$$\frac{dP}{dt} = 0\,2P - 0\,002P^2 \quad , t \geq 0$$

$$\frac{dP}{dt} = 002P\left(100 - P\right) \quad \left(L = 100\right) \quad \lim_{t\to\infty} P(t) = 100$$

---

a) $P(0) = 5$ $\qquad P = \dfrac{L}{1+ce^{-LKt}}$

$$5 = \frac{100}{1+ce^{-100(002)(0)}}$$

$$5 = \frac{100}{1+c} \rightarrow 5(1+c) = 100$$

$$1+c = 20 \qquad c = 19$$

$$\boxed{P(t) = \frac{100}{1+19e^{-100(002)t}}}$$

---

b) $P(0) = 60$ $\qquad P(t) = \dfrac{100}{1+\left(\frac{2}{3}\right)e^{-100(002)t}}$

$$60 = \frac{100}{1+ce^{-100(002)(0)}}$$

$$60 = \frac{100}{1+c} \qquad 60 + 60c = 100$$

$$60c = 40 \qquad c = \frac{2}{3}$$

c) $P(0) = 120$

$$P(t) = \frac{100}{1 + \left(-\frac{1}{6}\right)e^{-100(0.02)t}}$$

$$120 = \frac{100}{1+c}$$

$$120 + 120c = 100$$

$$120c = -20$$

$$c = -\frac{1}{6}$$

---

4) zoo problem $\quad \frac{dP}{dt} = \frac{1}{4}(220 - P) \quad P(0) = 20$

$$\int \frac{dP}{220-P} = \int \frac{1}{4}\, dt$$

$$\frac{1}{-1} \ln|220-P| = \frac{1}{4}t + c$$

$$\ln|220-P| = -\frac{1}{4}t + c$$

e^ $\quad$ e^

$$220 - P = ce^{-\frac{1}{4}t}$$

$\rightarrow$ Plug in $P(0) = 20$

$$220 - 20 = ce^{-\frac{1}{4}(0)}$$

$$200 = c$$

$$220 - P = 200e^{-\frac{1}{4}t}$$

$$-220 \qquad\qquad -220$$

$$-P = 200e^{-\frac{1}{4}t} - 220$$

$$\boxed{P = -200e^{-\frac{1}{4}t} + 220}$$

2nd zoologist $\quad \frac{dQ}{dt} = \frac{1}{500} Q (220 - Q) \quad$ *Logistic growth*

$\frac{dQ}{dt}$? when Q grows most rapidly $\rightarrow$ this occurs if $Q = \frac{L}{2} = \frac{220}{2} = 110$

$$\left.\frac{dQ}{dt}\right|_{Q=110} = \frac{1}{500}(110)(220 - 110)$$

c.) Euler's method approx. $Q(10)$ start at $t=0$ (2 steps)

$$\frac{dQ}{dt} = \frac{1}{500}Q(220-Q)$$

old	dx	$m = \frac{dy}{dx}$	$dy = m \cdot dx$	new
$(0, 20)$	5	$\frac{1}{500}(20)(220-20)$ $\frac{1}{25}(200) = 8$	$8 \cdot 5 = 40$	$(5, 60)$
$(5, 60)$	5	$\frac{1}{500}(60)(220-60)$ $\frac{3}{25}(160) = \frac{480}{25}$	5.96 $\cancel{\frac{480}{25}} \cdot \frac{5}{1} = 96$ 55	$(10, 156)$

$$\boxed{Q(10) \approx 156}$$

Baby Bird FRQ $\quad \frac{dB}{dt} = \frac{1}{5}(100-B) \quad B(0)=20$

a) $\left.\frac{dB}{dt}\right|_{B=40} = \frac{1}{5}(100-40) = \frac{60}{5} = 12$

$\left.\frac{dB}{dt}\right|_{B=70} = \frac{1}{5}(100-70) = \frac{30}{5} = 6$

the bird is gaining weight faster when weight is 40 grams.

---

b) $\frac{d}{dt}\left(\frac{dB}{dt}\right) = \frac{d}{dt}\left(\frac{1}{5}(100-B)\right)$

$$\frac{d^2B}{dt^2} = -\frac{1}{5}\frac{dB}{dt} = -\frac{1}{5}\cdot\frac{1}{5}(100-B)$$

$$= -4 + \frac{B}{25}$$

this implies B is concave down for $B < 100$, but the graph in the pic is concave up for some values of $B < 100$

c) $$\frac{dB}{dt} = \frac{1}{5}(100 - B) \qquad B(0) = 20$$

$$\int \frac{dB}{100 - B} = \int \frac{1}{5}\, dt$$

$$\frac{1}{-1} \ln|100 - B| = \frac{1}{5}t + c$$

$$\ln|100 - B| = -\frac{1}{5}t + c$$

e^ e^

$$100 - B = ce^{-\frac{1}{5}t}$$

$$100 - 20 = ce^{-\frac{1}{5}(0)}$$

$$80 = c$$

$$100 - B = 80e^{-\frac{1}{5}t}$$

$$-B = 80e^{-\frac{1}{5}t} - 100$$

$$\boxed{B = -80e^{-\frac{1}{5}t} + 100}$$