

More Convolved Cray U-Sub Notes

Evaluate each indefinite integral.

1) $\int \frac{x^2 - 1}{x^2 + 1} dx$

rational expression
and degree num \geq degree
denominator }
↳ Long DIVISION

$$\begin{array}{r} 1 \\ x^2+1 \overline{) x^2-1} \\ \underline{-(x^2+1)} \\ -2 \end{array} = \int \left(1 - \frac{2}{x^2+1} \right) dx$$

$$= x - 2 \tan^{-1} x + C$$

2) $\int (\sin x + \cos x)^2 dx$

expand it ~

$$= \int (\sin^2 x + 2 \sin x \cos x + \cos^2 x) dx$$

$$= \int (1 + 2 \sin x \cos x) dx$$

$$= \int 1 dx + \int 2 \sin x \cos x dx$$

$u = \sin x$
 $\frac{du}{dx} = \cos x$
 $du = \cos x dx$

$$+ \int 2u du$$

$$= x + \frac{2u^2}{2} + C$$

$$= x + \sin^2 x + C$$

3) $\int \frac{2}{x^2 - 6x + 10} dx$

* complete
the
square

$$\int \frac{2}{x^2 - 6x + 9 + 10 - 9} dx$$

$$\int \left(\frac{2}{(x-3)^2 + 1} \right) dx$$

$$\int \frac{2}{(x-3)^2 + 1} dx$$

$$2 \tan^{-1}(x-3) + C$$

$$4) \int \frac{3x+2}{\sqrt{1-x^2}} dx$$

$$\int \frac{3x}{\sqrt{1-x^2}} dx + \int \frac{2}{\sqrt{1-x^2}} dx$$

$$u = 1-x^2$$

$$\frac{du}{dx} = -2x$$

$$\frac{du}{-2} = x dx$$

$$\int \frac{1}{2} u^{-\frac{1}{2}} du$$



$$-3\sqrt{1-x^2} + 2\sin^{-1}x + C$$

$$6) \int \frac{e^x+4}{e^x} dx$$

$$\int \frac{e^x}{e^x} dx + \int \frac{4}{e^x} dx$$

$$\int 1 dx + \int 4e^{-x} dx$$

$$= X - 4e^{-x} + C$$

$$5) \int \frac{x^5 - 35x}{x^2 + 6} dx$$

Long Division

$$\begin{array}{r} x^3 - 6x \\ x^2 + 6 \overline{) x^5 - 35x} \\ \underline{-(x^5 + 6x^3)} \\ -6x^3 - 35x \\ \underline{-(-6x^3 - 36x)} \\ x \end{array}$$

$$\int \left(x^3 - 6x + \frac{x}{x^2+6} \right) dx$$

$$\int \frac{x}{x^2+6} dx$$

$$u = x^2 + 6$$

$$\frac{du}{dx} = 2x$$

$$\frac{du}{2} = x dx$$

$$\frac{1}{2} \int \frac{1}{u} du$$

$$\frac{1}{2} \ln|u| + C$$

$$= \frac{x^4}{4} - \frac{6x^2}{2} + \frac{1}{2} \ln|x^2+6| + C$$

$$7) \int \frac{x+1}{x-1} dx$$

$$\begin{array}{r} x-1 \overline{) x+1} \\ \underline{-(x-1)} \\ 2 \end{array}$$

$$\int \left(1 + \frac{2}{x-1} \right) dx$$

$$= x + 2 \ln|x-1| + C$$

$$8) \int \frac{1}{x^2 + 10x + 26} dx$$

$$\int \frac{1}{x^2 + 10x + 25 + 26 - 25} dx$$

$$\int \frac{1}{(x+5)^2 + 1} dx$$

$$= \tan^{-1}(x+5) + C$$

$$9) \int \cot x dx$$

$$= \int \frac{\cos x}{\sin x} dx$$

$$u = \sin x$$

$$\frac{du}{dx} = \cos x$$

$$du = \cos x dx$$

$$= \int \frac{1}{u} du$$

$$= \ln|u| + C$$

$$= \ln|\sin x| + C$$

$$10) \int \tan^2 x dx$$

$$\int (1 + \sec^2 x) dx$$

$$= -x + \tan x + C$$

* make it look like
need the 1 $\frac{1}{\sqrt{1-x^2}}$

$$11) \int \frac{1}{\sqrt{4-x^2}} dx$$

$$\int \frac{1}{\sqrt{4(1-\frac{x^2}{4})}} dx$$

$$\int \frac{1}{2\sqrt{1-(\frac{x}{2})^2}} dx$$

$$u = \frac{x}{2} \rightarrow \frac{du}{dx} = \frac{1}{2} \rightarrow 2 du = dx$$

$$\frac{2}{2} \int \frac{1}{\sqrt{1-u^2}} du$$

$$= \sin^{-1}(u) + c$$

$$= \sin^{-1}\left(\frac{x}{2}\right) + c$$

→ Prefer it to look like $\frac{1}{1+x^2}$

$$12) \int \frac{3}{16+x^2} dx$$

$$\frac{3}{16} \int \frac{1}{1+\frac{x^2}{16}} dx$$

$$= \frac{3}{16} \int \frac{1}{1+(\frac{x}{4})^2} dx$$

$$= \frac{3}{16} 4 \tan^{-1}\left(\frac{x}{4}\right) + c$$

13) $\int \frac{1}{9+25x^2} dx$

$\frac{1}{9} \int \frac{1}{1+\frac{25x^2}{9}} dx$

$\frac{1}{9} \int \frac{1}{1+(\frac{5x}{3})^2} dx$

$= \frac{1}{9} \cdot \frac{3}{5} \tan^{-1}\left(\frac{5x}{3}\right) + C$

needs to be "1"

14) $\int (x+1)\sqrt{2-x} dx$

$u = 2-x \rightarrow x = 2-u$
 $\frac{du}{dx} = -1$
 $-du = dx$

$-\int (2-u+1)\sqrt{u} du$

$-\int (3-u)u^{\frac{1}{2}} du$

$-\int (3u^{\frac{1}{2}} - u^{\frac{3}{2}}) du$

$= -3 \frac{2}{3} u^{\frac{3}{2}} + \frac{2}{5} u^{\frac{5}{2}} + C$

$= -2(2-x)^{\frac{3}{2}} + \frac{2}{5}(2-x)^{\frac{5}{2}} + C$

$$15) \int \frac{2x+1}{\sqrt{x+4}} dx$$

$$\int \frac{2x}{\sqrt{x+4}} dx + \int \frac{1}{\sqrt{x+4}} dx$$

$$u = x+4 \leftrightarrow x = u-4$$

$$\frac{du}{dx} = 1 \rightarrow du = dx$$

$$\int \frac{2u-8}{\sqrt{u}} du + \int \frac{1}{\sqrt{x+4}} dx$$

$$\int \frac{2u}{\sqrt{u}} du - 8 \int \frac{1}{\sqrt{u}} du + \int \frac{1}{\sqrt{x+4}} dx$$

$$= \int 2u^{\frac{1}{2}} du - 8 \int u^{-\frac{1}{2}} du + \int (x+4)^{-\frac{1}{2}} dx$$

$$= 2 \cdot \frac{2}{3} u^{\frac{3}{2}} - 8 \cdot 2 u^{\frac{1}{2}} + 2(x+4)^{\frac{1}{2}} + C$$

$$= \frac{4}{3} (x+4)^{\frac{3}{2}} - 16(x+4)^{\frac{1}{2}} + 2(x+4)^{\frac{1}{2}} + C$$

$$16) \int \frac{1}{x\sqrt{4x^2-9}} dx$$

$$\int \frac{1}{x\sqrt{9(\frac{4x^2}{9}-1)}} dx$$

$$\int \frac{1}{3x\sqrt{(\frac{2x}{3})^2-1}} dx$$

$$u = \frac{2x}{3} \leftrightarrow \frac{3u}{2} = x$$

$$\frac{du}{dx} = \frac{2}{3}$$

$$\frac{3}{2} du = dx$$

$$\frac{3}{2} \int \frac{1}{3x\sqrt{u^2-1}} du$$

$$\Rightarrow \frac{3}{2} \int \frac{1}{\frac{9}{2} u \sqrt{u^2-1}} du$$

$$\Rightarrow \frac{3}{2} \cdot \frac{2}{9} \int \frac{1}{u \sqrt{u^2-1}} du$$

$$\Rightarrow \frac{1}{3} \sec^{-1} u + C$$

$$\Rightarrow \frac{1}{3} \sec^{-1} \left(\frac{2x}{3} \right) + C$$

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Date _____ Period _____

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$x - 2\tan^{-1} x + C$

2) $\int (\sin x + \cos x)^2 dx$

$x + \sin^2 x + C$

3) $\int \frac{2}{x^2 - 6x + 10} dx$

$2\tan^{-1}(x - 3) + C$

$$4) \int \frac{3x+2}{\sqrt{1-x^2}} dx$$

$$-3\sqrt{1-x^2} + 2\sin^{-1} x + C$$

$$5) \int \frac{x^5 - 35x}{x^2 + 6} dx \quad \frac{1}{4}x^4 - 3x^2 + \frac{1}{2} \cdot \ln |x^2 - 16| + C$$

$$6) \int \frac{e^x + 4}{e^x} dx$$

$$x - 4e^{-x} + C$$

$$7) \int \frac{x+1}{x-1} dx$$

$$x + 2 \ln |x-1| + C$$

$$8) \int \frac{1}{x^2 + 10x + 26} dx$$

$$\tan^{-1}(x+5) + C$$

$$9) \int \cot x dx$$

$$\ln |\sin x| + C$$

$$10) \int \tan^2 x dx$$

$$\tan x - x + C$$

$$11) \int \frac{1}{\sqrt{4-x^2}} dx \quad \sin^{-1} \frac{x}{2} + C$$

$$12) \int \frac{3}{16+x^2} dx$$

$$\frac{3}{4} \cdot \tan^{-1} \frac{x}{4} + C$$

$$13) \int \frac{1}{9 + 25x^2} dx = \frac{1}{15} \cdot \tan^{-1} \frac{5x}{3} + C$$

$$14) \int (x+1)\sqrt{2-x} dx$$
$$= -2(2-x)^{\frac{3}{2}} + \frac{2}{5}(2-x)^{\frac{5}{2}} + C$$

$$15) \int \frac{2x+1}{\sqrt{x+4}} dx = \frac{4}{3}(x+4)^{\frac{3}{2}} - 14(x+4)^{\frac{1}{2}} + C$$

$$16) \int \frac{1}{x\sqrt{4x^2-9}} dx = \frac{1}{3} \cdot \sec^{-1} \frac{2x}{3} + C$$