AB Calculus Optimization Day 3 Homework Name: Key

1. Find two positive numbers such that the sum of the first and twice the second is 100 and their product is a maximum.

$x + 2y = 100$ maximize xy

$x = 100 - 2y$ $\quad \cancel{x(100-2y)}\ (100-2y)y$

$\rightarrow 100y - 2y^2 = P$

$100 - 4y = \dfrac{dP}{dy}$ $\quad 100 - 4y = 0$

$-4y = -100$

$y = 25$

$+ \overset{\text{max}}{|} -$ at 25

$\boxed{y = 25 \quad x = 50}$

2. If 40 passengers hire a special car on a train, they will be charged $8 each. This fare will be reduced by $0.10 for each passenger, if the number of passengers is over 40. What number of passengers will produce the most revenue for the railroad?

max Revenue

$(40)8 = 320$ $\quad$ Revenue $= (40 + p)(8 - .10p)$

$R = 320 - 4p + 8p - .10p^2$

$= 320 + 4p - .10p^2$

$\boxed{p = 20}$

$\dfrac{dR}{dp} = 4 - .20p$ $\quad \dfrac{-4}{-.20} = \dfrac{-.20p}{-.20}$

$0 = 4 - .20p$

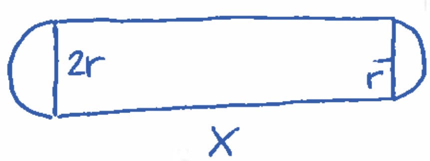
$p = 20$

$+ \overset{\text{max}}{|}$ at 20

$\boxed{\text{Total passengers } 40 + 20 = 60}$

3. An athletic field is to be built in the shape of a rectangle x units long capped by semicircular regions of radius r at the two ends. The field is to be bounded by a 400-m running track. What values of x and r will give the rectangle the largest possible area?

$C_{circle} = 2\pi r$



Perimeter $= x + x + \pi r + \pi r$

$400 = 2x + 2\pi r$

$200 = x + \pi r$

$200 - \pi r = x$

$A_{rectangle} = x(2r)$

$= (200 - \pi r)(2r)$

$A = 400r - 2\pi r^2$

$\dfrac{dA}{dr} = 400 - 4\pi r$

$0 = 400 - 4\pi r$

$\dfrac{-400}{-4\pi} = \dfrac{-4\pi r}{-4\pi}$

$\boxed{r = \dfrac{100}{\pi}\text{ m}}$

$x = 200 - \pi\left(\dfrac{100}{\pi}\right)$

$\boxed{x = 100\text{ m}}$

4. An offshore well is located in the ocean at a point W which is six miles from the closes shore point A on a straight shoreline. The oil is to be piped to a shore point B that is eight miles from A by piping it on a straight line under water from W to some shore point P between A and B and then on to B via a pipe along the shoreline. If the cost of laying pipe is \$100,000 per mile underwater and \$75,000 per mile over land, how far from A should the point P be located to minimize the cost of laying the pipe? What will the cost be?

$x^2 + 6^2 = y^2$

$x^2 + 36 = y^2$

$y = \sqrt{x^2+36}$

Cost = $100{,}000y + 75{,}000(8-x)$

$C = 100{,}000\sqrt{x^2+36} + 75000(8-x)$

$\frac{dC}{dx} = 100{,}000\left(\frac{1}{2}\right)(x^2+36)^{-\frac{1}{2}}(2x) + (-75{,}000)$

$0 = \frac{100{,}000x}{\sqrt{x^2+36}} - 75000$

$75{,}000 = \frac{100{,}000x}{\sqrt{x^2+36}}$

$75{,}000\sqrt{x^2+36} = 100{,}000x$

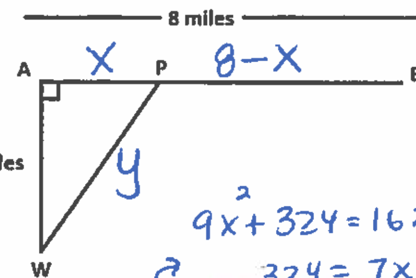
$3\sqrt{x^2+36} = 4x \rightarrow 9(x^2+36) = 16x^2$

$9x^2 + 324 = 16x^2$

$324 = 7x^2$

$\boxed{x = 6.803 \text{ miles}}$

$\boxed{C = \$996{,}862.70}$



5. A function f is continuous on the closed interval $[-3,3]$ such that $f(-3) = 4$ and $f(3) = 1$. The functions f' and f'' have the properties given in the table below.

ccu/inc, max, ccu/dec, p.o.i, ccd/dec

x	$-3 < x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x < 3$
$f'(x)$	positive	does not exist	negative	0	negative
$f''(x)$	positive	does not exist	positive	0	negative

a) What are the x-coordinates of all relative maximum and minimum points of f on the interval $(-3,3)$? Justify your answer. f has relative max @ $x = -1$ b/c $f'(x)$ DNE @ $x=-1$ ~~f has~~ and $f'(x)$ changes signs from + to −. no rel. min. b/c $f'(x)$ never changes − to +.

b) What are the x-coordinates of all points of inflection of f on the interval [-3, 3]? Justify your answer.

f has p.o.i. @ $x = 1$ b/c $f''(x) = 0$ @ $x=1$ and $f''(x)$ changes signs

c) For what values of x is the graph concave down? Justify your answer.

$f(x)$ is CCD on $1 < x < 3$ b/c $f''(x) < 0$

d) On the axes provided, sketch a graph that satisfies the given conditions of f. Answers may vary slightly.

Remember $f(-3) = 4$

$f(3) = 1$