Calculus AB Key

Limits and Continuity

A calculator may not be used.

I. Multiple Choice

Choose the best answer for each of the following questions.

Questions 1-3, are based on the function shown in the graph and defined below:



1. The function is defined on [-1,5]

 A. if B. if C. if D. for each in [-1,5]

The function is not continuous but since it passes the vertical line test is is defined for each value of in the given domain. (D)

2. The function has a removable discontinuity at

 A. B. C. D. none of these

A removable discontinuity occurs when there is a point discontinuity that can be “filled” so the removeable discontinuity is at . (C)

3. The function is continuous over which of the following intervals?

 A. B. C. D.

The function is continuous only over the closed interval [0,1] for all other options given have an endpoint that should not be included. (B)

1. Find the limit.

A. B. C. D. infinite

Evaluate the degree of the numerator and denominator, take lead coefficients of each since the degrees are the same.: (A)

1. Evaluate the limit , if it exists:

 A. B. C. 0 D.

. Rationalize the numerator, cancel out the denominator (D)

1. Consider the following graphs of and given below.



Find:

A -1 B. C. 1 D. Does not exist

***This question is commonly missed by students***; in order to solve this properly one must consider the product of the limit as x is approaching 2 both on the left and on the right. Since

And the limit for the left and right are equal

Therefor (B)

1. Find the limit.

A. B. C. D.

 Use direct substitution: (C)

8. Given the function: ,

what value of k will make this piecewise function continuous?

A. B. C. D. .

 (A)

9. Let which of the following statements is (are) true?

A. I only B. II only C. III only D. I and II

Only the 1st and 2nd statements are true.

Since the function is not continuous at (D)

10. is a continuous function on a closed interval , where

 and is in the closed interval , then which of the following statements must be true?

 A.

B.

C.

D.

The conditions given in the question are those required for the intermediate value theorem, therefor the third statement, which is the conclusion of that theorem, is the correct one. (C)

11. Let , which of the following statements is true?

 A. has a jump discontinuity at

 B. has a removable discontinuity at

 C. has a vertical asymptote at

D.

 and , so there is a jump discontinuity. (A)

12. For which of the following pairs of functions for and is infinite?

 A. ;

 B. ;

 C. ;

 D. ;

In order for a rational function to have a infinite limit as , the degree of the numerator must be larger then the degree of the denominator. (D)

II Free Response.

13. The function is defined by for .

 Let be the function defined by

1. Determine if is a continuous function over the closed interval [-10,10].

Use the definition of continuity to explain your answer.

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| Over the interval [-10,10] the only possible discontinuity is at where the piecewise function changes.  and .Since the left-sided limit and the right-sided limit are **not** equal, is **not** continuous. |   1: identifies possible point(s) of  discontinuity3 1: finds left and right sided limits 1: uses definition of continuity for  conclusion.  |

14. The function is defined as , the function is defined

 as , let , and let be the function defined by

 where is a constant.

1. Find the limit of approaches 1. Show all work that leads to your answer. Is continuous over the closed interval of [-2,2]? Explain.

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| Since was factored out there is a hole in the at that point. Therefor the function is **not** continuous over the given closed interval of [-2,2]. |  1:  3 1: work that leads to answer 1: Conclusion with reason |
|  |  |

b) What value of will make continuous at Justify your answer.

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| From part a above :, |   1: uses limit from part a 2 1: solves for k |
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