AB Calculus Optimization Day 1 Homework   Name: Key

1. Find two positive numbers such that their product is 192 and the sum of the first plus three times the second is a minimum.

$x \cdot y = 192 \qquad x + 3y = S$

$\frac{192}{y} + 3y = S$

$S' = \frac{-192}{y^2} + 3$

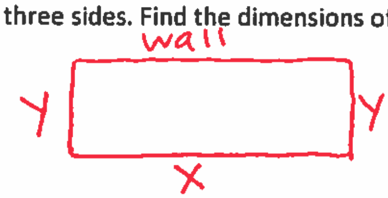
$-3y^2 = -192$

$y^2 = 64$

$y = 8$, so $x = 24$

$\boxed{8 \,\&\, 24}$

2. A gardener wants to make a rectangular enclosure using a wall as one side and 120 m of fencing for the other three sides. Find the dimensions of the garden that will maximize the area of the garden.



$x + 2y = 120 \rightarrow x = 120 - 2y$

$A = xy \rightarrow (120 - 2y)y = A$

$A = -2y^2 + 120y$

$A' = -4y + 120$

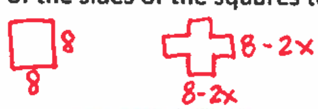
$-120 = -4y$

$y = 30$

, so $x = 60$

$\boxed{60\text{ m by }30\text{ m}}$

3. A tinsmith wishes to make an open top box from a square piece of tin which measures 8 in. by 8 in. To accomplish this task, he proposes to cut equal square pieces from each corner of the tin and fold up the tin to form sides. Determine the length of the sides of the squares to be cut from the corners so that the box will have the greatest possible volume.



$V = x \cdot (8 - 2x) \cdot (8 - 2x)$

$V = x(64 - 32x + 4x^2)$

$V = 64x - 32x^2 + 4x^3$

$V' = 64 - 64x + 12x^2 \rightarrow \div 4$

$0 = 4(16 - 16x + 3x^2)$

$0 = 4(3x - 4)(x - 4)$

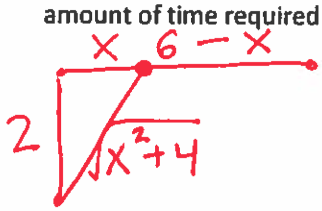
$\frac{16 \pm \sqrt{256 - 4(3)(16)}}{6}$

$\frac{16 \pm 8}{6} = \frac{24}{6} = 4 \quad \frac{8}{6} = \frac{4}{3}$

$+ \underset{4/3}{\overset{\text{max}}{|}} - \underset{4}{\overset{\text{min}}{|}} +$

$\boxed{x = \frac{4}{3}\text{ in}}$

4. A person in a rowboat, two miles from the nearest point on a straight shoreline wishes to reach a house six miles farther down the shore. If the person can row at a rate of 3 mi/h and walk at a rate of 5 mi/hr. find the least amount of time required to reach the house. How far from the house should the person land the rowboat?



$D = RT$ or $T = \frac{D}{R}$

$T_{Row} = \frac{\sqrt{x^2+4}}{3}$

$T_{Walk} = \frac{6 - x}{5}$

$T = \frac{\sqrt{x^2+4}}{3} + \frac{6-x}{5} \rightarrow \boxed{T(1.5) = 1.7\overline{33}\text{ hours}}$

$\frac{1}{3} \cdot \frac{1}{2}(x^2+4)^{-\frac{1}{2}}(2x) - \frac{1}{5} \rightarrow \frac{1x}{3\sqrt{x^2+4}} - \frac{1}{5} = 0$

$\boxed{6 - \frac{3}{2} = 4.5\text{ miles}}$

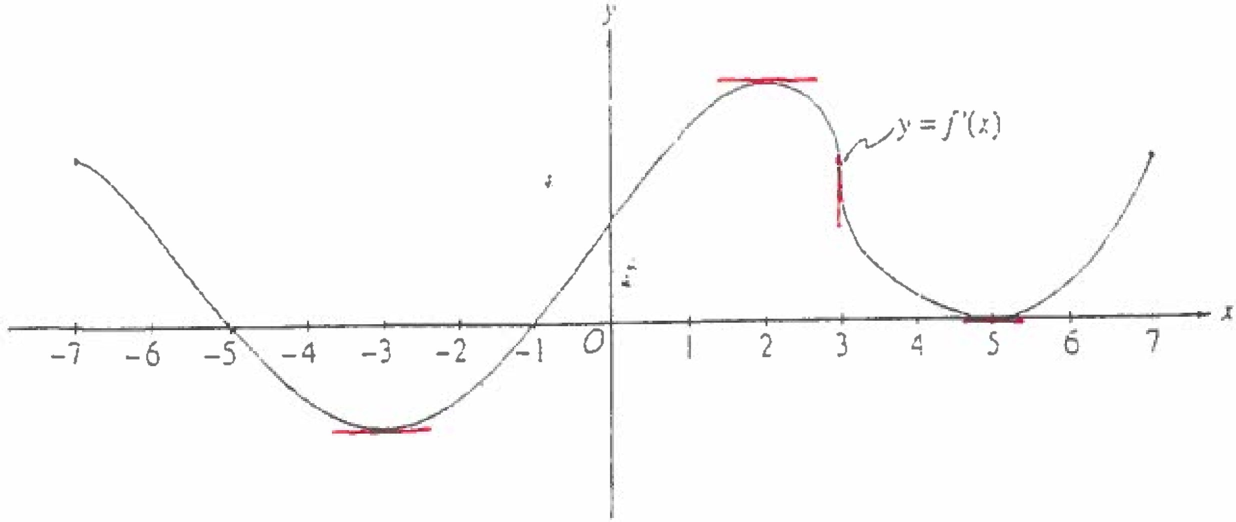
$5x = 3\sqrt{x^2+4}$

$25x^2 = 9x^2 + 36$

$16x^2 = 36$

$x^2 = \frac{36}{16} \quad x = \frac{6}{4} = \frac{3}{2}$

5.



The figure above shows the graph of f', the derivative of the function f, for $-7 \le x \le 7$. The graph of f' has horizontal tangent lines at $x = -3$, $x = 2$, and $x = 5$, and a vertical tangent line at $x = 3$.

a) Find all the values of x, for $-7 < x < 7$, at which f attains a relative minimum. Justify your answer.

$x = -1$ b/c $f'(x)$ changes from − to +

b) Find all the values of x, for $-7 < x < 7$, at which f attains a relative maximum. Justify your answer.

$x = -5$ b/c $f'(x)$ " " + to −

c) Find all the values of x, for $-7 < x < 7$, at which $f''(x) < 0$.

$x = (-7,-3) \cup (2,3) \cup (3,5)$ b/c $f''(x)$ is the slope of tangent line to $f'(x)$ and $f'(x)$ is decreasing

d) Find all the values of x, for $-7 < x < 7$, at which f has a point of inflection.

$x = -3, 2, 5$ b/c $f''(x)$ changes signs

6. Determine if the Mean Value Theorem applies to the function $f(x) = |x|$ over the interval $[-2, 2]$. If it does not, explain why. If it does, find the value of c guaranteed by the theorem.

No. B/C $f(x)=|x|$ is not diff. on $(-2,2)$

↳ not diff. @ $x=0$