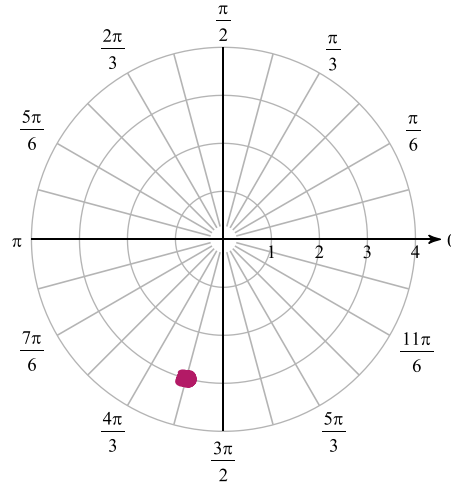
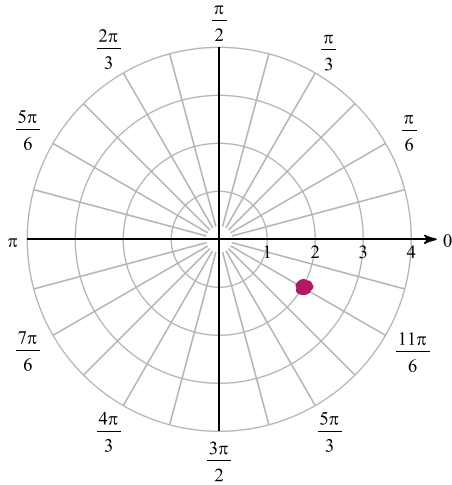


Intro to Polar Form Notes

Plot the point with the given polar coordinates.

1) $(-2, \frac{5\pi}{6})$

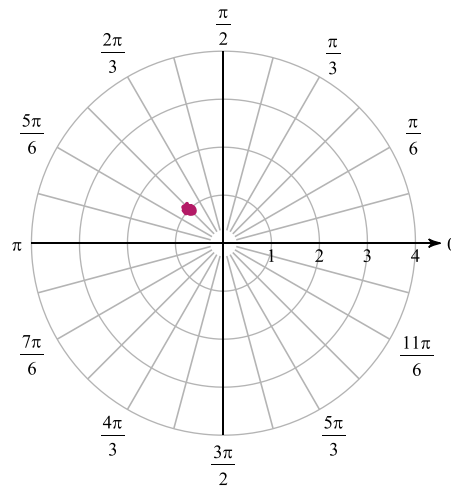
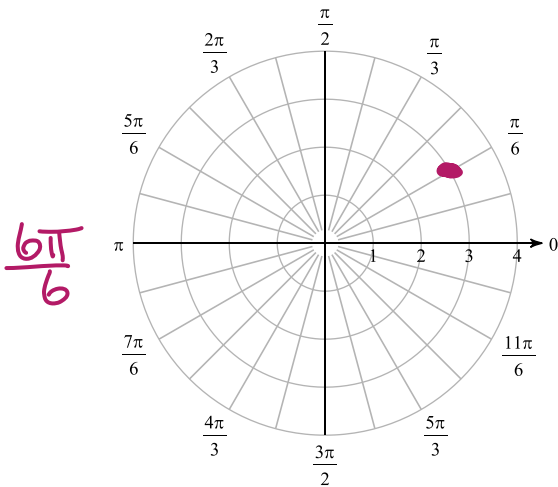
2) $(3, -\frac{7\pi}{12})$



Find 3 other polar coordinates that describe the same point as the provided polar coordinates.

3) $(3, -\frac{11\pi}{6})$

4) $(-1, -\frac{\pi}{4})$



$(3, \frac{\pi}{6})$
 $(-3, \frac{7\pi}{6})$

$(-3, -\frac{5\pi}{6})$

$(-1, \frac{7\pi}{4})$
 $(1, -\frac{5\pi}{4})$

$(1, \frac{3\pi}{4})$

Convert each pair of polar coordinates to rectangular coordinates.

5) $(4, -\frac{11\pi}{6})$

1st quad
↓

$x = r \cos \theta = 4 \cos(-\frac{11\pi}{6}) = \frac{4\sqrt{3}}{2} \text{ or } 2\sqrt{3}$

$y = r \sin \theta = 4 \sin(-\frac{11\pi}{6}) = 4(\frac{1}{2}) = 2$

$(2\sqrt{3}, 2)$

6) $(-2, \frac{4\pi}{3})$

$x = -2 \cos(\frac{4\pi}{3}) = -2(-\frac{1}{2}) = 1$

$y = -2 \sin(\frac{4\pi}{3}) = -2(-\frac{\sqrt{3}}{2}) = \sqrt{3}$

$(1, \sqrt{3})$

Convert each pair of rectangular coordinates to polar coordinates where $r > 0$ and $0 \leq \theta < 2\pi$.

7) $(-2, 2\sqrt{3})$ $x^2 + y^2 = r^2$

$(-2)^2 + (2\sqrt{3})^2 = r^2$ $(4, \frac{2\pi}{3})$

$4 + 12 = r^2$

$\tan \theta = \frac{2\sqrt{3}}{-2}$

$16 = r^2$

$r = 4$

$\tan \theta = -\sqrt{3}$

$\theta = \frac{2\pi}{3}$

8) $(-\sqrt{2}, -\sqrt{2})$

$(2, \frac{5\pi}{4})$

$(-\sqrt{2})^2 + (-\sqrt{2})^2 = 2 + 2 = 4$

$\sqrt{4} = 2 = r$

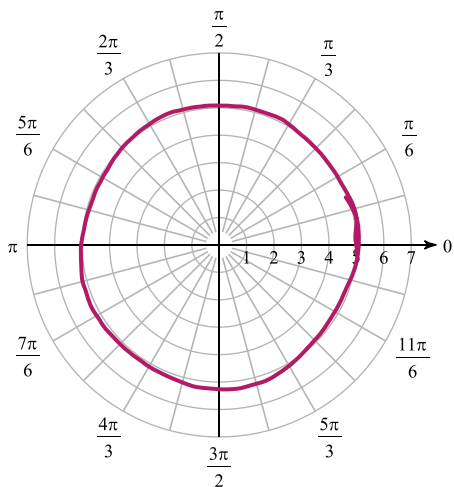
$\tan \theta = \frac{-\sqrt{2}}{-\sqrt{2}}$

$\tan \theta = 1$

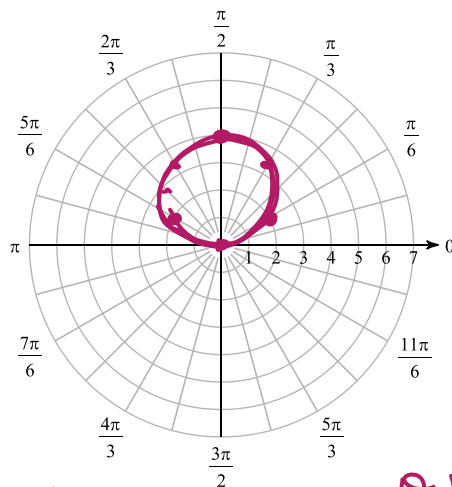
$\theta = \frac{5\pi}{4}$

Consider each polar equation. Classify the curve; and sketch the graph.

9) $r = 5$ circle



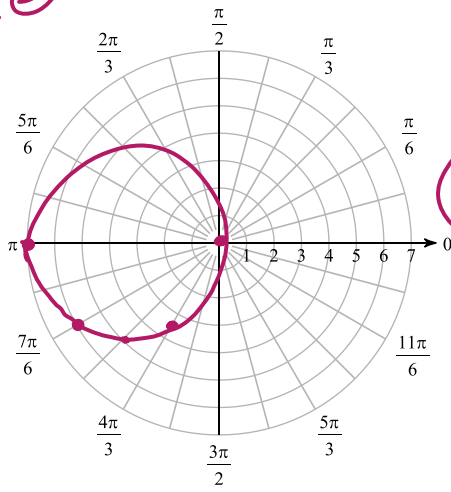
10) $r = 4 \sin \theta$ circle



θ	$r = 4 \sin \theta$	θ	$r = 4 \sin \theta$
0	0	$\pi/4$	$4(\frac{\sqrt{2}}{2}) = 2\sqrt{2}$
$\pi/2$	4	$\pi/6$	$4(\frac{1}{2}) = 2$
π	0	$\pi/3$	$4(\frac{\sqrt{3}}{2}) = 2\sqrt{3}$
$3\pi/2$	-4	$2\pi/3$	$= 2\sqrt{3}$
2π	0	$5\pi/6$	2

circle

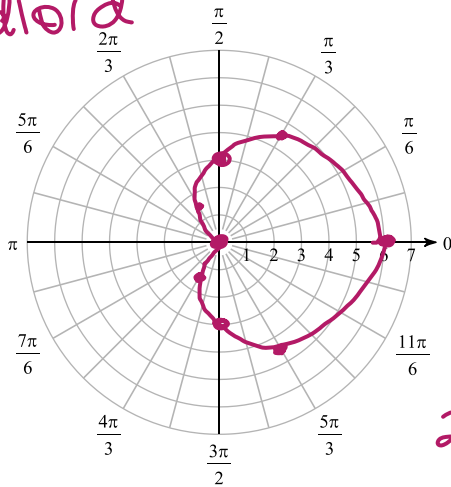
11) $r = -7\cos\theta$



θ	$r = -7\cos\theta$
0	-7
$\pi/6$	$-7\cos\frac{\pi}{6} = -\frac{7\sqrt{3}}{2}$
$\pi/3$	$-7\cos\frac{\pi}{3} = -7(\frac{1}{2}) = -3.5$
$\pi/4$	$-7\cos\frac{\pi}{4} = -\frac{7\sqrt{2}}{2}$
$\pi/2$	0

cardioid

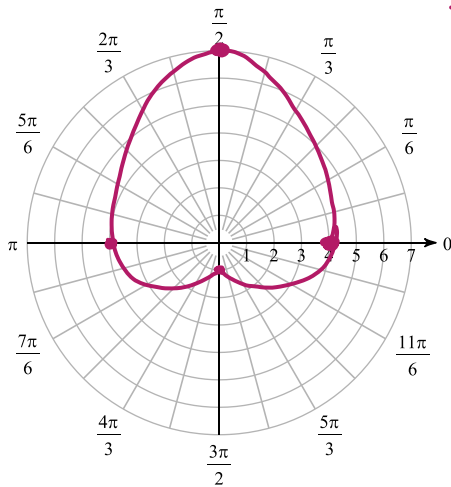
12) $r = 3 + 3\cos\theta$



θ	$3 + 3\cos\theta$	$3 + 3\cos\theta$
0	6	
$\pi/3$	3	$3 + 3(\frac{1}{2}) = 4.5$
$\pi/2$	3	
$2\pi/3$	0	$3 + 3(-\frac{1}{2}) = 1.5$
π	0	
$3\pi/2$	3	$3 + 3(-\frac{1}{2}) = 1.5$
2π	6	$3 + 3(\frac{1}{2}) = 4.5$

limacon

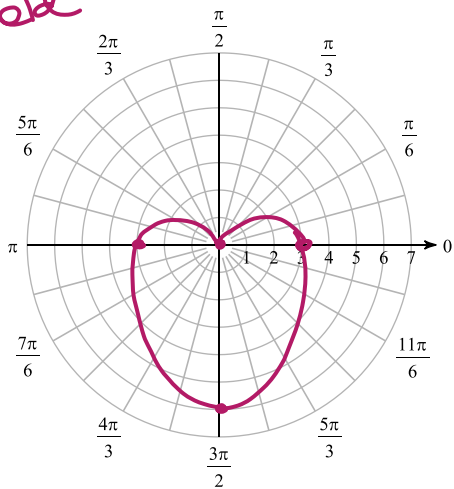
13) $r = 4 + 3\sin\theta$



θ	$r = 4 + 3\sin\theta$
0	4
$\pi/2$	7
π	4
$3\pi/2$	1

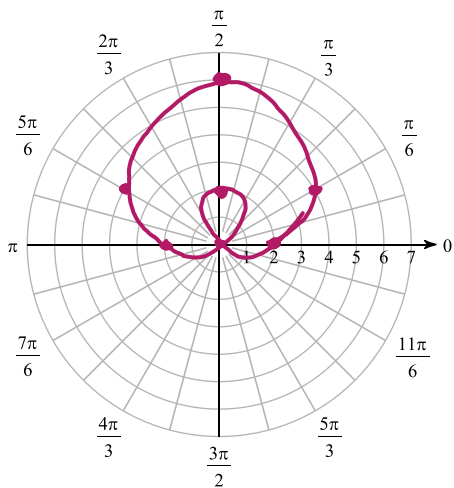
14) $r = 3 - 3\sin\theta$

Cardioid



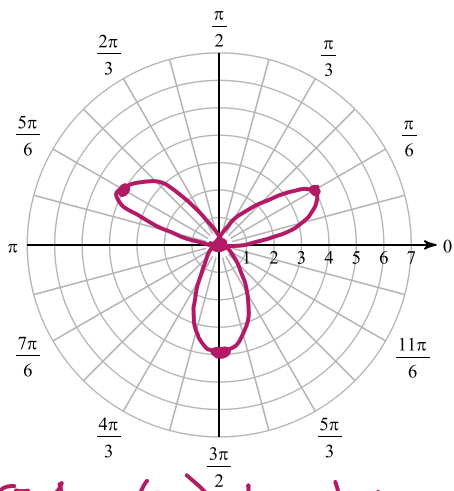
θ	$3 - 3\sin\theta$
0	3
$\pi/2$	0
π	3
$3\pi/2$	6

15) $r = 2 + 4\sin\theta$



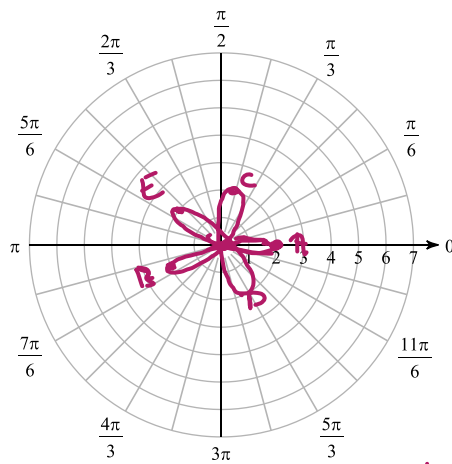
θ	$2 + 4\sin\theta$	θ	$2 + 4\sin\theta$
0	2	$\pi/6$	4
$\pi/2$	6	$5\pi/6$	4
π	2	$7\pi/6$	0
$3\pi/2$	-2	$11\pi/6$	0
2π	2		

16) $r = 4\sin(3\theta)$



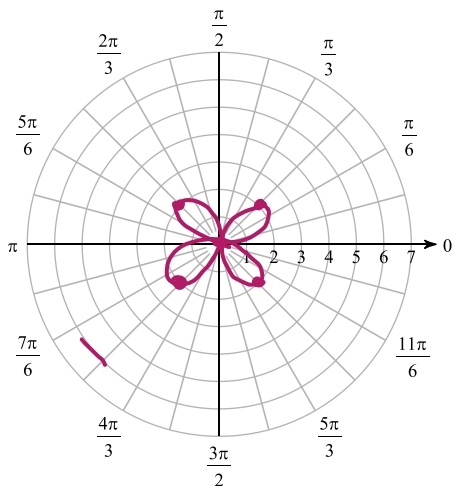
θ	$r = 4\sin(3\theta)$	θ	$4\sin(3\theta)$
0	0	$4\pi/6$	0
$\pi/6$	4	$5\pi/6$	4
$2\pi/6$	0	$6\pi/6$	0
$3\pi/6$	-4		

17) $r = 2\cos(5\theta)$



θ	$2\cos(5\theta)$	θ	$2\cos(5\theta)$	θ	$2\cos(5\theta)$
0	2	$4\pi/10$	2	$8\pi/10$	2
$\pi/10$	0	$5\pi/10$	0	$9\pi/10$	0
$2\pi/10$	-2	$6\pi/10$	-2	$10\pi/10$	-2
$3\pi/10$	0				

18) $r = 2\sin(2\theta)$



θ	$2\sin(2\theta)$
0	0
$\pi/4$	2
$2\pi/4$	0
$3\pi/4$	-2
$4\pi/4$	0
$5\pi/4$	2
$6\pi/4$	0
$7\pi/4$	-2
$8\pi/4$	0

$\frac{2\pi}{2} = \frac{(\pi)}{4}$

Convert each equation from rectangular to polar form.

19) $y = -x\sqrt{3}$

$$r \sin \theta = -r \cos \theta \sqrt{3}$$

$$\sin \theta = -\cos \theta \sqrt{3}$$

$$\tan \theta = -\sqrt{3}$$

$$\theta = \frac{2\pi}{3}$$

20) $y = 5x$

$$r \sin \theta = 5r \cos \theta$$

$$\sin \theta = 5 \cos \theta$$

$$\tan \theta = 5$$

21) $(x+3)^2 + (y-1)^2 = 10$

$$\underline{x^2} + \underline{6x} + \underline{9} + \underline{y^2} - \underline{2y} + \underline{1} = \underline{10}$$

$$\frac{r^2}{r} + \frac{6r \cos \theta}{r} - \frac{2r \sin \theta}{r} = \frac{0}{r}$$

$$r + 6 \cos \theta - 2 \sin \theta = 0$$

$$r = 2 \sin \theta - 6 \cos \theta$$

22) $y = \frac{x^2}{3}$

$$r \sin \theta = \frac{(r \cos \theta)^2}{3}$$

$$\frac{3r \sin \theta}{r} = \frac{r^2 \cos^2 \theta}{r}$$

$$\frac{3 \sin \theta}{\cos^2 \theta} = \frac{r \cos^2 \theta}{\cos^2 \theta}$$

$$3 \frac{\sin \theta}{\cos \theta} \frac{1}{\cos \theta} = r$$

$$3 \tan \theta \sec \theta = r$$

Convert each equation from polar to rectangular form.

$$23) \theta = \frac{3\pi}{4}$$

$$\tan \theta = \tan \frac{3\pi}{4}$$

$$\frac{y}{x} = -1$$

$$y = -x$$

$$24) r = 4\cos \theta + 2\sin \theta$$

$$r^2 = 4r\cos \theta + 2r\sin \theta$$

$$x^2 + y^2 = 4x + 2y$$

$$x^2 - 4x + 4 + y^2 - 2y + 1 = 0 + 4 + 1$$

$$(x-2)^2 + (y-1)^2 = 5$$

$$25) r^2 = 4\csc(2\theta)$$

$$r^2 = \frac{4}{\sin(2\theta)}$$

$$r^2 \sin(2\theta) = 4$$

$$r^2 2\sin \theta \cos \theta = 4$$

$$2r\sin \theta \cdot r\cos \theta = 4$$

$$2y \cdot x = 4 \quad \text{or} \quad xy = 2$$

$$26) r^2 = 3\sec(2\theta)$$

$$r^2 = \frac{3}{\cos(2\theta)}$$

$$r^2 \cos(2\theta) = 3$$

$$r^2 (\cos^2 \theta - \sin^2 \theta) = 3$$

$$r^2 \cos^2 \theta - r^2 \sin^2 \theta = 3$$

$$x^2 - y^2 = 3$$