1(x-5)$ ✓

38.



Graph of f

The graph of f is shown above for $0 \le x \le 4$. What is the value of $\int_0^4 f(x)\,dx$?

Integrals 1 Test Review

(A) -1

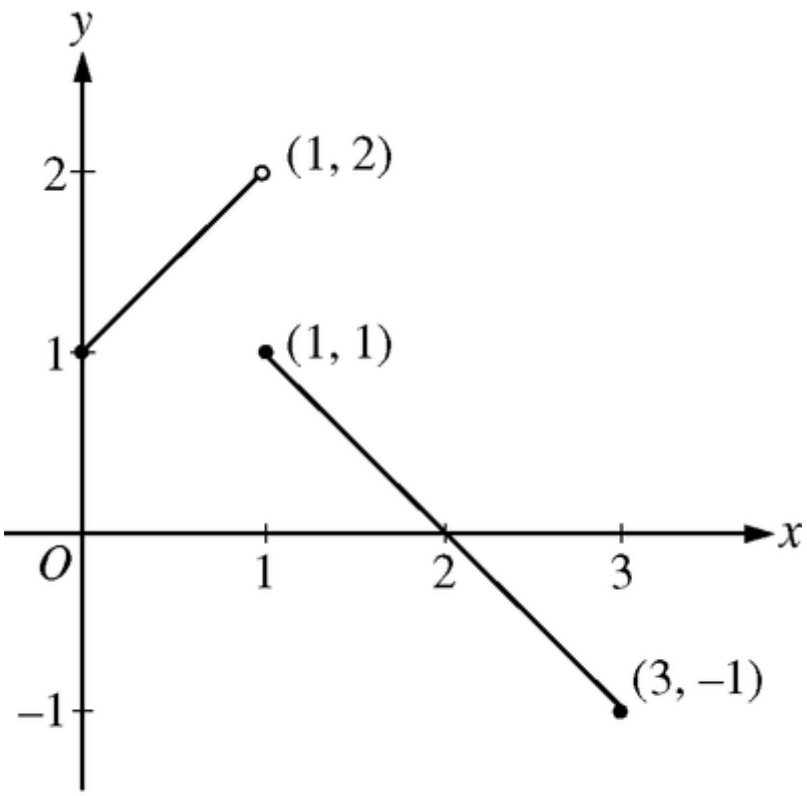
(B) 0 ✓

(C) 2

(D) 6

(E) 12

39.



Graph of f

The graph of the function f consists of two line segments, as shown in the figure above. The value of $\int_0^3 |f(x)|\,dx$ is

Integrals 1 Test Review

(A) $-\frac{3}{2}$

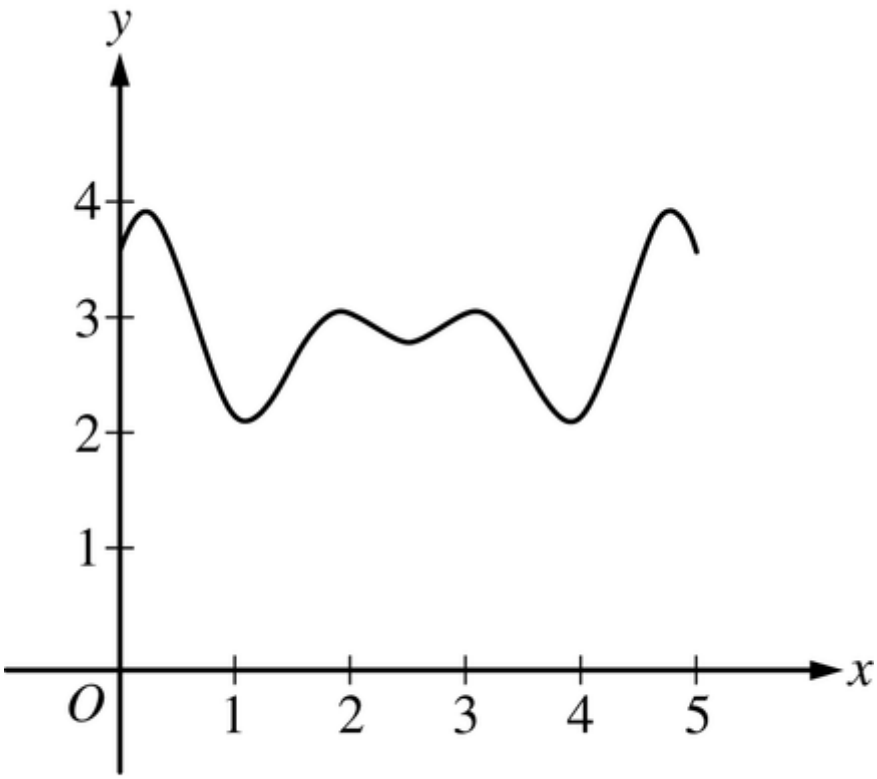
(B) $\frac{1}{2}$

(C) $\frac{3}{2}$

(D) $\frac{5}{2}$ ✔

(E) nonexistent

40.



Graph of f'

The graph of f', the derivative of f, is shown in the figure above. If $f(0) = 20$, which of the following could be the value of $f(5)$?

Integrals 1 Test Review

(A) 15

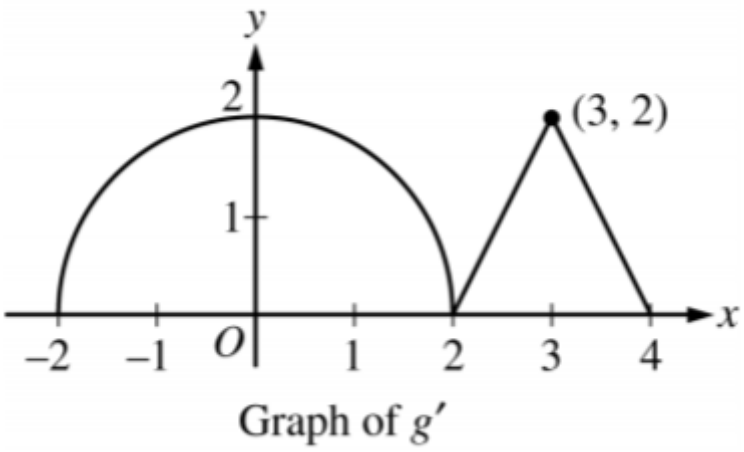
(B) 20

(C) 25

(D) 35 ✔

(E) 40

41.



Graph of g'

The graph of g', the first derivative of the function g, consists of a semicircle of radius 2 and two line segments, as shown in the figure above. If $g(0) = 1$, what is $g(3)$?

(A) $\pi + 1$

(B) $\pi + 2$ ✔

(C) $2\pi + 1$

(D) $2\pi + 2$

Integrals 1 Test Review

42. $\int_{-2}^{1} (8x^3 - 3x^2)\, dx =$

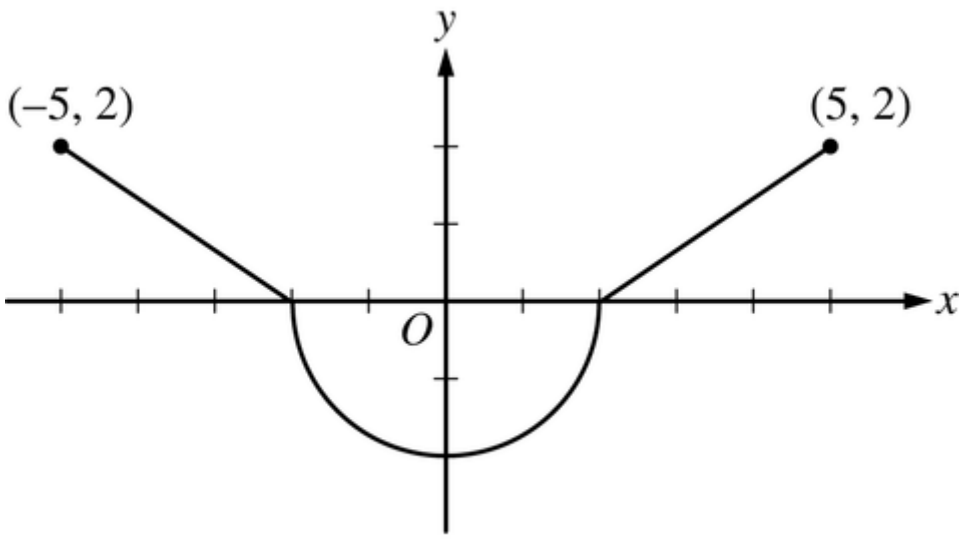
(A) -561

(B) -90

(C) -39 ✓

(D) 81

43.



Graph of f'

The graph of f', the derivative of a function *f*, consists of two line segments and a semicircle, as shown in the figure above. If $f(2) = 1$, then $f(-5) =$

Integrals 1 Test Review

(A) $2\pi - 2$ ✓

(B) $2\pi - 3$

(C) $2\pi - 5$

(D) $6 - 2\pi$

(E) $4 - 2\pi$

---

44. $\int (e^x + e)\, dx =$

(A) $e^x + C$

(B) $2e^x + C$

(C) $e^x + e + C$

(D) $e^{x+1} + ex + C$

(E) $e^x + ex + C$ ✓

---

45. $\int 2^x\, dx =$

Integrals 1 Test Review

(A) $2^x + C$

(B) $(ln\ 2)\ 2^x + C$

(C) $\frac{2^x}{\ln 2} + C$ ✓

(D) $\frac{2^{x+1}}{x+1} + C$

46.

x	0	2	4	6
$f(x)$	-22	-6	2	2
$f'(x)$	10	6	2	-2

Selected values of the twice-differentiable function f and its derivative f' are given in the table above. What is the value of $\int_0^6 f'(x)\,dx$?

(A) -12

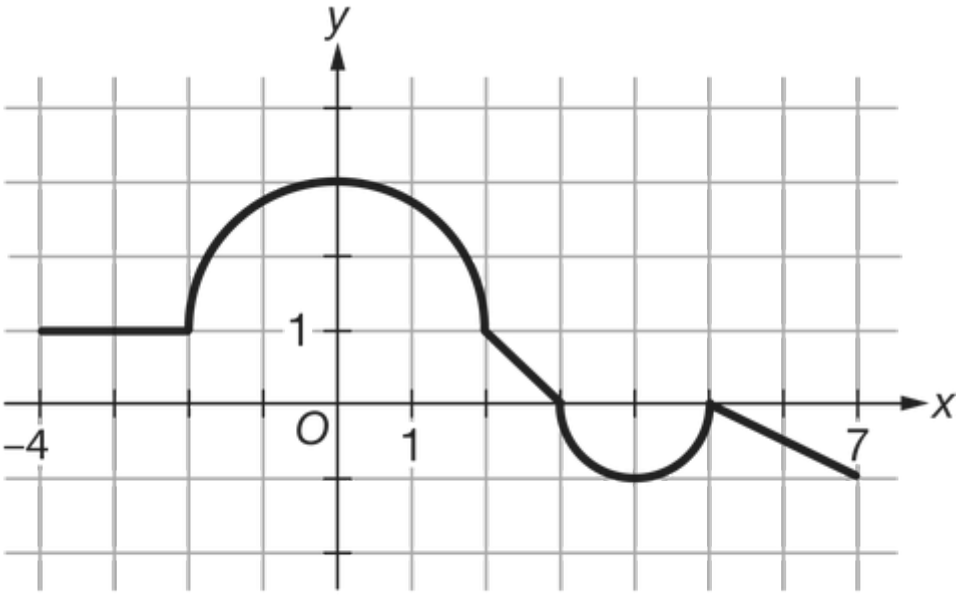
(B) 12

(C) 24 ✓

(D) 36

Integrals 1 Test Review

47.



Graph of f

The graph of the function f on the interval $-4 \leq x \leq 7$ consists of three line segments and two semicircles, as shown in the figure above. What is the value of $\int_{-4}^{7} f(x)\,dx$?

(A) $\frac{3}{2}\pi + \frac{3}{2}$

(B) $\frac{3}{2}\pi + \frac{11}{2}$ ✓

(C) $\frac{5}{2}\pi + \frac{7}{2}$

(D) $\frac{5}{2}\pi + \frac{15}{2}$

---

48. If $\int_{-1}^{3} (2g(x) + 4)\,dx = 22$ and $\int_{10}^{-1} g(x)\,dx = 12$, then $\int_{3}^{10} g(x)\,dx =$

Integrals 1 Test Review

(A) -21

(B) -15 ✓

(C) -9

(D) 9

49.

x	0	1	2	3	4	5	6
$f(x)$	0	5	2	−1	−2	0	3

The function f is continuous on the closed interval [0, 6] and has values as shown in the table above. Using the intervals [0,2], [2,4], and [4,6], what is the approximation of $\int_0^6 f(x)dx$ obtained from a midpoint Riemann sum?

(A) 0

(B) 3

(C) 4

(D) 6

(E) 8 ✓

50. The average value of a function f over the interval $[-1, 2]$ is -4, and the average value of f over the interval $[2, 7]$ is 8. What is the average value of f over the interval $[-1, 7]$?

Integrals 1 Test Review

(A) $\frac{1}{2}$

(B) 2

(C) $\frac{7}{2}$ ✓

(D) 14

---

51. Let f be the function given by $f(x) = \int_{10}^{x} \left(-t^2 + 2t + 3\right) dt$. On what intervals is f increasing?

(A) $(-\infty, 1]$

(B) $[-1, 3]$ ✓

(C) $[1, \infty)$

(D) $(-\infty, -1]$ and $[3, \infty)$

---

52. Which of the following is a left Riemann sum approximation of $\int_{1}^{7} (4 \ln x + 2)\, dx$ with n subintervals of equal length?

Integrals 1 Test Review

(A) $\sum_{k=1}^{n}\left(4\ln\left(1+\frac{k-1}{n}\right)+2\right)\frac{1}{n}$

(B) $\sum_{k=1}^{n}\left(4\ln\left(\frac{6k}{n}\right)+2\right)\frac{6}{n}$

(C) $\sum_{k=1}^{n}\left(4\ln\left(1+\frac{6(k-1)}{n}\right)+2\right)\frac{6}{n}$ ✓

(D) $\sum_{k=1}^{n}\left(4\ln\left(1+\frac{6k}{n}\right)+2\right)\frac{6}{n}$

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53. Which of the following is a left Riemann sum approximation of $\int_{2}^{8}\cos\left(x^2\right)\,dx$ with n subintervals of equal length?

(A) $\sum_{k=1}^{n}\left(\cos\left(2+\frac{k-1}{n}\right)^2\right)\frac{1}{n}$

(B) $\sum_{k=1}^{n}\left(\cos\left(\frac{6k}{n}\right)^2\right)\frac{6}{n}$

(C) $\sum_{k=1}^{n}\left(\cos\left(2+\frac{6(k-1)}{n}\right)^2\right)\frac{6}{n}$ ✓

(D) $\sum_{k=1}^{n}\left(\cos\left(2+\frac{6k}{n}\right)^2\right)\frac{6}{n}$
