AP Calculus Vectors in the Plane Day 1 Homework **Name:** Key

1. Find *dy/dx* and d^2y/dx^2 in terms of t for the parametric function $x = t + 1,\ \ y = t^2 + 3t$.

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t+3}{1} \rightarrow 2t+3$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}(2t+3)}{1} \rightarrow 2$$

2. Find the points where the tangent to the curve $x = 1 - t,\ \ y = t^3 - 3t$ is horizontal and vertical.

$\frac{dx}{dt} = -1$

$\frac{dy}{dt} = 3t^2 - 3$

H: $3t^2 - 3 = 0 \quad t = \pm 1$

$x = 1-1 \quad y = 1^3 - 3(1) \rightarrow (0,-2)$ H.T

$x = 1+1 \quad y = -1+3 \rightarrow (2,2)$

V: $-1 \neq 0$ No V.T.

3. Find the length of the curve $x = \sqrt{t},\ \ y = 3t - 1 \quad 0 \le t \le 1$.

$$L = \int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt \approx$$

$\frac{dx}{dt} = \frac{1}{2\sqrt{t}} \qquad \frac{dy}{dt} = 3.249$

4. Let $u = \langle 3, -2 \rangle$ and $v = \langle -2, 5 \rangle$. For each of the following, find the component form of the vector and the magnitude of the vector.

a) -2v $= \langle 4, -10 \rangle$

b) 2u – 3v $= \langle 6, -4 \rangle + \langle 6, -15 \rangle = \langle 12, -19 \rangle$

c) $u \cdot v$ $\quad 3 \cdot -2 + -2 \cdot 5 = -6 - 10 = -16$

5. Find the sum of the vectors $\overline{AB}$ and $\overline{CD}$ in component form where A = (1, -1), B = (2, 0), C = (-1, 3), and D = (-2, 2).

$\overrightarrow{AB} = \langle 1, 1 \rangle$

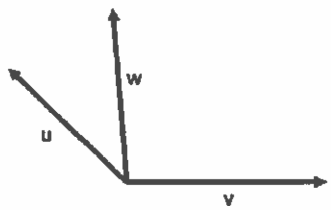
$\overrightarrow{CD} = \langle -1, -1 \rangle$

$\overrightarrow{AB} + \overrightarrow{CD} = \langle 0, 0 \rangle$

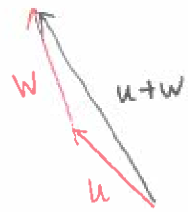
6. Find the component form of the unit vector that makes and angle $\theta = \frac{2\pi}{3}$ with the positive x-axis.

$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ on unit circle $\quad v \rightarrow \left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$

7. Sketch the indicated vector using the vectors below.

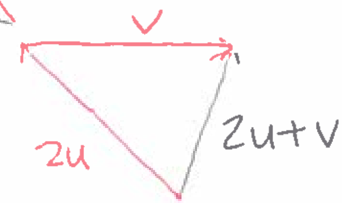


a) **u + w**



b) **u − v**

c) 2u + v



8. Find the unit vectors that are tangent and normal to the curve $x = \ln(t-1)$, $y = t - 1$ where t = 3.

$x'(t) = \frac{1}{t-1}(1) = \frac{1}{t-1} \qquad y'(t) = 1$

$x'(3) = \frac{1}{2} \qquad T: \left\langle \frac{1}{2}, 1 \right\rangle$

$y'(3) = 1 \qquad \sqrt{\left(\frac{1}{2}\right)^2 + 1^2} \rightarrow \sqrt{\frac{1}{4} + \frac{4}{4}} \rightarrow \frac{\sqrt{5}}{2}$

unit vector tan $\left\langle \frac{\frac{1}{2}}{\frac{\sqrt{5}}{2}}, \frac{1}{\frac{\sqrt{5}}{2}} \right\rangle$

$\rightarrow \left\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$

$\stackrel{?}{=} \left\langle -\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}} \right\rangle$

Normal: $\left\langle \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}} \right\rangle$ or $\left\langle -\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right\rangle$

$\theta = 70°$

9. An airplane, flying in the direction 20° east of north at 325 mph in still air, encounters a 40 mph tailwind acting in the direction 40° west of north. The airplane maintains its compass heading but, because of the wind, acquires a new ground speed and direction. What are they?

$\theta = 130°$

Airplane $\langle 325\cos 70°, 325\sin 70° \rangle$

Tailwind $\langle 40\cos 130°, 40\sin 130° \rangle$

New: $\langle 85.445, 336.042 \rangle \rightarrow$ speed: $\sqrt{85.445^2 + 336.042^2}$

$= 346.735$ mph

Direction: $\tan^{-1}\left(\frac{336.042}{85.445}\right) \rightarrow 75.734°$ (14.266° E of N)