$\int (uv)' = \int u\,dv + \int v\,du \rightarrow uv = \int u\,dv + \int v\,du \rightarrow uv - \int v\,du = \int u\,dv$

BC Calculus Integration by Parts Day 1 Homework Name: Key

1. Evaluate the following integrals.

a) $\int x \sin x\,dx$

$u = x \quad dv = \int \sin x\,dx$

$du = dx \quad v = -\cos x$

$\int u\,dv = uv - \int v\,du$

$= x(-\cos x) - \int -\cos x\,dx$

$\boxed{\rightarrow -x\cos x + \sin x + C}$

b) $\int x e^{9x}\,dx$

$u = x \quad \int dv = \int e^{9x}\,dx$

$du = dx \quad v = \frac{1}{9}e^{9x}$

$\int u\,dv = uv - \int v\,du$

$\int x e^{9x}\,dx = x\left(\frac{1}{9}e^{9x}\right) - \int \frac{1}{9}e^{9x}\,dx$

$\boxed{= \frac{1}{9}xe^{9x} - \frac{1}{81}e^{9x} + C}$

c) $\int x^3 \sin(3x)\,dx$

$u = x^3 \quad dv = \sin(3x)\,dx$

$du = 3x^2\,dx \quad v = -\frac{1}{3}\cos(3x)$

$\int x^3 \sin(3x)\,dx = -\frac{1}{3}x^3\cos(3x) - \int -\frac{1}{3}\cos(3x) \cdot 3x^2\,dx$

so again → look on next page for solution

d) $\int x^2 e^{x/2}\,dx$

next page

e) $\int \arccos x\,dx$

next page

f) $\int \frac{x^2 + 3x}{x}\,dx$

$\int (x + 3)\,dx$

$\boxed{= \frac{x^2}{2} + 3x + C}$

g) $\int \frac{\sec^2(\ln x)}{x}\,dx$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{1}{x}\,dx$

$\int \sec^2(u)\,du$

$= \tan u + C \rightarrow \boxed{\tan(\ln x) + C}$

h) $\int \frac{x}{x^2 + 5}\,dx$

$u = x^2 + 5$

$\frac{du}{dx} = 2x$

$\frac{1}{2}du = x\,dx$

$\frac{1}{2}\int \frac{1}{u}\,du \rightarrow \frac{1}{2}\ln|u| + C$

$\boxed{\rightarrow \frac{1}{2}\ln|x^2 + 5| + C}$

i) $\int \frac{1}{x\sqrt{x}}\,dx \rightarrow \int \frac{1}{x^{\frac{3}{2}}}\,dx \rightarrow \int x^{-\frac{3}{2}}\,dx$

$-2x^{-\frac{1}{2}} + C = \boxed{\frac{-2}{\sqrt{x}} + C}$

j) $\int e^x \sec(e^x)\tan(e^x)\,dx$

$u = e^x \quad du = e^x\,dx$

$\int \sec u \tan u\,du$

$= \sec u + C \rightarrow \boxed{\sec(e^x) + C}$

1.)

c) $\int x^3 \sin(3x)\, dx$

$u = x^3$ $\quad +$ $\quad dv = \sin(3x)\,dx$

$3x^2$ $\quad -$ $\quad -\frac{1}{3}\cos 3x$

$6x$ $\quad +$ $\quad -\frac{1}{9}\sin 3x$

6 $\quad -$ $\quad \frac{1}{27}\cos 3x$

$\frac{1}{81}\sin 3x$

$$\boxed{\int x^3 \sin(3x)\,dx = -\frac{1}{3}x^3\cos(3x) + \frac{1}{3}x^2\sin(3x) + \frac{2}{9}x\cos(3x) - \frac{2}{27}\sin(3x) + C}$$

d.) $\int x^2 e^{x/2}\, dx$

$u = x^2$ $\quad +$ $\quad dv = e^{\frac{x}{2}}\,dx$

$2x$ $\quad -$ $\quad 2e^{\frac{x}{2}}$

2 $\quad +$ $\quad 4e^{\frac{x}{2}}$

$8e^{\frac{x}{2}}$

$$\boxed{\int x^2 e^{\frac{x}{2}}\,dx = 2x^2e^{\frac{x}{2}} - 8xe^{\frac{x}{2}} + 16e^{\frac{x}{2}} + C}$$

e.) $\int \arccos x\, dx$

$\int u\,dv = uv - \int v\,du$

$\int \cos^{-1}x\,dx = x\cos^{-1}x + \int \frac{+x}{\sqrt{1-x^2}}\,dx$

↳ u-substitution

$u = \cos^{-1}x$ $\quad dv = dx$

$du = \frac{-1}{\sqrt{1-x^2}}\,dx$ $\quad v = x$

New u-sub

$u = 1 - x^2$

$\frac{du}{dx} = -2x$

↳ $-\frac{1}{2}du = x\,dx$

$-\frac{1}{2}\int \frac{1}{\sqrt{u}}\,du$

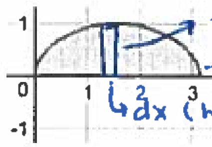
↳ $u^{-\frac{1}{2}}$

$-\frac{1}{2}\cdot 2u^{\frac{1}{2}} + C$

Final answer:

$$\boxed{x\cos^{-1}x - \sqrt{1-x^2} + C}$$

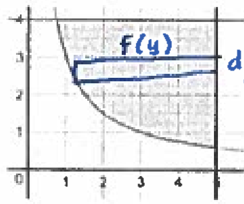
2. Find the volume of the solid formed by revolving the region in the first quadrant bounded by the graph of $f(x) = \sqrt{\sin x}$ about the x-axis. $V = \pi r^2 h \rightarrow V = \pi \int f(x)^2 dx$



$V = \pi \int_0^{\pi} (\sqrt{\sin x})^2 dx \rightarrow \pi \int_0^{\pi} \sin x \, dx$

$\rightarrow -\pi \cos x \Big|_0^{\pi} \rightarrow -\pi\cos\pi + \pi\cos 0 \rightarrow \pi + \pi = 2\pi$

3. Find the volume of the solid formed by revolving the region bounded by the graphs of $xy = 3$, $y = 4$, and $x = 5$ about the line $x = 5$.



$V = \pi \int_{3/5}^{4} \left(5 - \frac{3}{y}\right)^2 dy$

↳ intersects w/ $x=5$

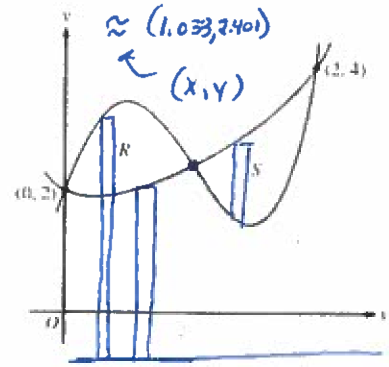
$5 \cdot y = 3 \rightarrow y = 3/5$

$xy = 3 \quad x = \frac{3}{y}$

↳ calculate w/ calculator

(3 decimals) → $\boxed{128.291}$

4. Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2-2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure to the right.



a) Find the sum of the areas of regions R and S.

Calculate intersection w/ Calculator

$\text{Area} = \int_0^x (g(x) - f(x))\,dx + \int_x^2 (f(x) - g(x))\,dx$

$\boxed{\approx \cancel{1.949}}$ $\boxed{\approx 2.004 \text{ better}}$

b) Region S is the base of a solid whose cross sections perpendicular to the x-axis are squares. Find the volume of the solid. $V = \int_a^b (\text{area})\,dx$ Area square = base^2

$V = \int_x^2 (f(x) - g(x))^2 dx$ $\boxed{\approx \cancel{1.292}}$ $\boxed{\approx 1.283 \text{ better}}$

c) Find the volume of the solid generated by revolving the region R about the line $y = -1$.

$V = \pi \int_0^x \left((g(x) - (-1))^2 - (f(x) - (-1))^2 \right) dx$ $\boxed{\approx \cancel{21.918}}$ $\boxed{\approx 22.523 \text{ better}}$

5. Evaluate the following limits.

a) $\lim_{x\to\infty} \frac{3}{x} \rightarrow \boxed{0}$

b) $\lim_{x\to\infty} \frac{3x^2 - 2x}{x} \boxed{\rightarrow \infty}$

c) $\lim_{x\to\infty} \frac{3x^2 - 2x^4 + x}{3x^4 - 3x^2 + 2} \rightarrow \boxed{-\frac{2}{3}}$

d) $\lim_{x\to\infty} 2e^{-x} \rightarrow \frac{2}{e^x} \rightarrow \boxed{0}$