Find the derivative of each function

a) $y=(2x^4-3)(x^2+1)$

$y'=(2x^4-3)(2x)+(x^2+1)(8x^3)$

$y'=4x^5-6x+8x^5+8x^3$

$\boxed{y'=12x^5+8x^3-6x}$

b) $y=\dfrac{4x^5+x^2+4}{5x^2-2}$

$y'=\dfrac{(5x^2-2)(20x^4+2x)-(4x^5+x^2+4)(10x)}{(5x^2-2)^2}$

$y'=100x^6+10x^3-40x^4-4x-40x^6-10x^3-40x$

$\boxed{y'=\dfrac{60x^6-40x^4-44x}{(5x^2-2)^2}}$

c) $f(x)=\dfrac{1}{(x^2+2)^3}$

$f(x)=(x^2+2)^{-3}$

f	d
x^{-3}	$-3x^{-4}$
x^2+2	$2x$

$\dfrac{-3}{(x^2+2)^4}\cdot 2x$

$\boxed{y'=\dfrac{-6x}{(x^2+2)^4}}$

d) $f(x)=\tan^2\left(\sqrt{x^3+5x}\right)$

x^2	$2x$
$\tan x$	$\sec^2 x$
$\sqrt{x}$	$\dfrac{1}{2\sqrt{x}}$
x^3+5x	$3x^2+5$

$\cancel{2}\left(\tan\left(\sqrt{x^3+5x}\right)\right)\cdot\sec^2\left(\sqrt{x^3+5x}\right)\cdot\dfrac{1}{\cancel{2}\sqrt{x^3+5x}}\cdot(3x^2+5)$

$y'=\dfrac{\tan\sqrt{x^3+5x}\cdot\sec^2\left(\sqrt{x^3+5x}\right)\cdot(3x^2+5)}{\sqrt{x^3+5x}}$

2. Find the point where $f(x) = x^2 - 6x + 11$ has a horizontal tangent line.

$f'(x) = 2x - 6$

$0 = 2x - 6$

$6 = 2x$

$x = 3$

$f(3) = 3^2 - 18 + 11 = -9 + 11 = 2$

$\boxed{(3,2)}$

3. Evaluate $\lim_{h \to 0} \frac{(x+h)^8 - x^8}{h}$

limit definition of derivative

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f(x) = x^8$

$\boxed{f'(x) = 8x^7}$

Major Assessment Review Session Solutions

4. Use the table to find the derivatives of the following at $x=4$.

	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
$x=4$	3	-2	7	1
$x=7$	5	-1	4	6

a) $f(x)g(x)$

$fg' + gf' \rightarrow 3(1) + 7(-2)$

$3 - 14 = -11$

b) $\dfrac{f(x)}{g(x)}$ $\qquad \dfrac{gf' - fg'}{g^2}$

$\dfrac{7(-2) - 3(1)}{49} = \boxed{\dfrac{-17}{49}}$

c) $3f(x) + 4g(x)^2$

$3f' + 4 \cdot 2g \cdot g' = 3(-2) + 8 \cdot 7 \cdot 1$

$-6 + 56 = 50$

d) $f(g(x))$ $\qquad f'(g) \cdot g'$

$f'(7) \cdot 1 = -1 \cdot 1 = -1$

5 Given $s(t) = \frac{1}{3}t^3 - \frac{7}{2}t^2 + 10t + 5$ is the position function for a particle moving along the x-axis, find the following.

CALC for Arithmetic

a) displacement from $t=1$ to $t=4$

-1.5

$s(4) - s(1) = -1.5$

b) times when the particle is at rest.

$t=2 \quad t=5$

$s'(t) = v(t) = t^2 - 7t + 10 = 0$

$(t-5)(t-2) = 0$

$t=5 \quad t=2$

c) avg velocity from $t=1$ to $t=4$

-0.5

$\dfrac{s(4) - s(1)}{4-1} = \dfrac{-1.5}{3} = -0.5$

d) instantaneous velocity at $t=3$

-2

$v(3) = 3^2 - 7(3) + 10 = -2$

e) value of t when the acceleration is 0.

$t = 7/2$

$a(t) = 2t - 7$

$0 = 2t - 7$

$t = \frac{7}{2}$

6. Find the derivative

a) $x^3 - 3x^2y + 4y = 12$

$3x^2 - 3x^2y' + y(-6x) + 4y' = 0$

$-3x^2y' + 4y' = 6xy - 3x^2$

$y'(-3x^2+4) = 6xy - 3x^2$

$\boxed{y' = \dfrac{6xy - 3x^2}{-3x^2+4}}$

b) $\sin x + 2\cos 2y = 1$

$\cos x + 2(-\sin(2y)) \cdot 2y' = 0$

$\cos x = 4\sin(2y)y'$

$\boxed{y' = \dfrac{\cos x}{4\sin(2y)}}$

6. Find the derivative

c) $y = \sqrt{\cos^{-1} x}$

$\sqrt{x} \quad \dfrac{1}{2\sqrt{x}}$

$\cos^{-1} x \quad \dfrac{-1}{\sqrt{1-x^2}}$

$\boxed{y' = \dfrac{1}{2\sqrt{\cos^{-1} x}} \cdot \dfrac{-1}{\sqrt{1-x^2}}}$

d) $y = \sin(\tan^{-1} x)$

$y' = \cos(\tan^{-1} x) \cdot \dfrac{1}{1+x^2}$

$\boxed{y' = \dfrac{\cos(\tan^{-1} x)}{1+x^2}}$

7. Find the derivative

a) $y = \dfrac{e^x}{4+x}$

$$y' = \frac{(4+x)e^x - e^x(1)}{(4+x)^2}$$

$$\boxed{y' = \frac{4e^x + xe^x - e^x}{(4+x)^2} = \frac{3e^x + xe^x}{(4+x)^2}}$$

b) $y = 2^{\frac{4}{x}}$

f	d
2^x	$\ln 2 \cdot 2^x$
$\frac{4}{x}$	$\frac{-4}{x^2}$

$$y' = \ln 2 \cdot 2^{\frac{4}{x}} \cdot \frac{-4}{x^2}$$

$$\boxed{y' = \frac{-4 \ln 2 \cdot 2^{\frac{4}{x}}}{x^2}}$$

7. Find the derivative

c) $y = \ln(\sin x)$

$$y' = \frac{1}{\sin x} \cdot \cos x$$

$$y' = \frac{\cos x}{\sin x} = \cot x$$

d) $y = \ln\left(\dfrac{2x+6}{2x-9}\right)$

$$y = \ln(2x+6) - \ln(2x-9)$$

$$y' = \frac{1}{2x+6} \cdot 2 - \frac{1}{2x-9} \cdot 2$$

$$\boxed{y' = \frac{2}{2x+6} - \frac{2}{2x-9}}$$

8. If $f(x) = x^4 + 2x^3 - x^2 + 5$ and $f(2) = 33$,

find $(f^{-1})'(33)$. original $(2, 33)$ inverse $(33, 2)$

m 1 inverse pt

$x = y^4 + 2y^3 - y^2 + 5$

$1 = 4y^3 y' + 6y^2 y' - 2y y'$

$1 = 4(2)^3 y' + 6(2)^2 y' - 2(2) y'$

$1 = 32y' + 24y' - 4y'$

$1 = 52y'$ $\boxed{y' = \frac{1}{52}}$

m 2 original pt

$f'(x) = 4x^3 + 6x^2 - 2x$

$f'(2) = 32 + 24 - 4 = 52$

$\boxed{(f^{-1})'(33) = \frac{1}{52}}$

9. If $f(x) = -\cos x$, find $f^{(42)}(x)$.

find 42^{nd} derivative

$-\cos x$ R 0

$\sin x$ R 1

$\cos x$ R 2

$-\sin x$ R 3

$\frac{42}{4} = 10$ R 2

$\boxed{f^{(42)}(x) = \cos x}$

10. If $f(x) = e^{2x}$, find the equation of the tangent line to $f(x)$ at the point $\left(\frac{1}{2}\ln 5, 5\right)$.

$f'(x) = 2e^{2x}$

$f'\left(\frac{1}{2}\ln 5\right) = 2e^{2\frac{1}{2}\ln 5} = 2e^{\ln 5} = 2 \cdot 5 = 10$

$$\boxed{y - 5 = 10\left(x - \frac{1}{2}\ln 5\right)}$$