BC Calculus Integration by Parts Day 2 Homework Name: Key

1. Evaluate the following integrals.

a) $\int 3x^3 e^{2x}\,dx$

$u = 3x^3$ $dv = e^{2x}dx$

$9x^2$ (−) $\frac{1}{2}e^{2x}$

$18x$ (+) $\frac{1}{4}e^{2x}$

18 (−) $\frac{1}{8}e^{2x}$

Ran out of space → 0 (+) $\frac{1}{16}e^{2x}$

$\rightarrow \frac{3}{2}x^3e^{2x} - \frac{9}{4}x^2e^{2x} + \frac{9}{4}xe^{2x} - \frac{9}{8}e^{2x} + C$

b) $\int \frac{\sin x}{\sqrt{\cos x}}\,dx$ $u = \cos x$ $\frac{du}{dx} = -\sin x$

$-du = \sin x\,dx$

$\rightarrow -\int \frac{1}{\sqrt{u}}\,du \rightarrow -\int u^{-\frac{1}{2}}\,du$

$\rightarrow -2u^{\frac{1}{2}} + C$

$\rightarrow -2\sqrt{\cos x} + C$

c) $\int \frac{2x^2+3x-2}{x}\,dx$

$\rightarrow \int \left(2x + 3 - 2\cdot\frac{1}{x}\right)dx$

$\rightarrow x^2 + 3x - 2\ln|x| + C$

d) $\int \frac{5}{1+x^2}\,dx \rightarrow 5\int \frac{1}{1+x^2}\,dx$

$\rightarrow 5\tan^{-1}x + C$

e) $\int \frac{5x}{1+x^2}\,dx$ $u = 1+x^2$

$\frac{du}{dx} = 2x$

$\frac{1}{2}du = x\,dx$

$\frac{5}{2}\int \frac{1}{u}\,du$

$\rightarrow \frac{5}{2}\ln|u| + C \rightarrow \frac{5}{2}\ln|1+x^2| + C$

f) $\int \frac{\ln x}{x^3}\,dx$ $u = \ln x$ $dv = \frac{1}{x^3}dx \rightarrow x^{-3}dx$

$du = \frac{1}{x}dx$ $v = -\frac{1}{2}x^{-2}$

$\rightarrow -\frac{1}{2}x^{-2}\cdot\ln x + \int +\frac{1}{2}x^{-2}\cdot\frac{1}{x} \rightarrow x^{-1}\,dx$ $\frac{1}{2}\int x^{-3}$

$\rightarrow -\frac{1}{2}x^{-2}\cdot\ln x - \frac{1}{4}x^{-2} + C$

g) $\int \sin x\, 3^{\cos x}\,dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$-du = \sin x\,dx$

$-\int 3^u\,du$

$\rightarrow -\frac{3^{\cos x}}{\ln 3} + C$

h) $\int \frac{1}{x^2+4x+13}\,dx$

Completing the Square

$(x^2+4x+4) + 13 - 4$

$\int \left((x+2)^2 + 9\right)dx$

$u = x+2$ $\rightarrow \int (u^2+9)\,du$

$du = dx$

$\rightarrow \int \frac{1}{9\left(\frac{1}{9}u^2+1\right)}\,du \rightarrow \frac{1}{9}\int \frac{1}{\left(\frac{u}{3}\right)^2+1}\,du$

$\rightarrow w = \frac{u}{3} \rightarrow \frac{dw}{du} = \frac{1}{3} \rightarrow 3dw = du$

$\rightarrow 3\cdot\frac{1}{9}\int \frac{dw}{w^2+1} \rightarrow \frac{1}{3}\tan^{-1}(w) + C \rightarrow \frac{1}{3}\tan^{-1}\left(\frac{u}{3}\right) + C$

$\frac{1}{3}\tan^{-1}\left(\frac{x+2}{3}\right) + C$

i) $\int e^x \sin x\,dx$

$u = e^x$ $dv = \sin x\,dx$

$du = e^x dx$ $v = -\cos x$

new: $u = e^x$ $du = e^x dx$ $dv = \cos x\,dx$ $v = \sin x$

$\rightarrow -e^x\cos x + \int e^x\cos x\,dx$ $\sin x e^x - \int e^x \sin x\,dx$

$\rightarrow -e^x\cos x + \sin x e^x - \int e^x\sin x\,dx$

$= \int e^x \sin x\,dx$

$\rightarrow \frac{1}{2}(e^x\cos x + \sin x e^x) + C = \int e^x \sin x\,dx$

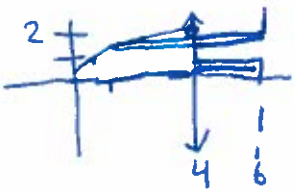
or $x^2-1 = x+1 \rightarrow x^2 - x - 2 \rightarrow (x-2)(x+1)$

2. Find the volume of the solid whose base is bound by the graphs of $y = x^2 - 1$ and $y = x + 1$ whose cross sections taken perpendicular to the x-axis are isosceles right triangles with a leg in the base region.

$$V = \int_{-1}^{2} \frac{1}{2}(x+1-(x^2-1))^2\,dx \approx 4.05$$

$\frac{1}{2}bh \rightarrow b=h \rightarrow \frac{1}{2}b^2$

3. Find the volume of the solid generated by revolving the region bounded by the graphs of $y = \sqrt{x}$, $y = 0$, and $x = 4$ about the line $x = 6$.



$$V = \pi\int_0^2 \left((6-y^2)^2 - (6-4)^2\right)dy \quad y^2 = x$$

≈ 120.637

4. An object moves along a straight line. For $0 \le t \le 5$, the velocity of the object is given by $v(t) = -2 + (t^2 + 3t)^{6/5} - t^3$, and the position of the object is given by $s(t)$. It is known that $s(0) = 10$.

a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the object is 2.

$|v(t)| = 2$ ↳ $|v(t)|$

↳ graph & calculate intersections

$t \approx 3.128, 3.473$

b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the object at time $t = 5$.

$$s(t) = 10 + \int_0^t v(x)\,dx \rightarrow s(5) = 10 + \int_0^5 v(x)\,dx$$

≈ -9.207

c) Find all times t in the interval $0 \le t \le 5$ at which the object changes direction. Justify your answer.

$v(t) = 0$ ↳ calculator

$t \approx 3.318, .536$ b/c $v(t)$ changes sign

d) Is the speed of the object increasing or decreasing at time $t = 4$? Give a reason for your answer.

Speeding up b/c $v(t) < 0$ & $a(t) < 0$

*Remember when looking @ a velocity graph, objects slow down when moving towards x-axis and speed up when moving away.