AP Calculus Parametric Functions Homework **Name:** Key

1. For each parametric function, find *dy/dx* and *d²y/dx²* in terms of t.

a) $x = 4\sin t,\quad y = 2\cos t$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \qquad \frac{dx}{dt} = 4\cos t \qquad \frac{dy}{dt} = -2\sin t$$

$$\frac{dy}{dx} = \frac{-2\sin t}{4\cos t} = \boxed{-\frac{1}{2}\tan t}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\frac{dy}{dx}}{\frac{dx}{dt}} \rightarrow \frac{-\frac{1}{2}\sec^2 t}{4\cos t} \rightarrow \boxed{\frac{-1}{8}\sec^3 t}$$

b) $x = t^2 - 1,\quad y = e^{t^3}$

$$\frac{dx}{dt} = 2t \qquad \frac{dy}{dt} = 3t^2 e^{t^3}$$

$$\frac{dy}{dx} = \frac{3t^2 e^{t^3}}{2t} \quad \boxed{= \frac{3}{2}te^{t^3}}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\frac{dy}{dx}}{\frac{dx}{dt}} = \frac{\left(\frac{3}{2}t\right)(3t^2)e^{t^3} + \frac{3}{2}e^{t^3}}{2t} \quad \boxed{= \frac{\frac{3}{2}e^{t^3}(3t^3+1)}{2t}}$$

c) $x = t^2 - 3t,\quad y = t^3$

$$\frac{dx}{dt} = 2t - 3 \qquad \frac{dy}{dt} = 3t^2 \qquad \boxed{\frac{dy}{dx} = \frac{3t^2}{2t-3}}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{(2t-3)(6t)-(3t^2)(2)}{(2t-3)^2}}{2t-3} \rightarrow \frac{12t^2 - 18t - 6t^2}{(2t-3)^3} \rightarrow \boxed{\frac{6t^2-18t}{(2t-3)^3}}$$

2. Find the points where the tangent to the curve is horizontal and vertical.

a) $x = 2 + \cos t,\quad y = -1 + \sin t$

$$\frac{dx}{dt} = -\sin t \qquad \frac{dy}{dt} = \cos t \qquad \frac{dy}{dx} = \frac{\cos t}{-\sin t}$$

H.T $\cos t = 0$

$t = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = 2 + 0 = 2$

$y = -1 + \sin\frac{\pi}{2} = 0$

$-1 + \sin\frac{3\pi}{2} = -2$

$\boxed{\text{HT } (2,0)\ (2,-2)}$

V.T. $-\sin t = 0 \quad t = 0, \pi$

$x = 2 + \cos 0 = 3$

$2 + \cos\pi = 1$

$y = -1 + 0 = -1$

$\boxed{\text{VT } (3,-1)\ (1,-1)}$

b) (Calculator) $x = 2 - t,\ y = t^3 - 4t$

$$\frac{dx}{dt} = -1 \qquad \frac{dy}{dt} = 3t^2 - 4 \qquad \frac{dy}{dx} = -3t^2 + 4$$

No VT

HT $-3t^2 + 4 = 0$

$4 = 3t^2$

$\frac{4}{3} = t^2$

$\pm\sqrt{\frac{4}{3}} = t$

$x = 2 - \left(\pm\sqrt{\frac{4}{3}}\right)$

$y = \left(\pm\sqrt{\frac{4}{3}}\right)^3 - 4\left(\pm\sqrt{\frac{4}{3}}\right)$

HT

$(3.155, 3.079)$

$(.845, -3.079)$

3. Find the length of the curve.

a) $x = \cos t,\quad y = t + \sin t \quad 0 \le t \le \pi$

$$\frac{dx}{dt} = -\sin t \qquad \frac{dy}{dt} = 1 + \cos t \qquad L = \int_0^{\pi} \sqrt{(-\sin t)^2 + (1+\cos t)^2}\,dt \ \boxed{= 4}$$

b) $x = \frac{1}{3}t^3,\quad y = \frac{1}{2}t^2 \quad 0 \le t \le 1$

$$\frac{dx}{dt} = t^2 \qquad \frac{dy}{dt} = t \qquad L = \int_0^1 \sqrt{(t^2)^2 + (t)^2}\,dt \ \boxed{\approx .609}$$

3\. c) Find the length of the curve $x = \ln(\sec t + \tan t) - \sin t$, $y = \cos t$ $\quad 0 \le t \le \frac{\pi}{3}$

$\frac{dx}{dt} = \frac{1}{\sec t + \tan t}(\sec t \tan t + \sec^2 t) - \cos t \rightarrow \frac{\sec t\,\cancel{(\tan t + \sec t)}}{\cancel{\sec t + \tan t}} - \cos t \rightarrow \sec t - \cos t$

$\frac{dy}{dt} = -\sin t$ $\qquad L = \int_0^{\pi/3} \sqrt{(\sec t - \cos t)^2 + (-\sin t)^2}\, dt$ $\boxed{\approx .693}$

4\. A particle moves along the curve $xy = 10$. If $x = 2$ and $\frac{dy}{dt} = 3$, what is the value of $\frac{dx}{dt}$? if $x = 2$, $y = 5$

$\frac{d}{dt}(xy = 10) \rightarrow x\frac{dy}{dt} + \frac{dx}{dt}y = 0 \qquad 2(3) + \frac{dx}{dt}(5) = 0$ $\qquad \boxed{\frac{dx}{dt} = -\frac{6}{5}}$

5\. A curve C is defined by the parametric equations $x = t^3$ and $y = t^2 - 5t + 2$. Write an equation of the line tangent to the graph of C at the point $(8, -4)$. $8 = t^3 \quad t = 2$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \qquad \frac{dx}{dt} = 3t^2 \qquad \frac{dy}{dt} = 2t - 5 \qquad \frac{2t-5}{3t^2} \rightarrow \frac{2(2)-5}{3(2)^2} = \frac{-1}{12}$ $\qquad \boxed{y + 4 = \frac{-1}{12}(x - 8)}$

6\. The position of an object moving in the xy-plane is given by the parametric equations $x = t^3 - \frac{3}{2}t^2 - 18t + 5$ and $y = t^3 - 6t^2 + 9t + 4$. Find the values of t when the object is at rest. (Note: For the object to be at rest, both the x and y components must be 0).

$\frac{dx}{dt} = 3t^2 - 3t - 18$

$3(t^2 - t - 6)$

$3(t - 3)(t + 2)$

$t = 3, -2$

$\frac{dy}{dt} = 3t^2 - 12t + 9$

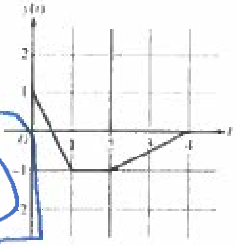
$= 3(t^2 - 4t + 3)$

$3(t - 3)(t - 1)$

$t = 3, 1$

$\boxed{t = 3}$

7\. (Calculator) At time t, the position of a particle moving in the xy-plane is given by the parametric functions $(x(t), y(t))$, where $\frac{dx}{dt} = t^2 + \sin(3t^2)$. The graph of y, consisting of three line segments, is shown to the right. At $t = 0$, the particle is at the point $(5, 1)$.



a) Find the position of the particle at $t = 3$.

$x(3) = 5 + \int_0^3 t^2 + \sin(3t^2)\,dt$ $\boxed{\approx 14.377}$ $\qquad \boxed{y(3) = -\frac{1}{2}}$ $\qquad \boxed{(14.377, -.5)}$

b) Find the slope of the line tangent to the path of the particle at $t = 3$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{1}{2}}{3^2 + \sin(3 \cdot 3^2)}$ → slope from graph $\approx \boxed{.05}$

c) Find the speed of the particle at $t = 3$.

$\text{Speed} = \sqrt{(3^2 + \sin(3 \cdot 3^2))^2 + (\frac{1}{2})^2}$ $\approx \boxed{9.969}$

d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

$\text{Total distance} = \int_0^1 \sqrt{(t^2 + \sin(3t^2))^2 + (-2)^2}\, dt + \int_1^2 \sqrt{(t^2 + \sin(3t^2))^2 + (0)^2}\, dt$

$\boxed{\approx 4.35}$