2. For $t \geq 0$, a particle moving in the xy-plane has the position vector $\langle x(t), y(t) \rangle$ at time t, where $\frac{dx}{dt} = -1 + e^{\sin t}$ and $\frac{dy}{dt} = \cos(t^2)$. At time $t = 2$, the position of the particle is $(5, 7)$.

(a) Find the acceleration vector of the particle at time $t = 2$.

$a(t) = \langle x''(t), y''(t) \rangle$

$a(2) = \langle x''(2), y''(2) \rangle$

$= \langle -1.033, 3.027 \rangle$

For practice

$x''(t) = \cos t \, e^{\sin t}$

$y''(t) = -\sin(t^2) \, 2t$

(b) Find the total distance traveled by the particle over the time interval $1.8 \leq t \leq 2$.

$$\int_{1.8}^{2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt \approx 360$$

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(c) Find the x-coordinate of the position of the particle at time $t = 1$.

$x(2) = 5$

$$x(1) + \int_1^2 (-1 + e^{\sin t})\,dt = x(2)$$

$$x(1) = x(2) - \int_1^2 (-1 + e^{\sin t})\,dt$$

$x(1) = 3395$

(d) At time $t = \sqrt{\frac{7\pi}{2}}$, the line tangent to the path of the particle is horizontal. Find the particle's speed at time $t = \sqrt{\frac{7\pi}{2}}$. Determine whether the particle is moving to the left or to the right at that time. Give a reason for your answer.

→ it's not moving vertically, so $\frac{dx}{dt}$ determines motion

$\frac{dy}{dx} = 0$ so $\frac{dy}{dt} = 0$

$$\text{Speed}\Big|_{t=\sqrt{\frac{7\pi}{2}}} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\Bigg|_{t=\sqrt{\frac{7\pi}{2}}} = 0.159$$

$$\frac{dx}{dt}\Big|_{t=\sqrt{\frac{7\pi}{2}}} = -0.159 \quad \left(\text{left at } t=\sqrt{\tfrac{7\pi}{2}}\right)$$

since $-0.159 < 0$

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