AP Calculus Name________________________________

More Convoluted Cray U-Sub Notes

Date________________ Period____

Evaluate each indefinite integral.

1) $\int \frac{x^2 - 1}{x^2 + 1}\, dx$

rational expression and degree num ≥ degree denominator

↳ Long Division

$$\begin{array}{r} 1 \\ x^2+1 \overline{\big) x^2 - 1} \\ -(x^2+1) \\ \hline -2 \end{array}$$

$$= \int \left(1 - \frac{2}{x^2+1}\right) dx$$

$$\boxed{= x - 2\tan^{-1} x + C}$$

2) $\int (\sin x + \cos x)^2\, dx$ expand it

$$= \int (\sin^2 x + 2\sin x \cos x + \cos^2 x)\, dx$$

$$= \int (1 + 2\sin x \cos x)\, dx$$

$$= \int 1\, dx + \int 2\sin x \cos x\, dx$$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x\, dx$

$$+ \int 2u\, du$$

$$x \quad + \frac{2u^2}{2} + C$$

$$\boxed{= x + \sin^2 x + C}$$

3) $\int \frac{2}{x^2 - 6x + 10}\, dx$

* complete the square

$$\int \frac{2}{\underbrace{x^2 - 6x + 9} + \underbrace{10 - 9}} dx$$

$$\int \left(\frac{2}{\underbrace{(x-3)^2} + \underbrace{1}}\right) dx$$

$$\int \frac{2}{(x-3)^2 + 1}\, dx$$

$$\boxed{2\tan^{-1}(x-3) + C}$$

4) $\int \frac{3x+2}{\sqrt{1-x^2}}\,dx$

$$\int \frac{3x}{\sqrt{1-x^2}}\,dx + \int \frac{2}{\sqrt{1-x^2}}\,dx$$

$u = 1 - x^2$

$\frac{du}{dx} = -2x$

$\frac{du}{-2} = x\,dx$

$\frac{3}{-2}\int u^{-\frac{1}{2}}\,du$

$\frac{-3}{2}\;2\;u^{\frac{1}{2}}$

$$\boxed{-3\sqrt{1-x^2} + 2\sin^{-1}x + C}$$

5) $\int \frac{x^5-35x}{x^2+6}\,dx$

Long Division

$$\begin{array}{r|l} & x^3 - 6x \\ \hline x^2+6 & x^5 - 35x \\ & -(x^5 + 6x^3) \\ \hline & -6x^3 - 35x \\ & -(-6x^3 - 36x) \\ \hline & x \end{array}$$

$$\int \left(x^3 - 6x + \frac{x}{x^2+6}\right) dx$$

$$\int \frac{x}{x^2+6}\,dx$$

$u = x^2+6$

$\frac{du}{dx} = 2x$

$\frac{du}{2} = x\,dx$

$\frac{1}{2}\int \frac{1}{u}\,du$

$\frac{1}{2}\ln|u| + C$

$$\boxed{= \frac{x^4}{4} - \frac{6x^2}{2} + \frac{1}{2}\ln|x^2+6| + C}$$

6) $\int \frac{e^x+4}{e^x}\,dx$

$$\int \frac{e^x}{e^x}\,dx + \int \frac{4}{e^x}\,dx$$

$$\int 1\,dx + \int 4e^{-x}\,dx$$

$$\boxed{= x - 4e^{-x} + C}$$

7) $\int \frac{x+1}{x-1}\,dx$

$$\begin{array}{r} 1 \\ x-1 \overline{)\, x+1} \\ \underline{-(x-1)} \\ 2 \end{array}$$

$$\int \left(1 + \frac{2}{x-1}\right) dx$$

$$\boxed{= x + 2\ln|x-1| + C}$$

8) $\int \frac{1}{x^2 + 10x + 26}\,dx$

$$\int \frac{1}{x^2+10x\underline{+25}\;+26-25}\,dx$$

$$\int \frac{1}{(x+5)^2+1}\,dx$$

$$\boxed{= \tan^{-1}(x+5) + C}$$

9) $\int \cot x\,dx$

$$= \int \frac{\cos x}{\sin x}\,dx$$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x\,dx$

$$= \int \frac{1}{u}\,du$$

$$= \ln|u| + c$$

$$\boxed{= \ln|\sin x| + C}$$

10) $\int \tan^2 x\,dx$

$$\int (-1 + \sec^2 x)\,dx$$

$$\boxed{= -x + \tan x + C}$$

* make it look like $\frac{1}{\sqrt{1-x^2}}$

need the 1

11) $\int \frac{1}{\sqrt{4-x^2}}\,dx$

$$\int \frac{1}{\sqrt{4(1-\frac{x^2}{4})}}\,dx$$

$$\int \frac{1}{2\sqrt{1-(\frac{x}{2})^2}}\,dx$$

$$u=\frac{x}{2} \rightarrow \frac{du}{dx}=\frac{1}{2} \rightarrow 2du=dx$$

$$\frac{2}{2}\int \frac{1}{\sqrt{1-u^2}}\,du$$

$$=\sin^{-1}(u)+c$$

$$\boxed{=\sin^{-1}\left(\underline{\tfrac{x}{2}}\right)+c}$$

→ Prefer it to look like $\frac{1}{1+x^2}$

12) $\int \frac{3}{16+x^2}\,dx$

$$\frac{3}{16}\int \frac{1}{1+\frac{x^2}{16}}\,dx$$

$$=\frac{3}{16}\int \frac{1}{1+(\frac{x}{4})^2}\,dx$$

$$\boxed{=\frac{3}{16}\,4\tan^{-1}\left(\frac{x}{4}\right)+c}$$

needs to be "1"

13) $\int \frac{1}{9+25x^2}\,dx$

$$\frac{1}{9}\int \frac{1}{1+\frac{25x^2}{9}}\,dx$$

$$\frac{1}{9}\int \frac{1}{1+\left(\frac{5x}{3}\right)^2}\,dx$$

$$\boxed{= \frac{1}{9}\,\frac{3}{5}\tan^{-1}\left(\frac{5x}{3}\right) + C}$$

14) $\int (x+1)\sqrt{2-x}\,dx$

$u = 2-x \longrightarrow x = 2-u$

$\frac{du}{dx} = -1$

$-du = dx$

$$-\int (2-u+1)\sqrt{u}\,du$$

$$-\int (3-u)u^{\frac{1}{2}}\,du$$

$$-\int \left(3u^{\frac{1}{2}} - u^{\frac{3}{2}}\right)du$$

$$= -3\,\frac{2}{3}u^{\frac{3}{2}} + \frac{2}{5}u^{\frac{5}{2}} + C$$

$$\boxed{= -2(2-x)^{\frac{3}{2}} + \frac{2}{5}(2-x)^{\frac{5}{2}} + C}$$

15) $\int \frac{2x+1}{\sqrt{x+4}}\,dx$

$$\int \frac{2x}{\sqrt{x+4}}dx + \int \frac{1}{\sqrt{x+4}}dx$$

$u = x+4 \leftrightarrow x = u-4$

$\frac{du}{dx} = 1 \rightarrow du = dx$

$$\int \frac{2u-8}{\sqrt{u}}\,du + \int \frac{1}{\sqrt{x+4}}dx$$

$$\int \frac{2u}{\sqrt{u}}du - 8\int \frac{1}{\sqrt{u}}du + \int \frac{1}{\sqrt{x+4}}dx$$

$$= \int 2u^{\frac{1}{2}}du - 8\int u^{-\frac{1}{2}}du + \int (x+4)^{-\frac{1}{2}}dx$$

$$= 2\,\frac{2}{3}u^{\frac{3}{2}} - 8\;2u^{\frac{1}{2}} + 2(x+4)^{\frac{1}{2}} + c$$

$$\boxed{= \frac{4}{3}(x+4)^{\frac{3}{2}} - 16(x+4)^{\frac{1}{2}} + 2(x+4)^{\frac{1}{2}} + c}$$

16) $\int \frac{1}{x\sqrt{4x^2-9}}\,dx$

$$\int \frac{1}{x\sqrt{9\left(\frac{4x^2}{9}-1\right)}}dx$$

$$\int \frac{1}{3\cancel{x}\sqrt{\left(\frac{2x}{3}\right)^2-1}}dx$$

$u = \frac{2x}{3} \leftrightarrow \frac{3u}{2} = x$

$\frac{du}{dx} = \frac{2}{3}$

$\frac{3}{2}du = dx$

$$\frac{3}{2}\int \frac{1}{3\,\frac{3}{2}u\sqrt{u^2-1}}du$$

$$\Rightarrow \frac{3}{2}\int \frac{1}{\frac{9}{2}u\sqrt{u^2-1}}du$$

$$\Rightarrow \frac{3}{2}\;\frac{2}{9}\int \frac{1}{u\sqrt{u^2-1}}du$$

$$\Rightarrow \frac{1}{3}\sec^{-1}u + c$$

$$\boxed{\Rightarrow \frac{1}{3}\sec^{-1}\left(\frac{2x}{3}\right) + c}$$

AP Calculus Name________________________________

More Convoluted Cray U-Sub Notes

Date________________ Period____

Evaluate each indefinite integral.

1) $\int \frac{x^2 - 1}{x^2 + 1}\, dx$

$x - 2\tan^{-1} x + C$

2) $\int (\sin x + \cos x)^2\, dx$

$x + \sin^2 x + C$

3) $\int \frac{2}{x^2 - 6x + 10}\, dx$

$2\tan^{-1} (x - 3) + C$

4) $\int \frac{3x + 2}{\sqrt{1 - x^2}}\, dx$

$-3\sqrt{1 - x^2} + 2\sin^{-1} x + C$

5) $\int \frac{x^5 - 35x}{x^2 + 6}\, dx$ $\frac{1}{4}x^4 - 3x^2 + \frac{1}{2} \cdot \ln \left|x^2 - 16\right| + C$

6) $\int \frac{e^x + 4}{e^x}\, dx$

$x - 4e^{-x} + C$

7) $\int \frac{x + 1}{x - 1} \, dx$

$x + 2\ln \left|x - 1\right| + C$

8) $\int \frac{1}{x^2 + 10x + 26} \, dx$

$\tan^{-1} \left(x + 5\right) + C$

9) $\int \cot x \, dx$

$\ln \left|\sin x\right| + C$

10) $\int \tan^2 x \, dx$

$\tan x - x + C$

11) $\int \frac{1}{\sqrt{4 - x^2}}\, dx$ $\sin^{-1} \frac{x}{2} + C$

12) $\int \frac{3}{16 + x^2}\, dx$

$\frac{3}{4} \cdot \tan^{-1} \frac{x}{4} + C$

13) $\int \frac{1}{9 + 25x^2}\, dx$ $\frac{1}{15} \cdot \tan^{-1} \frac{5x}{3} + C$

14) $\int (x + 1)\sqrt{2 - x}\, dx$

$-2(2 - x)^{\frac{3}{2}} + \frac{2}{5}(2 - x)^{\frac{5}{2}} + C$

15) $\int \frac{2x+1}{\sqrt{x+4}}\,dx$ $\frac{4}{3}(x+4)^{\frac{3}{2}} - 14(x+4)^{\frac{1}{2}} + C$

16) $\int \frac{1}{x\sqrt{4x^2-9}}\,dx$ $\frac{1}{3} \cdot \sec^{-1}\frac{2x}{3} + C$