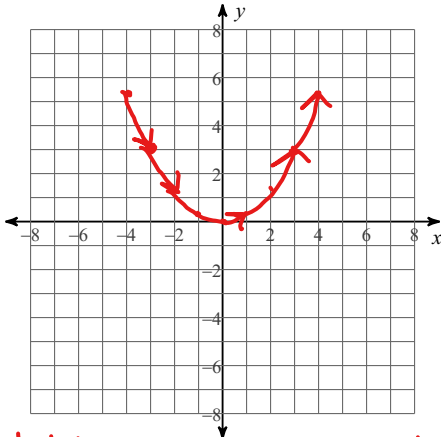


Parametric Equations Notes

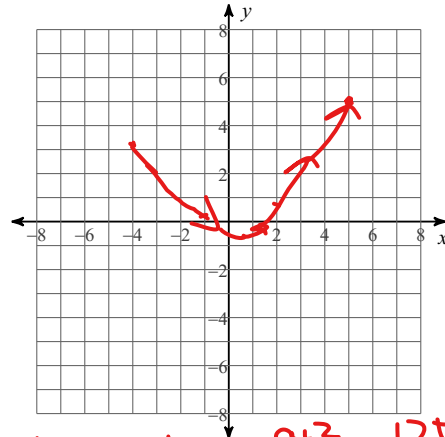
Sketch the curve for each pair of parametric equations.

1) $x = t, y = \frac{t^2}{3}, -4 \leq t \leq 4$



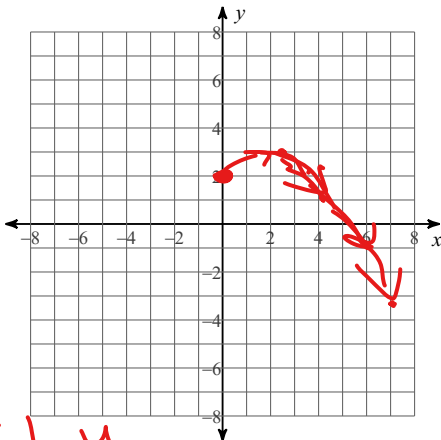
t	x	y	t	x	y
-4	-4	16/3	1	1	1/3
-3	-3	3	2	2	4/3
-2	-2	4/3	3	3	3
-1	-1	1/3	4	4	16/3
0	0	0			

2) $x = 3t + 2, y = \frac{9t^2}{5} + \frac{12t}{5} + \frac{4}{5}, -2 \leq t \leq 1$



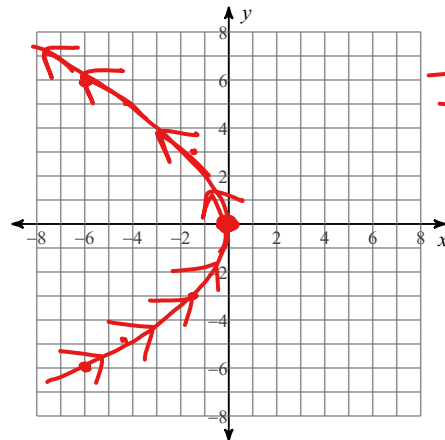
t	x	y = $\frac{9t^2}{5} + \frac{12t}{5} + \frac{4}{5}$
-2	-4	$\frac{36}{5} - \frac{24}{5} + \frac{4}{5} = \frac{16}{5}$
-1	-1	$\frac{9}{5} - \frac{12}{5} + \frac{4}{5} = \frac{1}{5}$
0	2	$\frac{4}{5}$
1	5	$\frac{9}{5} + \frac{12}{5} + \frac{4}{5} = 5$

3) $x = \sqrt{7t}, y = -\frac{7t}{4} + 2 + \sqrt{7t}$



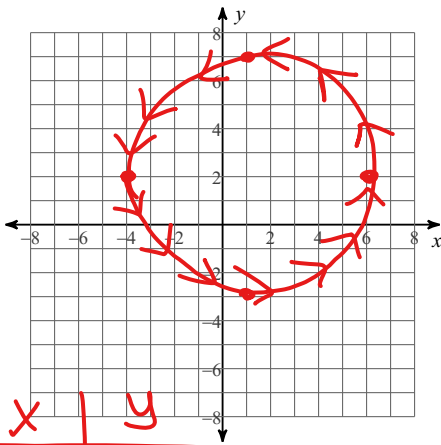
t	x	y
0	0	2
1	$\sqrt{7} \sim 2.6$	$-\frac{7}{4} + 2 + \sqrt{7} \sim 2.9$
2	$\sqrt{14} \sim 3.7$	$-\frac{14}{4} + 2 + \sqrt{14} \sim 2.2$
5	$\sqrt{35} \sim 5.9$	$-\frac{35}{4} + 2 + \sqrt{35} \sim -1$
7	7	$-\frac{49}{4} + 2 + 7 \sim -3.25$

4) $x = -\frac{t^2}{6}, y = t$



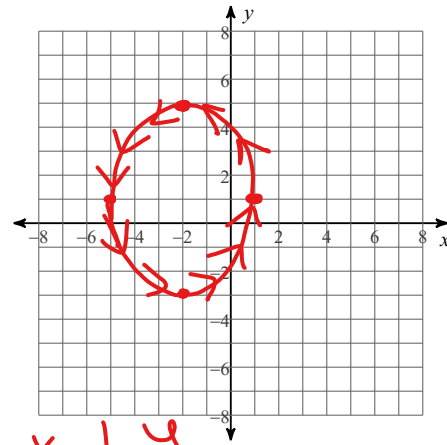
t	x	y
-3	-3/2	-3
-2	-2/3	-2
-1	-1/6	-1
0	0	0
1	-1/6	1
2	-2/3	2
3	-3/2	3
4	-8/3	4
5	-25/6	5
6	-6	6

5) $x = 5\cos t + 1$, $y = 5\sin t + 2$



t	x	y
0	6	2
$\pi/2$	1	7
π	-4	2
$3\pi/2$	1	-3

6) $x = 3\cos t - 2$, $y = 4\sin t + 1$



t	x	y
0	1	1
$\pi/2$	-2	5
π	-5	1
$3\pi/2$	-2	-3

Write each pair of parametric equations in rectangular form. → y in terms of x

7) $x = 2t$, $y = -\frac{t^2}{6}$

$\frac{x}{2} = t$ ↓

$$y = -\frac{\left(\frac{x}{2}\right)^2}{6} = \frac{-x^2}{4 \cdot 6} = -\frac{x^2}{24}$$

Write each pair of parametric equations in rectangular form. State any restrictions on the domain.

8) $x = \sqrt{5t}$, $y = -\frac{5t}{4}$

$x^2 = 5t$ (indicated by a dashed red arrow pointing to the $5t$ term in the second equation)

$$y = -\frac{x^2}{4} \quad x \geq 0$$

Write each pair of parametric equations in rectangular form.

9) $x = \frac{9t^2}{5} - 6t + 4, y = -3t + 3$

$$y - 3 = -3t$$

$$\frac{y - 3}{-3} = t$$

$$x = \frac{9 \left(\frac{y - 3}{-3} \right)^2}{5} - 6 \left(\frac{y - 3}{-3} \right) + 4$$

$$x = \frac{y^2 - 6y + 9}{5} + 2(y - 3) + 4$$

$$x = \frac{y^2}{5} - \frac{6y}{5} + \frac{9}{5} + 2y - 6 + 4$$

$$x = \frac{y^2}{5} + \frac{4y}{5} - \frac{1}{5}$$

11) $x = 3\sin 2t - 2, y = 3\cos 2t + 1$

$$x + 2 = 3\sin 2t$$

$$\frac{x + 2}{3} = \sin 2t$$

$$y - 1 = 3\cos 2t$$

$$\frac{y - 1}{3} = \cos 2t$$

$$\sin^2(2t) + \cos^2(2t) = 1$$

$$\left(\frac{x + 2}{3} \right)^2 + \left(\frac{y - 1}{3} \right)^2 = 1$$

$$(x + 2)^2 + (y - 1)^2 = 9$$

13) $x = 2\sec t, y = 2\tan t$

$$\frac{x}{2} = \sec t, \frac{y}{2} = \tan t$$

$$\left(\frac{y}{2} \right)^2 + 1 = \left(\frac{x}{2} \right)^2$$

10) $x = 5\sin t - 1, y = 5\cos t + 1$

$$\frac{x + 1}{5} = \sin t, \frac{y - 1}{5} = \cos t$$

$$\sin^2 t + \cos^2 t = 1$$

$$\left(\frac{x + 1}{5} \right)^2 + \left(\frac{y - 1}{5} \right)^2 = 1$$

$$(x + 1)^2 + (y - 1)^2 = 25$$

12) $x = 3\sin t + 2, y = 4\cos t + 2$

$$\frac{x - 2}{3} = \sin t, \frac{y - 2}{4} = \cos t$$

$$\left(\frac{x - 2}{3} \right)^2 + \left(\frac{y - 2}{4} \right)^2 = 1$$

$$\frac{(x - 2)^2}{9} + \frac{(y - 2)^2}{16} = 1$$

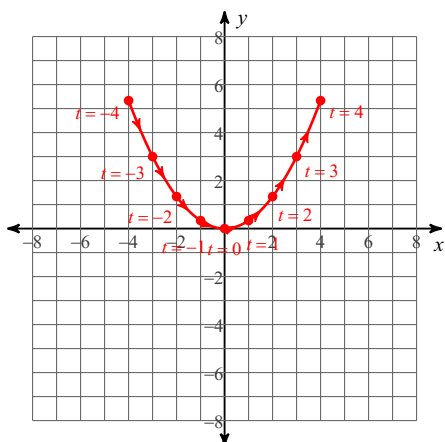
$$\frac{y^2}{4} + 1 = \frac{x^2}{4}$$

$$1 = \frac{x^2}{4} - \frac{y^2}{4}$$

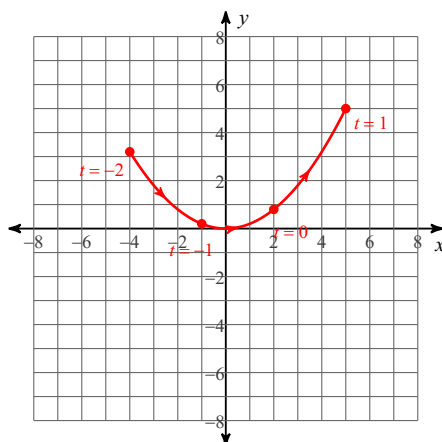
Parametric Equations Notes

Sketch the curve for each pair of parametric equations.

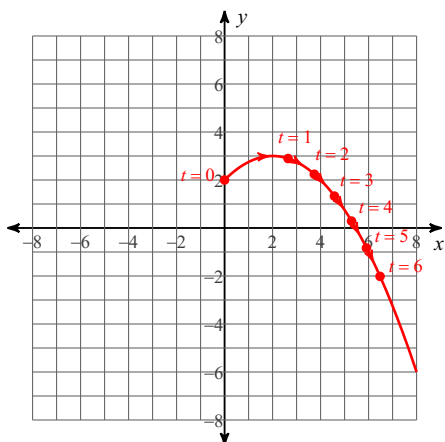
1) $x = t, y = \frac{t^2}{3}, -4 \leq t \leq 4$



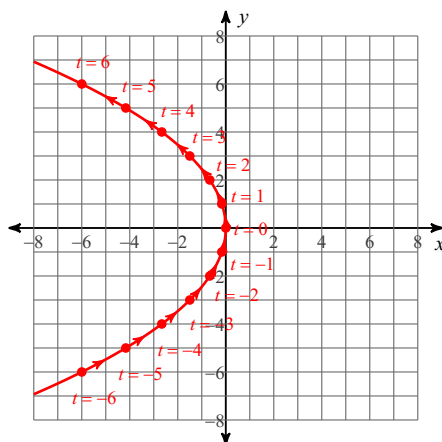
2) $x = 3t + 2, y = \frac{9t^2}{5} + \frac{12t}{5} + \frac{4}{5}, -2 \leq t \leq 1$



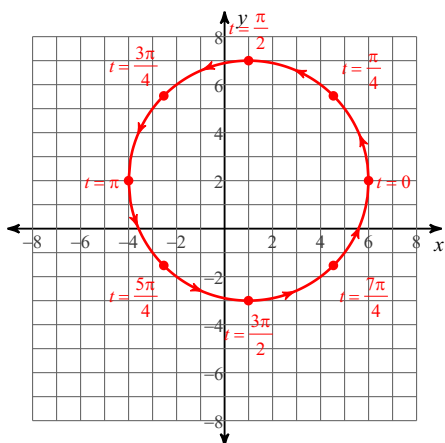
3) $x = \sqrt{7t}, y = -\frac{7t}{4} + 2 + \sqrt{7t}$



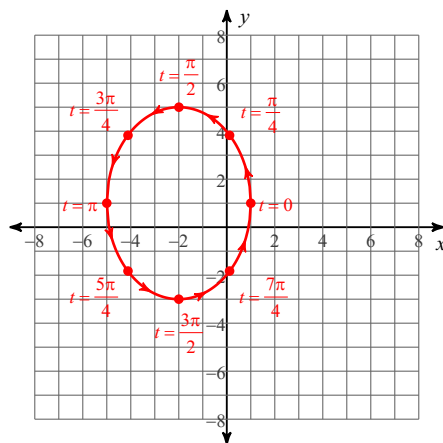
4) $x = -\frac{t^2}{6}, y = t$



5) $x = 5\cos t + 1$, $y = 5\sin t + 2$



6) $x = 3\cos t - 2$, $y = 4\sin t + 1$



Write each pair of parametric equations in rectangular form.

7) $x = 2t$, $y = -\frac{t^2}{6}$

$$y = -\frac{x^2}{24}$$

Write each pair of parametric equations in rectangular form. State any restrictions on the domain.

8) $x = \sqrt{5t}$, $y = -\frac{5t}{4}$

$$y = -\frac{x^2}{4}, x \geq 0$$

Write each pair of parametric equations in rectangular form.

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$$x = \frac{y^2}{5} + \frac{4y}{5} - \frac{1}{5}$$

10) $x = 5\sin t - 1, y = 5\cos t + 1$

$$\frac{(x+1)^2}{25} + \frac{(y-1)^2}{25} = 1$$

11) $x = 3\sin 2t - 2, y = 3\cos 2t + 1$

$$\frac{(x+2)^2}{9} + \frac{(y-1)^2}{9} = 1$$

12) $x = 3\sin t + 2, y = 4\cos t + 2$

$$\frac{(x-2)^2}{9} + \frac{(y-2)^2}{16} = 1$$

13) $x = 2\sec t, y = 2\tan t$

$$\frac{x^2}{4} - \frac{y^2}{4} = 1$$