Parametric/Vector Review

1.) $x = t^2 + 5 \qquad y = e^{2t}$

$\frac{dy}{dt} = 2e^{2t} \qquad \frac{dx}{dt} = 2t \qquad \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2e^{2t}}{2t} = \frac{e^{2t}}{t}$

$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\frac{dy}{dx}}{dx/dt} \rightarrow \frac{\frac{t(2e^{2t}) - e^{2t}(1)}{t^2}}{2t} \rightarrow \frac{2te^{2t} - e^{2t}}{2t^3}$

2.) $x = 2 - t^2 \qquad y = t^3 - 4t$

$\frac{dx}{dt} = -2t \qquad \frac{dy}{dt} = 3t^2 - 4$

$-2t = 0$

$t = 0$

$x = 2 - 0 = 2$

$y = 0 - 0 = 0$

V.T @ $(2, 0)$

$3t^2 - 4 = 0$

$t = \pm\sqrt{4/3}$

$x = 2 - \left(\pm\sqrt{\frac{4}{3}}\right)^2 = 2 - \frac{4}{3} = \frac{2}{3}$

$y = \left(\pm\sqrt{\frac{4}{3}}\right)^3 - 4\left(\pm\sqrt{\frac{4}{3}}\right)$

$= \pm\frac{4}{3}\sqrt{\frac{4}{3}} \mp 4\sqrt{\frac{4}{3}} = \pm\frac{8}{3}\sqrt{\frac{4}{3}}$

H.T @ $\left(\frac{2}{3}, \frac{8}{3}\sqrt{\frac{4}{3}}\right) \left(\frac{2}{3}, \frac{-8}{3}\sqrt{\frac{4}{3}}\right.$

3.) $x = 8\cos t$ $y = 8t\sin t$ $0 \le t \le \pi/2$

$\frac{dx}{dt} = -8\sin t$ $\frac{dy}{dt} = 8\sin t + 8t\cos t$

$$L = \int_0^{\pi/2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt \approx 14.950$$

4.) $u = \langle 2, -1 \rangle$ $v = \langle -5, 7 \rangle$

(a) $3u = \langle 6, -3 \rangle$ $3u + v = \langle 1, 4 \rangle$

$v = \langle -5, 7 \rangle$

(b) $|3u+v| \rightarrow \sqrt{1^2 + 4^2} = \sqrt{17}$

(c) $u \cdot v \rightarrow (2)(-5) + (-1)(7) = -10 - 7 = -17$

(d) $u \cdot v = |u||v| \cos\theta$ $\cos\theta = \frac{-17}{\sqrt{5}\sqrt{74}} \rightarrow \theta \approx 152.103°$

$-17 = \sqrt{5} \cdot \sqrt{74} \cos\theta$ (take $\cos^{-1}$)

5.) $x = 2t^3 - 3t$ $y = -5t^2$

$\frac{dx}{dt} = 6t^2 - 3$ $\frac{dy}{dt} = -10t$

$\left.\frac{dx}{dt}\right|_{t=1} = 3$ $\left.\frac{dy}{dt}\right|_{t=} -10$

Tangent $\langle 3, -10 \rangle$ mag: $\sqrt{9+100} = \sqrt{109}$

unit vector tangent $\langle \frac{3}{\sqrt{109}}, \frac{-10}{\sqrt{109}} \rangle$ or $\langle \frac{-3}{\sqrt{109}}, \frac{10}{\sqrt{109}} \rangle$

unit vector perpendicular $\langle \frac{10}{\sqrt{109}}, \frac{3}{\sqrt{109}} \rangle$ or $\langle \frac{-10}{\sqrt{109}}, \frac{-3}{\sqrt{109}} \rangle$

6.) $r(t) = (\sec t)i + (\tan t)j$

a) $v(t) = r'(t) = (\sec t \tan t)i + (\sec^2 t)j$

$v(\frac{\pi}{6}) = (\sec\frac{\pi}{6} \tan\frac{\pi}{6})i + (\sec^2\frac{\pi}{6})j$

$= \left(\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{3}\right)i + \left(\frac{2}{\sqrt{3}}\right)^2 j$

$= \frac{2}{3}i + \frac{4}{3}j$

6) b $a(t)=v'(t)=(\sec t\tan t\tan t+\sec^3 t)i+(2\sec t\sec t\tan t)j$

$=(\sec t\tan^2 t+\sec^3 t)i+(2\sec^2 t\tan t)j$

c speed $=\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}$

$\text{speed}\Big|_{\pi/6}=\sqrt{\left(\frac{2}{3}\right)^2+\left(\frac{4}{3}\right)^2}=\sqrt{\frac{4+16}{9}}=\sqrt{\frac{20}{9}}=\frac{\sqrt{20}}{3}$

d $\left\langle \frac{\frac{2}{3}}{\frac{\sqrt{20}}{3}}, \frac{\frac{4}{3}}{\frac{\sqrt{20}}{3}} \right\rangle \rightarrow \left\langle \frac{2}{\sqrt{20}}, \frac{4}{\sqrt{20}} \right\rangle$

7.) $r(t)=(\sec t)i+(\tan t)j$

$r'(t)=(\sec t\tan t)i+(\sec^2 t)j$

$r'(-1)=-2.882i+3.426j$

$r(-1)=1.851i-1.557j$

→ But use exact values from calculator

$y+1.557=\frac{3.426}{-2.882}(x-1.851)$

T: $y+1.557=-1.188(x-1.851)$

N: $y+1.557=.841(x-1.851)$

8.) $\int \left((6-6t)i + (3\sqrt{t})j\right) dt$

$\rightarrow \left(6t - 3t^2 + c\right)i + \left(2t^{\frac{3}{2}} + c\right)j$

9.) $r(t) = ?$ $\quad \frac{dr}{dt} = (-\sin t)i + (\cos t)j \quad r(0) = j$

$\int \frac{dr}{dt} \rightarrow \quad r(t) = (\cos t + c)i + (\sin t + c)j$

$r(0) = 0i + 1j$

$\cos 0 + c = 0 \qquad \sin 0 + c = 1$

$1 + c = 0 \qquad 0 + c = 1$

$c = -1 \qquad c = 1$

$r(t) = (\cos t - 1)i + (\sin t + 1)j$

10.) $x(t) = e^t \cos t \quad y(t) = e^t \sin t$

ⓐ slope @ $t = \frac{\pi}{4}$? $x'\left(\frac{\pi}{4}\right) = -23.141 \quad y'\left(\frac{\pi}{4}\right) = -23.141$

$$\text{Slope @ } t = \frac{\pi}{4} = \frac{y'\left(\frac{\pi}{4}\right)}{x'\left(\frac{\pi}{4}\right)} = 1 \quad \left(\text{this is } \frac{dy}{dx}\right)$$

ⓑ $\text{speed} = \sqrt{(x'(t))^2 + (y'(t))^2}$

$$\text{Speed @ } t = 3 = \sqrt{(x'(3))^2 + (y'(3))^2} \approx 28.405$$

(use math 8 twice within $\sqrt{\ }$ and you won't have to do by hand)

ⓒ Distance Traveled $= \displaystyle\int_0^3 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt \approx 26.991$

(again → really careful or do some stuff by hand)

ⓓ $y'(3) = -17.050$ so vertical movement is down since $y'(3) < 0$.

11.) $\frac{dx}{dt} = \cos(t^2)$ $\frac{dy}{dt} = \sin(t^3)$

$t=3$ $\langle x(t), y(t)\rangle \to \langle 4,7\rangle$

$$x(1) + \int_1^3 \cos(t^2)\,dt = x(3)$$

$$x(1) = 4 - \int_1^3 \cos(t^2)\,dt \approx 4.202$$

$$y(1) + \int_1^3 \sin(t^3)\,dt = y(3)$$

$$y(1) = 7 - \int_1^3 \sin(t^3)\,dt \approx 6.777$$

$(4.202, 6.777)$ @ $t=1$

12.) $P \to \langle -400, 0\rangle$ $W \to \langle 50\cos 135, 50\sin 135\rangle$

New $\to \langle -400 + 50\cos 135, 0 + 50\sin 135\rangle$

$\to \langle -435.355, 35.355\rangle$

ground speed $= \sqrt{(-435.355)^2 + (35.355)^2} \approx 436.789$ mph

$$\theta = \tan^{-1}\left(\frac{35.356}{-435.355}\right) \approx -4.643°$$

Angle $= -4.643° + 180° \approx 175.357°$

4.) $a(t) = 2j$

$\int 2j \rightarrow (C)i + (2t + c)j$

$v(0) = 0i + 0j \Rightarrow 0 = c$ $\quad 0 = 2(0) + c$

$\rightarrow c = 0$

$v(t) = 2tj$

$\int 2tj \rightarrow (c)i + (t^2 + c)j$ $\quad r(0) = i$

$i = ci \rightarrow c = 1$ $\quad (0)^2 + c = 0$ $\quad c = 0$

$r(t) = i + t^2 j$

6.) $\frac{dx}{dt} = 3 + \cos(t^2)$ @ $t=2 \rightarrow$ position $\rightarrow (1,8)$

a $x(2) + \int_2^4 (3+\cos(t^2))\,dt = x(4)$

$x(4) \approx 7.133$

b $\left.\frac{dy}{dt}\right|_{t=2} = -7$ $\left.\frac{dx}{dt}\right|_{t=2} = 3 + \cos(4) \approx 2.346$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-7}{2.346} \approx -2.983$

$y - 8 = -2.983(x-1)$

c speed$\Big|_{t=2} = \sqrt{(3+\cos(4))^2 + (-7)^2} \approx 7.383$

d $\frac{dy}{dx}$ (for $t \geq 3$) $= 2t+1 = \frac{dy/dt}{3+\cos(t^2)}$

$\frac{dy}{dt} = (2t+1)(3+\cos(t^2))$

$a(4) = \left\langle \frac{d}{dt}\left.\frac{dx}{dt}\right|_{t=4}, \frac{d}{dt}\left.\frac{dy}{dt}\right|_{t=4} \right\rangle \approx \langle 2.303, 24.814 \rangle$