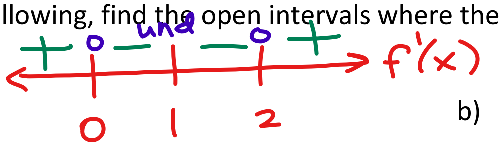
Critical pts when: $f'(x)=0$ or $f'(x)$ is undefined

AB Calculus Mean Value Theorem Day 2

Name: ____

1. For each of the following, find the open intervals where the function is increasing and decreasing and find all relative extrema.

a) $y=\dfrac{x^2}{2x-2}$



$$\frac{dy}{dx}=\frac{(2x-2)(2x)-(x^2)(2)}{(2x-2)^2}$$

$$=\frac{4x^2-4x-2x^2}{(2x-2)^2}$$

$$=\frac{2x^2-4x}{(2x-2)^2}\rightarrow\frac{2x(x-2)}{(2x-2)^2}$$

f(x) inc $(-\infty,0)\,(2,\infty)$
$f'(x)>0$

f(x) dec $(0,1)\,(1,2)$
$f'(x)<0$

rel max at $x=0$
$f'(x)$ changes + to −

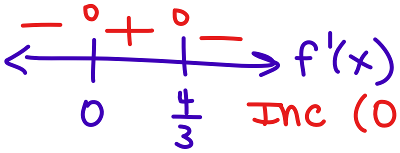
rel min at $x=2$ $f'(x)$ changes − to +

b) $y=-x^3+2x^2+4$

$$\frac{dy}{dx}=-3x^2+4x$$

$$0=-x(3x-4)$$

$x=0$ $\quad x=\frac{4}{3}$



Inc $(0,\frac{4}{3})$

Dec $(-\infty,0)(\frac{4}{3},\infty)$

Rel min at $x=0$ $f'(x)$ − to +

Rel max at $x=4/3$ $f'(x)$ + to −

2. For each of the following, find the absolute extrema over the given interval

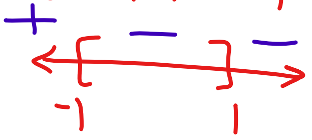
a) $y=-2x^2-4x+4$ over $[-1,1]$

$$\frac{dy}{dx}=-4x-4$$

$0=-4x-4$

$0=-4(x+1)$

C.P at $x=-1$

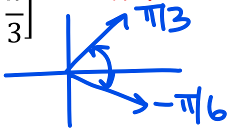


x	f(x)
-1	6
1	-2

abs max = 6 at $x=-1$

abs min = -2 at $x=1$

b) $y=2\sin^2 x$ over $\left[-\frac{\pi}{6},\frac{\pi}{3}\right]$



$y=2(\sin x)^2$

$$\frac{dy}{dx}=4\sin x\cos x$$

$0=4\sin x\cos x$

$\sin x=0$ $\quad\cos x=0$

$x=0$ $\quad$ ~~$x=\frac{\pi}{2},-\frac{\pi}{2}$~~, outside of domain

x	y
−π/6	1/2
0	0
π/3	3/2

abs max = 3/2 at $x=\frac{\pi}{3}$

abs min = 0 at $x=0$

3. For each of the following, show that the function f satisfies the hypothesis of the Mean Value Theorem on the given interval $[a,b]$. If it does, find each value of c in (a,b) guaranteed by the theorem.

a) $f(x)=\dfrac{x^2-1}{4x}$ over $[-1,1]$

Discontinuous at $x=0$

which is between $[-1,1]$

b) $y=x^3+10x^2+32x+33$ over $[-4,-2]$

polynomial so [,] and (,) ◡

$y(-2)=-8+10(4)+32(-2)+33$

$y(-4)=-64+10(16)+32(-4)+33$

$$\frac{y(-2)-y(-4)}{-2--4}=\frac{1-1}{2}=\frac{0}{2}=0$$

$$\frac{dy}{dx}=3x^2+20x+32$$

$0=(3x+8)(x+4)$

$x=-\frac{8}{3}\ ,-4$

$\boxed{c=-\frac{8}{3}\text{ only}}$ because $a<c<b$