

Verify the identity.

1.) $\frac{\sec x}{\cot x + \tan x} = \sin x$

$$\frac{\frac{1}{\cos x}}{\frac{\cos x \cancel{\cos x} \sin x}{\sin x \cancel{\cos x}} + \frac{\sin x}{\sin x}} = \frac{\frac{1}{\cos x}}{\frac{\sin x}{\sin x}} = \frac{1}{\cos x} \cdot \frac{\sin x \cos x}{1} = \frac{\sin x \cancel{\cos x}}{1} = \sin x = \text{RHS}$$

3.) $\frac{\cos x - \csc x}{\sin x - \sec x} = \cot x$

$$\frac{\frac{\sin x}{\sin x} \cdot \frac{\cos x}{1} - \frac{1}{\sin x}}{\frac{\cos x}{\cos x} \cdot \frac{\sin x}{1} - \frac{1}{\cos x}} = \frac{\frac{\sin x \cos x - 1}{\sin x}}{\frac{\cos x \sin x - 1}{\cos x}} = \frac{\sin x \cos x - 1}{\sin x} \cdot \frac{\cos x}{\cos x \sin x - 1} = \frac{\cos x}{\sin x} = \cot x = \text{RHS}$$

2.) $\frac{\sin x + \tan x}{\cos x + 1} = \tan x$

$$\frac{\frac{\cos x}{\cos x} \frac{\sin x}{1} + \frac{\sin x}{\cos x}}{\cos x + 1} = \frac{\cos x \sin x + \sin x}{\cos x (\cos x + 1)} = \frac{\sin x (\cos x + 1)}{\cos x (\cos x + 1)} = \frac{\sin x}{\cos x} = \tan x = \text{RHS}$$

4.) $\frac{\tan x - \sin x}{\tan x \sin x} = \frac{1 - \cos x}{\sin x}$

$$\frac{\frac{\tan x}{\tan x \sin x} - \frac{\sin x}{\tan x \sin x}}{\frac{1}{\sin x} - \frac{1}{\tan x}} = \frac{\frac{1}{\sin x} - \frac{\cos x}{\sin x}}{\frac{1}{\sin x} - \frac{\cos x}{\sin x}} = \frac{1 - \cos x}{\sin x} = \text{RHS}$$

$$5.) \frac{1 + \tan^2 x}{1 + \cot^2 x} = \tan^2 x$$

$$\frac{\sec^2 x}{\csc^2 x} = \frac{\frac{1}{\cos^2 x} \cdot \frac{\sin^2 x}{1}}{\frac{1}{\sin^2 x}}$$

$$= \tan^2 x = \text{RHS} \quad \checkmark$$

$$7.) \frac{\tan x}{\tan x + 1} = \frac{\sin x}{\sin x + \cos x}$$

$$\frac{\frac{\sin x}{\cos x}}{\frac{\sin x}{\cos x} + \frac{\cos x}{\cos x}}$$

$$= \frac{\sin x}{\sin x + \cos x} = \text{RHS} \quad \checkmark$$

$$6.) \frac{(\sec x + 1) \cos x}{(\sec x + 1)(\sec x - 1)} + \frac{\cos x (\sec x - 1)}{(\sec x + 1)(\sec x - 1)} = 2 \cot^2 x$$

$$\frac{\cos x \sec x + \cos x + \cos x \sec x - \cos x}{(\sec x + 1)(\sec x - 1)}$$

$$= \frac{2 \cos x \sec x}{\sec^2 x - 1} \rightarrow \frac{2}{\tan^2 x} = 2 \cot^2 x \quad \checkmark \text{ RHS}$$

$$8.) \frac{3 \tan x}{1 + \tan^2 x} = 3 \sin x \cos x$$

$$\frac{3 \tan x}{\sec^2 x}$$

$$\rightarrow \frac{3 \sin x \cos^2 x}{\cos x \cdot \frac{1}{\cos^2 x}} \rightarrow 3 \sin x \cos x = \text{RHS} \quad \checkmark$$

$$9.) \sec^2 x - \cos^2 x - \sin^2 x = \tan^2 x$$

$$\sec^2 x - (\cos^2 x + \sin^2 x)$$

$$\sec^2 x - 1$$

$$\tan^2 x + 1 - 1$$

$$\tan^2 x = \text{RHS} \quad \checkmark$$

10.)

$$(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = 2$$

$$\begin{aligned}
 & \sin^2 x + 2\sin x \cos x + \cos^2 x \\
 & + \sin^2 x - 2\sin x \cos x + \cos^2 x \\
 \hline
 & 1 + 1 = 2 \text{ RHS } \checkmark
 \end{aligned}$$

* If I don't see that $\rightarrow 2\sin^2 x + 2\cos^2 x \rightarrow 2(\sin^2 x + \cos^2 x) = 2 \cdot 1 = 2$

11.) $\sin^4 x - \cos^4 x = 2\sin^2 x - 1$

Diff of square

$$\begin{aligned}
 & (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) \\
 & \qquad \qquad \qquad \underline{\qquad 1 \qquad} \\
 & \sin^2 x - \cos^2 x \\
 & \sin^2 x - (1 - \sin^2 x) \\
 & \sin^2 x - 1 + \sin^2 x \\
 & = 2\sin^2 x - 1 = \text{RHS } \checkmark
 \end{aligned}$$

12.) $\frac{1+\sin x}{1+\sin x} \frac{\cos x}{1-\sin x} = \frac{1+\sin x}{\cos x}$

$$\frac{(1+\sin x) \cos x}{1-\sin^2 x}$$

$$\frac{(1+\sin x) \cos x}{\cos^2 x} = \frac{1+\sin x}{\cos x} = \text{RHS } \checkmark$$

13.) $\frac{\sin x}{1+\cos x} = \csc x - \cot x$

$$\begin{aligned}
 & \frac{\sin x (1-\cos x)}{(1+\cos x)(1-\cos x)} \\
 & = \frac{\sin x (1-\cos x)}{1-\cos^2 x} \\
 & = \frac{\sin x (1-\cos x)}{\sin^2 x} = \frac{1-\cos x}{\sin x}
 \end{aligned}$$

$\csc x - \cot x = \text{RHS } \checkmark$
 $\frac{1}{\sin x} - \frac{\cos x}{\sin x}$

14.) $(\cos x + 2 \cot x)(\sin x - 2 \tan x) = (\sin x + 2)(\cos x - 2)$

$$\begin{aligned}
 & \cos x \sin x - 2 \cos x \tan x + 2 \cot x \sin x - 4 \cot x \tan x \\
 & \cos x \cdot \sin x - 2 \cos x \frac{\sin x}{\cos x} + 2 \frac{\cos x}{\sin x} \sin x - 4 \\
 & \cos x \sin x - 2 \sin x + 2 \cos x - 4 \\
 & \sin x (\cos x - 2) \quad 2(\cos x - 2) \\
 & (\sin x + 2)(\cos x - 2) = \text{RHS } \checkmark
 \end{aligned}$$