AB Calculus Mean Value Theorem Day 1 Homework Name: ____________________

1. For each of the following, find the x-values of all relative extrema and the intervals on which the function is increasing and decreasing.

a) $y = 5x - x^2$

$y' = 5 - 2x$ Inc $(-\infty, \frac{5}{2})$

$x = \frac{5}{2}$ Dec $(\frac{5}{2}, \infty)$

$+ \mid -$

$5/2$

$\hookrightarrow$ rel. max

b) $y = \frac{2}{x}$ $y = 2x^{-1}$

$y' = \frac{-2}{x^2}$

Dec

$(-\infty, 0) \cup (0, \infty)$

No rel. extrema

$- \mid -$

0

c) $y = x^3 - 6x^2 + 4$

$y' = 3x^2 - 12x$ Inc:

$= 3x(x-4)$ $(-\infty, 0) \cup (4, \infty)$

$\rightarrow$ max $\rightarrow$ Min Dec:

$+ \mid - \mid +$ $(0, 4)$

$0 \quad 4$

d) $y = 2x + \cos x$

$y' = 2 - \sin x \rightarrow$ always $+$

$0 = 2 - \sin x$ Inc: $(-\infty, \infty)$

$-2 = -\sin x$

$\sin x = 2$ No critical pts. No rel. extrema

No real sol.

2. For each of the following, find the absolute extrema over the given interval

a) $y = x^3 - 3x + 1$ over $[-3, 3]$

$y' = 3x^2 - 3$ abs max

$= 3(x^2 - 1)$ 19 @ $x = 3$

$= 3(x-1)(x+1)$

abs min

-17 @ $x = -3$

$-17 \quad + \quad 3 \quad - \quad -1 \quad + \quad 19$

$-3 \quad -1 \quad 1 \quad 3$

b) $y = \sqrt{9 - x^2}$ over $[-3, 3]$

$y = (9 - x^2)^{\frac{1}{2}}$

$y' = \frac{-2x}{2\sqrt{9-x^2}} \rightarrow \frac{-x \rightarrow 0}{\sqrt{9-x^2} \rightarrow \pm 3}$

abs max

3 @ $x = 0$

abs min

0 @ $x = \pm 3$

$0 \quad [\ + \quad 3 \quad - \ ] \quad 0$

$-3 \quad 0 \quad 3$

3. For each of the following, show that the function f satisfies the hypothesis of the Mean Value Theorem on the given interval $[a, b]$. If it does, find each value of c in (a, b) guaranteed by the theorem.

a) $f(x) = x^2 + 2x - 1$ over $[-1, 1]$

$f(x)$ is cont. on $[-1, 1]$

$f(x)$ is diff. on $(-1, 1)$

$f(-1) = 1 - 2 - 1 = -2$

$f(1) = 1 + 2 - 1 = 2$

$\frac{2 - -2}{1 - -1} = \frac{4}{2} = 2$

$2x + 2 = 2$

$f'(x) = 2x + 2$

$2x = 0$

$x = 0$

b) (Calculator) $f(t) = 2t - t^2 - t^3$ over $[-2, 1]$

$f(t)$ is cont. $[-2, 1]$

$f(t)$ is diff $(-2, 1)$

$f(-2) = -4 - 4 + 8 = 0$

$f(1) = 2 - 1 - 1 = 0$

$\frac{0 - 0}{-2 - 1} = \frac{0}{-3} = 0$

$f'(t) = 2 - 2t - 3t^2$

$t = -1.21525$

$t = .5486$

4. Find $\frac{dy}{dx}$ for each of the following

a) $y = e^{\tan^3 5x}$

$e^{\tan^3 5x} \cdot 3\tan^2 5x \cdot \sec^2 5x \cdot 5$

$e^x \to e^x$

$x^3 \to 3x^2$

$= 15e^{\tan^3 5x} \tan^2 5x \cdot \sec^2 5x$

$\tan x \to \sec^2 x$

$5x \to 5$

b) $y = 6^{3x^2}$ $\to 6^x \to 6^x \cdot \ln 6$

$3x^2 \to 6x$

$6^{3x^2} \cdot \ln 6 \cdot 6x$

c) $y = \sin^{-1}(\ln x)$

$\sin^{-1} x \to \frac{1}{\sqrt{1-x^2}}$

$\ln x \to \frac{1}{x}$

$\frac{1}{\sqrt{1-(\ln x)^2}} \cdot \frac{1}{x}$

d) $y = \ln(\cos^2 x)$

$\ln x \to 1/x$

$x^2 \to 2x$

$\cos x \to -\sin x$

$\frac{1}{\cos^2 x} \cdot 2\cos x \cdot -\sin x \to -2\tan x$

$-\frac{2\cos \cdot \sin}{\cos^2}$

e) $y = \log_3(\sqrt{x})$

$3^y = \sqrt{x}$

$y \ln 3 = \ln x^{\frac{1}{2}} \to \frac{1}{2}\ln x$

$y' \cdot \ln 3 = \frac{1}{2x} \to y' = \frac{1}{\ln 3 \cdot 2x}$

f) $f(x) = -3x^4 e^{2x}$ $f'(x) = -3x^4 \cdot e^{2x} \cdot 2 + (-12x^3)e^{2x}$

g) $y = (\sec x)^x$

$\ln y = x \ln \sec x$

$\frac{y'}{y} = \frac{x \cdot \sec x \tan x}{\sec x} + \ln \sec x$

$y' = (\sec x)^x \left(\frac{x \tan x}{\sec x} + \ln \sec x\right)$

h) $y^3 - 3xy^3 = 2$ $3y^2 y' + [(-3x)(3y^2 y') + (-3)(y^3)] = 0$

$3y^2 y' - 9xy^2 y' - 3y^3 = 0$

$y' = \frac{3y^3}{3y^2 - 9xy^2} \to \frac{y}{1-3x}$

i) $f(x) = x^2 \tan^{-1} x$

$x^2 \cdot \frac{1}{1+x^2} + 2x \tan^{-1} x$

j) $e^{3y} + 3^x = \ln y$

$e^{3y} \cdot 3y' + 3^x \cdot \ln 3 = \frac{y'}{y}$

$y \cdot e^{3y} \cdot 3y' + 3^x \cdot \ln 3 \cdot y = y'$

$3^x \cdot \ln 3 \cdot y = y' - y \cdot e^{3y} \cdot 3y'$

$\frac{3^x \cdot \ln 3 \cdot y}{1 - ye^{3y} \cdot 3} = y'$

5. If $1 = x^3 + y^3$, find $\frac{d^2y}{dx^2}$.

$0 = 3x^2 + 3y^2 y'$

$y' = \frac{-3x^2}{3y^2} = -\frac{x^2}{y^2} = y'$

$y'' = \frac{y^2(-2x) + x^2(2y)y'}{y^4} \to \frac{\frac{-2xy^2}{y} - \frac{2x^4}{y}}{y^4} \to$

$\frac{-2xy^3 - 2x^4}{y^5}$