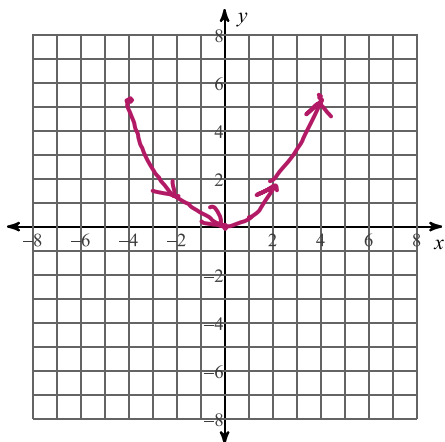


Parametric Equations Notes

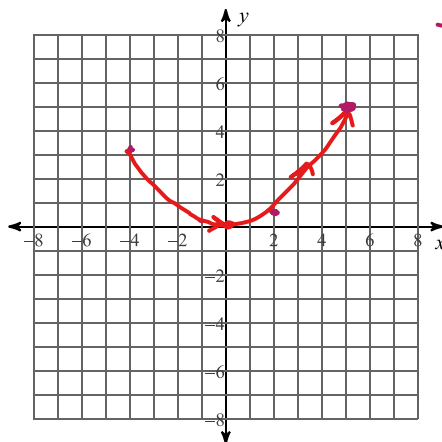
Sketch the curve for each pair of parametric equations.

1) $x = t, y = \frac{t^2}{3}, -4 \leq t \leq 4$



t	-4	-2	0	1	4
x	-4	-2	0	1	4
y	16/3	4/3	0	1/3	16/3

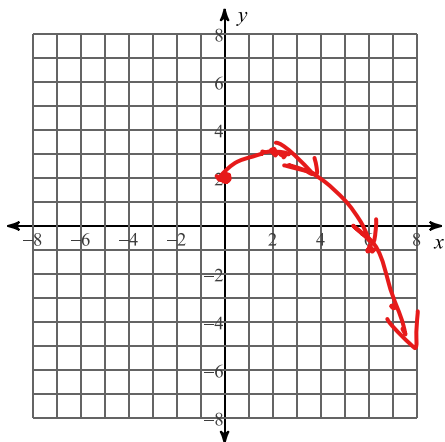
2) $x = 3t + 2, y = \frac{9t^2}{5} + \frac{12t}{5} + \frac{4}{5}, -2 \leq t \leq 1$



$\frac{1}{5}(9t^2 + 12t + 4)$
 $\frac{1}{5}(3t + 2)^2$
 $0 = 3t + 2$
 $t = -2/3$

t	-2	-1	0	1
x	-4	-1	2	5
y	16/5	1/5	4/5	5

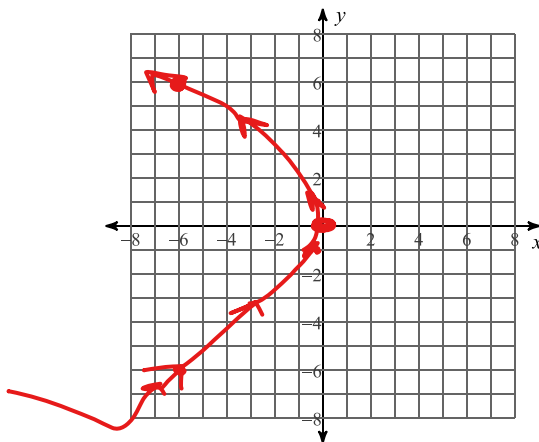
3) $x = \sqrt{7t}, y = -\frac{7t}{4} + 2 + \sqrt{7t}$



$-\frac{7}{4} + \frac{8}{4}$
 $-\frac{3}{4} + \sqrt{7}$
 $-\frac{35}{4} + \frac{8}{4}$
 $-\frac{27}{4}$

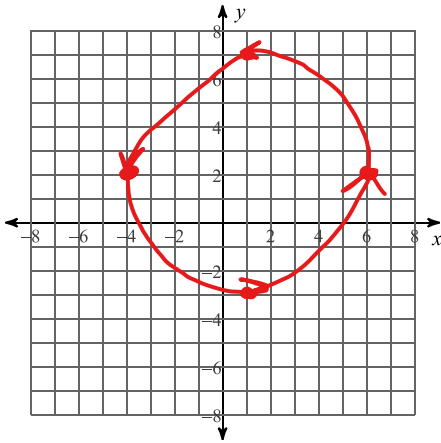
t	0	1	7	5	2
x	0	$\sqrt{7}$	7	$\sqrt{35}$	$\sqrt{14}$
y	2	$\frac{1}{4} + \sqrt{7}$	$-\frac{3}{4}$	$-\frac{7}{4} + \sqrt{35}$	

4) $x = -\frac{t^2}{6}, y = t$



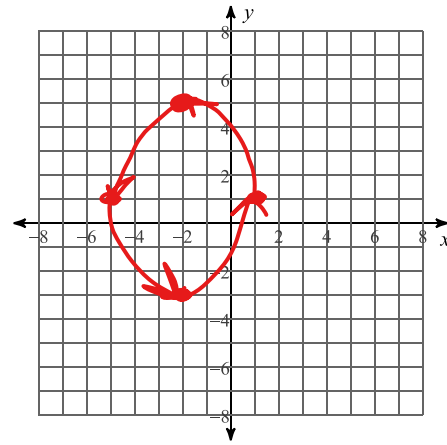
t	0	1	6	-6
x	0	-1/6	-6	-6
y	0	1	6	-6

5) $x = 5\cos t + 1$, $y = 5\sin t + 2$



t	0	$\pi/2$	π	$3\pi/2$	2π
x	6	1	-4	1	6
y	2	7	2	-3	2

6) $x = 3\cos t - 2$, $y = 4\sin t + 1$



t	0	$\pi/2$	π	$3\pi/2$	2π
x	1	-2	-5	-2	1
y	1	5	1	-3	1

Write each pair of parametric equations in rectangular form.

7) $x = 2t$, $y = -\frac{t^2}{6}$ *eliminate t*

$\frac{x}{2} = t$

$$y = -\frac{\left(\frac{x}{2}\right)^2}{6} \Rightarrow y = -\frac{x^2}{24}$$

Write each pair of parametric equations in rectangular form. State any restrictions on the domain.

8) $x = \sqrt{5t}$, $y = -\frac{5t}{4}$ *eliminate t*

$x^2 = 5t$

$$y = -\frac{x^2}{4}$$

$x \geq 0$

Write each pair of parametric equations in rectangular form.

9) $x = \frac{9t^2}{5} - 6t + 4, y = -3t + 3$

$$y - 3 = -3t$$

$$9t^2 = (-3t)^2$$

$$x = \frac{(y-3)^2}{5} + 2(y-3) + 4$$

10) $x = 5\sin t - 1, y = 5\cos t + 1$

$$\left(\frac{x+1}{5} = \sin t\right)^2 \quad \left(\frac{y-1}{5} = \cos t\right)^2$$

$$\frac{(x+1)^2}{25} = \sin^2 t \quad \frac{(y-1)^2}{25} = \cos^2 t$$

$$\frac{(x+1)^2}{25} + \frac{(y-1)^2}{25} = 1$$

11) $x = 3\sin 2t - 2, y = 3\cos 2t + 1$

$$\left(\frac{x+2}{3} = \sin 2t\right)^2 \quad \frac{y-1}{3} = \cos 2t$$

$$\frac{(x+2)^2}{9} = \sin^2 2t \quad \frac{(y-1)^2}{9} = \cos^2 2t$$

$$\frac{(x+2)^2}{9} + \frac{(y-1)^2}{9} = 1$$

12) $x = 3\sin t + 2, y = 4\cos t + 2$

$$\frac{x-2}{3} = \sin t$$

$$\frac{y-2}{4} = \cos t$$

$$\frac{(x-2)^2}{9} + \frac{(y-2)^2}{16} = 1$$

13) $x = 2\sec t, y = 2\tan t$

$$\left(\frac{x}{2} = \sec t\right)^2 \quad \left(\frac{y}{2} = \tan t\right)^2$$

$$\frac{x^2}{4} = \sec^2 t \quad \frac{y^2}{4} = \tan^2 t$$

$$\frac{x^2}{4} - \frac{y^2}{4} = 1$$

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \cot^2 x = \csc^2 x$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\tan^2 x - \sec^2 x = -1$$

$$\sec^2 x - \tan^2 x = 1$$