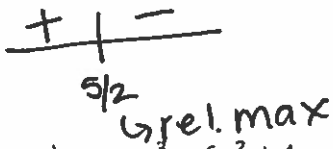


AB Calculus Mean Value Theorem Day 1 Homework

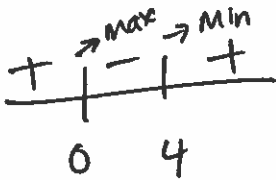
Name: _____

1. For each of the following, find the x-values of all relative extrema and the intervals on which the function is increasing and decreasing.

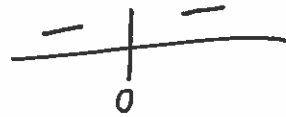
a) $y = 5x - x^2$
 $y' = 5 - 2x$ Inc $(-\infty, \frac{5}{2})$
 $x = \frac{5}{2}$ Dec $(\frac{5}{2}, \infty)$



c) $y = x^3 - 6x^2 + 4$
 $y' = 3x^2 - 12x = 3x(x-4)$
 Inc: $(-\infty, 0) \cup (4, \infty)$
 Dec: $(0, 4)$



b) $y = \frac{2}{x}$ $y = 2x^{-1}$
 $y' = -\frac{2}{x^2}$ Dec $(-\infty, 0) \cup (0, \infty)$
 No rel. extrema

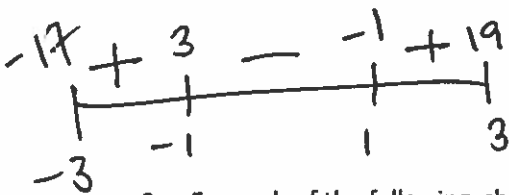


d) $y = 2x + \cos x$
 $y' = 2 - \sin x \rightarrow$ always +
 $0 = 2 - \sin x$ Inc: $(-\infty, \infty)$
 $-2 = -\sin x$

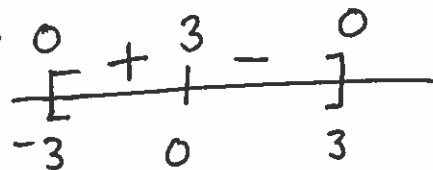
$\sin x = 2$ NO real sol. NO critical pts. NO rel. extrema

2. For each of the following, find the absolute extrema over the given interval

a) $y = x^3 - 3x + 1$ over $[-3, 3]$
 $y' = 3x^2 - 3 = 3(x^2 - 1) = 3(x-1)(x+1)$
 abs max 19 @ $x = 3$
 abs min -17 @ $x = -3$



b) $y = \sqrt{9 - x^2}$ over $[-3, 3]$
 $y = (9 - x^2)^{\frac{1}{2}}$
 $y' = \frac{-2x}{2\sqrt{9-x^2}} \rightarrow \frac{-x}{\sqrt{9-x^2}} \rightarrow \pm 3$
 abs max 3 @ $x = 0$
 abs min 0 @ $x = \pm 3$



3. For each of the following, show that the function f satisfies the hypothesis of the Mean Value Theorem on the given interval $[a, b]$. If it does, find each value of c in (a, b) guaranteed by the theorem.

a) $f(x) = x^2 + 2x - 1$ over $[-1, 1]$
 $f(x)$ is cont. on $[-1, 1]$
 $f(x)$ is diff. on $(-1, 1)$

$f(-1) = 1 - 2 - 1 = -2$
 $f(1) = 1 + 2 - 1 = 2$

$\frac{2 - (-2)}{1 - (-1)} = \frac{4}{2} = 2$

$f'(x) = 2x + 2$
 $2x + 2 = 2$
 $2x = 0$
 $x = 0$

b) (Calculator) $f(t) = 2t - t^2 - t^3$ over $[-2, 1]$

$f(t)$ is cont. $[-2, 1]$
 $f(t)$ is diff. $(-2, 1)$

$f(-2) = -4 - 4 + 8 = 0$
 $f(1) = 2 - 1 - 1 = 0$

$\frac{0 - 0}{-2 - 1} = \frac{0}{-3} = 0$

$f'(t) = 2 - 2t - 3t^2$

$t = -1.21525$

$t = .5486$

4. Find $\frac{dy}{dx}$ for each of the following

a) $y = e^{\tan^3 5x}$

$$e^x \rightarrow e^x$$

$$x^3 \rightarrow 3x^2$$

$$\tan x \rightarrow \sec^2 x$$

$$5x \rightarrow 5$$

$$e^{\tan^3 5x} \cdot 3 \tan^2 5x \cdot \sec^2 5x \cdot 5 = 15e^{\tan^3 5x} \tan^2 5x \sec^2 5x$$

b) $y = 6^{3x^2} \rightarrow 6^x \rightarrow 6^x \cdot \ln 6$

$$3x^2 \rightarrow 6x$$

$$6^{3x^2} \cdot \ln 6 \cdot 6x$$

c) $y = \sin^{-1}(\ln x)$

$$\sin^{-1} x \rightarrow \frac{1}{\sqrt{1-x^2}}$$

$$\ln x \rightarrow \frac{1}{x}$$

$$\frac{1}{\sqrt{1-(\ln x)^2}} \cdot \frac{1}{x}$$

d) $y = \ln(\cos^2 x)$

$$\ln x \rightarrow \frac{1}{x}$$

$$x^2 \rightarrow 2x$$

$$\cos x \rightarrow -\sin x$$

$$\frac{1}{\cos^2 x} \cdot 2 \cos x \cdot -\sin x \rightarrow -2 \tan x$$

e) $y = \log_3(\sqrt{x})$

$$3^y = \sqrt{x}$$

$$y \ln 3 = \ln x^{\frac{1}{2}} \rightarrow \frac{1}{2} \ln x$$

$$y' \ln 3 = \frac{1}{2x} \rightarrow y' = \frac{1}{\ln 3 \cdot 2x}$$

f) $f(x) = -3x^4 e^{2x}$

$$f'(x) = -3x^4 \cdot e^{2x} \cdot 2 + (-12x^3) e^{2x}$$

g) $y = (\sec x)^x$

$$\ln y = x \ln \sec x$$

$$\frac{y'}{y} = x \cdot \frac{\sec x \tan x}{\sec^2 x} + \ln \sec x$$

$$y' = (\sec x)^x \left(\frac{x \tan x \sec x}{\sec x} + \ln \sec x \right)$$

h) $y^3 - 3xy^3 = 2$

$$3y^2 y' - 9xy^2 y' - 3y^3 = 0$$

$$y' = \frac{3y^3}{3y^2 - 9xy^2} \rightarrow \frac{y}{1-3x}$$

i) $f(x) = x^2 \tan^{-1} x$

$$x^2 \cdot \frac{1}{1+x^2} + 2x \tan^{-1} x$$

j) $e^{3y} + 3^x = \ln y$

$$e^{3y} \cdot 3y' + 3^x \cdot \ln 3 = \frac{y'}{y}$$

$$y \cdot e^{3y} \cdot 3y' + 3^x \cdot \ln 3 \cdot y = y'$$

$$3^x \cdot \ln 3 \cdot y = y' - y \cdot e^{3y} \cdot 3y'$$

$$\frac{3^x \cdot \ln 3 \cdot y}{1 - y e^{3y} \cdot 3} = y'$$

$$\frac{-2xy^3 - 2x^4}{y^5}$$

5. If $1 = x^3 + y^3$, find $\frac{d^2y}{dx^2}$

$$0 = 3x^2 + 3y^2 y'$$

$$y' = \frac{-3x^2}{3y^2} = -\frac{x^2}{y^2} = y'$$

$$y'' = \frac{y^2(-2x) + x^2(2y)y'}{y^4} \rightarrow \frac{-2xy^2 - 2x^4}{y^4} \rightarrow$$