7a. *[3 marks]*

Let $f(x) = 3\sin\left(\frac{\pi}{2}x\right)$, for $0 \leqslant x \leqslant 4$.

(i) Write down the amplitude of f.

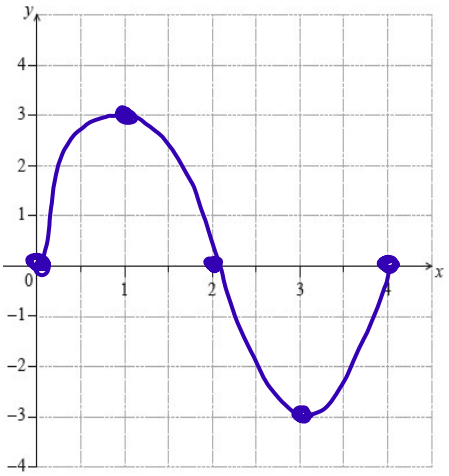
(ii) Find the period of f.

(i) 3

(ii) $\frac{2\pi}{\pi/2} = 4$

7b. *[4 marks]*

On the following grid sketch the graph of f.



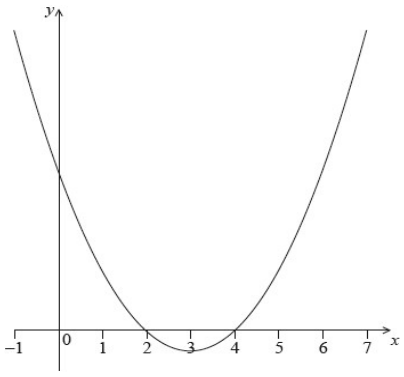
SIN → MID
MAX
MID
MIN
MID

count by $\frac{P}{4} = \frac{4}{4} = 1$

8a. *[2 marks]*

The following diagram shows part of the graph of a quadratic function f.

The vertex is at $(3, -1)$ and the x-intercepts at 2 and 4.



The function f can be written in the form $f(x) = (x-h)^2 + k$.

$f(x) = (x-3)^2 - 1$

Write down the value of h and of k.

$h = 3$

$k = -1$

8b. *[2 marks]*

$(x-2)(x-4)$

The function can also be written in the form $f(x) = (x - a)(x - b)$.

Write down the value of a and of b.

$a = 2$
$b = 4$

8c. *[2 marks]*

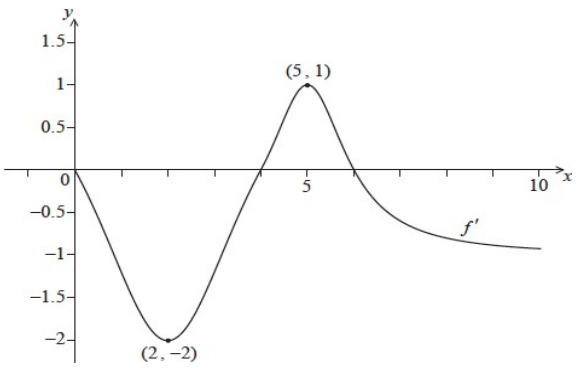
Find the y-intercept.

$x = 0 \rightarrow (0-2)(0-4)$

$y\text{-int} = 8$

9a. *[3 marks]*

Consider a function f, for $0 \leq x \leq 10$. The following diagram shows the graph of f', the derivative of f.



The graph of f' passes through $(2, -2)$ and $(5, 1)$, and has x-intercepts at 0, 4 and 6.

The graph of f has a local maximum point when $x = p$. State the value of p, and justify your answer.

$p = 6$ f' changes + to −

9b. *[1 mark]*

Write down $f'(2)$. -2

9c. *[4 marks]*

composition

Let $g(x) = \ln(f(x))$ and $f(2) = 3$.

Find $g'(2)$.

$g'(x) = \frac{1}{f(x)} \cdot f'(x)$

$g'(2) = \frac{1}{f(2)} \cdot f'(2) = \frac{1}{3} \cdot -2$ or $-\frac{2}{3}$

9d. *[4 marks]*

Verify that $\ln 3 + \int_2^a g'(x)\mathrm{d}x = g(a)$, where $0 \leq a \leq 10$.

$\ln 3 + g(x)\Big|_2^a$

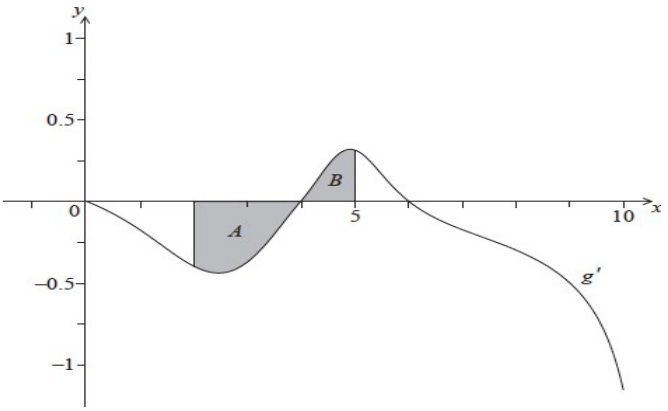
$g(2) = \ln(f(2)) = \ln 3$

$\ln 3 + g(a) - g(2)$

$\ln 3 + g(a) - \ln 3 = g(a)$ ✓

9e. *[4 marks]*

The following diagram shows the graph of g', the derivative of g.



$g(5) = g(2) + \int_2^5 g'(x)\,dx$

$g(5) = \ln 3 + (-.66) + .21$

$= \ln 3 - .45$

The shaded region A is enclosed by the curve, the x-axis and the line $x = 2$, and has area 0.66 units^2.

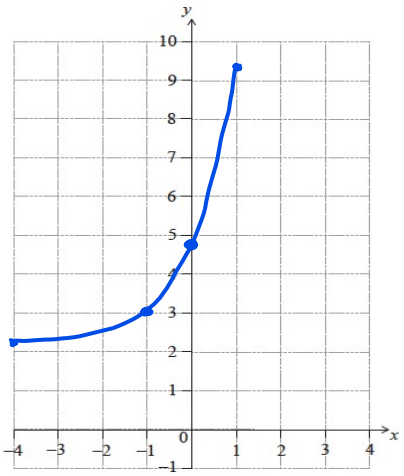
The shaded region B is enclosed by the curve, the x-axis and the line $x = 5$, and has area 0.21 units^2.

Find $g(5)$.

10a. *[3 marks]*

Let $f(x) = e^{x+1} + 2$, for $-4 \leq x \leq 1$.

On the following grid, sketch the graph of f.



$f(-4) = e^{-4+1} + 2 \approx 2.05$

$f(1) = e^{1+1} + 2 \approx 9.39$

$f(-1) = e^{0} + 2 = 3$

$f(0) = e + 2$

10b. *[3 marks]*

The graph of f is translated by the vector $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ to obtain the graph of a function g.

→ right

↳ down

Find an expression for $g(x)$.

Inside $x+1$ → take 3 away $x+1-3 = x-2$

outside 2 → take 1 away 1

$g(x) = e^{x-2} + 1$

11a. *[1 mark]*

The line L_1 passes through the points $\mathrm{A}(2, 1, 4)$ and $\mathrm{B}(1, 1, 5)$.

Show that $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

11b. *[1 mark]*

Hence, write down a direction vector for L_1;

11c. *[2 marks]*

Hence, write down a vector equation for L_1.