Pre-Calculus

Name_____

More Trig Identities Notes

Date_____

Verify the identify.

1.) $\dfrac{\sec x}{\cot x + \tan x} = \sin x$

$$\frac{\frac{1}{\cos x}}{\frac{\cos x}{\cos x}\cdot\frac{\cos x}{\sin x}+\frac{\sin x}{\cos x}\cdot\frac{\sin x}{\sin x}}$$

$$\frac{1}{\frac{\cos^2 x+\sin^2 x}{\cos x\sin x}}\cdot\frac{\cos x\sin x}{1}$$

$$\frac{\cos x\sin x}{\cos x}$$

$$=\sin x \checkmark \text{ RHS}$$

2.) $\dfrac{\sin x + \tan x}{\cos x + 1} = \tan x$

$$\frac{\frac{\cos x}{\cos x}\cdot\frac{\sin x}{1}+\frac{\sin x}{\cos x}}{\frac{\cos x+1}{1}}\cdot\frac{1}{\cos x+1}$$

$$\frac{\cos x\sin x+\sin x}{\cos x(\cos x+1)}$$

$$\frac{\sin x(\cos x+1)}{\cos x(\cos x+1)}=\tan x \checkmark \text{RHS}$$

3.) $\dfrac{\cos x - \csc x}{\sin x - \sec x} = \cot x$

$$\frac{\frac{\sin x}{\sin x}\cdot\frac{\cos x}{1}-\frac{1}{\sin x}}{\frac{\cos x}{\cos x}\cdot\frac{\sin x}{1}-\frac{1}{\cos x}}\rightarrow\frac{\frac{\sin x\cdot\cos x-1}{\sin x}}{\frac{\cos x\cdot\sin x-1}{\cos x}}$$

$$\frac{\sin x\cos x-1}{\sin x}\cdot\frac{\cos x}{\cos x\cdot\sin x-1}$$

$$=\cot x \rightarrow \text{RHS}\checkmark$$

4.) $\dfrac{\tan x - \sin x}{\tan x \ \sin x} = \dfrac{1 - \cos x}{\sin x}$

$$\frac{\tan x}{\tan x\sin x}-\frac{\sin x}{\tan\sin x}$$

$$\frac{1}{\sin x}-\frac{1}{\tan x}$$

$$\frac{1}{\sin x}-\frac{\cos x}{\sin x}=\frac{1-\cos x}{\sin x}\ \text{RHS}\checkmark$$

5.) $\dfrac{1+\tan^2 x}{1+\cot^2 x} = \tan^2 x$

$\dfrac{\sec^2 x}{\csc^2 x}$

$\dfrac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}} \rightarrow \dfrac{\sin^2 x}{\cos^2 x} \rightarrow \tan^2 x \rightarrow RHS$

$\dfrac{1}{\cos^2 x} \cdot \dfrac{\sin^2 x}{1}$

6.) $\dfrac{\cos x}{\sec x - 1} + \dfrac{\cos x}{\sec x + 1} = 2\cot^2 x$

$(\sec x + 1)$ $(\sec x - 1)$ $(\sec x - 1)$ $(\sec x + 1)$

$\dfrac{1+\cos x + 1 - \cos x}{\sec^2 x - 1}$

$\dfrac{2}{\tan^2 x} \rightarrow 2\cot^2 x \rightarrow RHS$

7.) $\dfrac{\tan x}{\tan x + 1} = \dfrac{\sin x}{\sin x + \cos x}$ $\cdot \frac{\frac{1}{\cos x}}{\frac{1}{\cos x}}$

✓ $LHS = \dfrac{\tan x}{\tan x + 1}$

8.) $\dfrac{3\tan x}{1+\tan^2 x} = 3\sin x \cos x$

$\dfrac{3\tan x}{\sec^2 x}$

$\dfrac{3\frac{\sin x}{\cos x} \cdot \frac{\cos^2 x}{1}}{\frac{1}{\cos^2 x}} \rightarrow 3\sin x \cos x \rightarrow RHS$ ✓

9.) $\sec^2 x - \cos^2 x - \sin^2 x = \tan^2 x$

$1 + \tan^2 x - 1(\cos^2 x + \sin^2 x)$

$1 + \tan^2 x - 1 = \tan^2 x$ ✓

10.) $(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = 2$

$\sin^2 x + 2\sin x\cos x + \cos^2 x$

$\sin^2 x - 2\sin x\cos x + \cos^2 x$

$1 + 0 + 1 = 2$

11.) $\sin^4 x - \cos^4 x = 2\sin^2 x - 1$

Diff of Squares

$(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)$

$\sin^2 x - \cos^2 x$

$\sin^2 x - (1 - \sin^2 x)$

$\sin^2 x - 1 + \sin^2 x$

$2\sin^2 x - 1 \rightarrow$ RHS ✓

12.) $\frac{\cos x}{1-\sin x} = \frac{1+\sin x}{\cos x}$

$\frac{(1+\sin x)\cos x}{1-\sin^2 x}$

$\frac{(1+\sin x)\cancel{\cos x}}{\cos^2 x}$

$\frac{1+\sin x}{\cos x} \rightarrow$ RHS ✓

13.) $\frac{\sin x}{(1+\cos x)} = \csc x - \cot x$

$\frac{(1-\cos x)\sin x}{1-\cos^2 x}$

$\frac{(1-\cos x)\sin x}{\sin^2 x}$

$\frac{1-\cos x}{\sin x} \rightarrow \frac{1}{\sin x} - \frac{\cos x}{\sin x}$

$= \csc x - \cot x$ RHS ✓

14.) $(\cos x + 2\cot x)(\sin x - 2\tan x) = (\sin x + 2)(\cos x - 2)$

$\cos x \sin x - 2\tan x \cos x + 2\cot x \sin x - 4\cot x \tan x$

$\cos x \cdot \sin x - 2\sin x + 2\cos x - 4$

$\sin x(\cos x - 2) + 2(\cos x - 2)$

$(\sin x + 2)(\cos x - 2) \rightarrow$ RHS ✓