AP Calculus Partial Fractions Homework Name: Key

1. Evaluate the following integrals.

a) $\int \frac{x-5}{x^2-4x+3}dx = \int\left(\frac{-1}{x-3}+\frac{2}{x-1}\right)dx$

$\frac{A}{x-3}+\frac{B}{x-1}=\frac{x-5}{(x-3)(x-1)}$

$A(x-1)+B(x-3)=x-5$

$x=1 \to -2B=-4 \to B=2$

$x=3 \to 2A=-2 \quad A=-1$

$\boxed{\to -\ln|x-3| + 2\ln|x-1| + C}$

b) $\int \frac{3x+2}{(5x-1)(x+1)}dx \to \int\left(\frac{13}{6(5x-1)}+\frac{1}{6(x+1)}\right)dx$

$\frac{A}{5x-1}+\frac{B}{x+1}=\frac{3x+2}{(5x-1)(x+1)}$

$A(x+1)+B(5x-1)=3x+2$

$A=-1 \to -6B=-1 \quad B=1/6$

$A=1/5 \to \frac{6}{5}A=\frac{13}{5} \quad A=13/6$

$\boxed{=\frac{13}{6}\cdot\frac{1}{5}\ln|5x-1| + \frac{1}{6}\ln|x+1| +}$

c) $\int \frac{x}{x^2-2x-3}dx = \int\left(\frac{3}{4(x-3)}+\frac{1}{4(x+1)}\right)dx$

$\frac{A}{x-3}+\frac{B}{x+1}=\frac{x}{(x-3)(x+1)}$

$x=3 \to 4A=3 \quad A=3/4$

$x=-1 \quad -4B=-1 \quad B=1/4$

$\boxed{\frac{3}{4}\ln|x-3| + \frac{1}{4}\ln|x+1| + C}$

d) $\int \frac{5x-7}{x^2-3x+2}dx$

$\frac{A}{x-1}+\frac{B}{x-2}=\frac{5x-7}{(x-1)(x-2)}$

$x=1 \to -A=-2 \to A=2$

$x=2 \to B=3$

$\int\left(\frac{2}{x-1}+\frac{3}{x-2}\right)dx = \boxed{2\ln|x-1| + 3\ln|x-2| + C}$

2. Determine whether the integral converges or diverges, and evaluate the integral if it converges.

a) $\int_4^\infty \frac{1}{\sqrt{x-3}}dx$ $\quad \lim_{b\to\infty}\int_4^b \frac{1}{\sqrt{x-3}}dx \to \lim_{b\to\infty}\left[2\sqrt{x-3}\right]_4^b \to \lim_{b\to\infty}\left(2\sqrt{b-3}-2\sqrt{4-3}\right)$

$\infty - 2 \to \boxed{\infty \text{ diverges}}$

b) $\int_{\frac{1}{4}}^\infty \frac{5}{4x-1}dx$ $\quad \lim_{a\to 1/4^+}\int_a^1 \frac{5}{4x-1}dx + \lim_{b\to\infty}\int_1^b \frac{5}{4x-1}dx$

$\to \lim_{a\to\frac{1}{4}^+}\left[5\cdot\frac{1}{4}\cdot\ln|4x-1|\right]_a^1 + \lim_{b\to\infty}\left[5\cdot\frac{1}{4}\cdot\ln|4x-1|\right]_1^b$

$\to \lim_{a\to\frac{1}{4}^+}\left(\frac{5}{4}\ln|4(1)-1| - \frac{5}{4}\ln|4a-1|\right) + \lim_{b\to\infty}\left(\frac{5}{4}\ln|4b-1| - \frac{5}{4}\ln|4(1)-1|\right)$

$\to -\frac{5}{4}\cdot(-\infty)$

$\to \frac{5}{4}\ln 3 + \infty + \infty - \frac{5}{4}\ln 3 \to \boxed{\infty \text{ diverges}}$

c) $\int_0^{16}\frac{1}{\sqrt[4]{x}}dx$

$\lim_{a\to 0^+}\int_a^{16} x^{-\frac{1}{4}}dx$

$\to \lim_{a\to 0^+}\left[\frac{4}{3}x^{\frac{3}{4}}\right]_a^{16} \to \lim_{a\to 0^+}\left(\frac{4}{3}(16)^{3/4} - \frac{4}{3}(a)^{3/4}\right) \to \frac{4}{3}(2)^3 - 0$

2^4

$\boxed{\to 32/3}$

d) $\int_0^\infty \frac{e^{-\frac{1}{x}}}{x^2}dx$

$u=-\frac{1}{x}$ $\frac{du}{dx}=\frac{1}{x^2}$

$du=\frac{1}{x^2}dx$

$\int e^u du \to e^u \to e^{-\frac{1}{x}}$

$\lim_{a\to0^+}\int_a^1 \frac{e^{-1/x}}{x^2}dx+\lim_{b\to\infty}\int_1^b \frac{e^{-1/x}}{x^2}dx \to \lim_{a\to0^+}\left[e^{-\frac{1}{x}}\right]_a^1+\lim_{b\to\infty}\left[e^{-\frac{1}{x}}\right]_1^b$

$\lim_{a\to0^+}\left(e^{-1}-\frac{1}{e^{1/a}}\right)+\lim_{b\to\infty}\left(\frac{1}{e^{1/b}}-e^{-1}\right)$

$\frac{1}{e}-0+1-\frac{1}{e}$

$\to 1$

3. Use one of the comparison tests to determine whether $\int_1^\infty \frac{1}{x^3-3}dx$ converges or diverges.

$\frac{1}{x^3}\leq\frac{1}{x^3-3}$ for $x\geq1$ $\int_1^\infty \frac{1}{x^3}dx$ converges to $\frac{1}{3-1}=\frac{1}{2}$ since p-series $a=1$, $p>1$

ugh. this is inconclusive. so, $\lim_{x\to\infty}\frac{\frac{1}{x^3-3}}{\frac{1}{x^3}}\to\frac{x^3}{x^3-3}\to1$ since $0<1<\infty$ and $\int_1^\infty\frac{1}{x^3}dx$ converges, $\boxed{\int_1^\infty\frac{1}{x^3-3}dx \text{ converges by LCT}}$

4. Use one of the comparison tests to determine whether $\int_1^\infty \frac{1}{x^2-3x+5}dx$ converges or diverges.

$\lim_{x\to\infty}\frac{\frac{1}{x^2-3x+5}}{\frac{1}{x^2}}\to\frac{x^2}{x^2-3x+5}\to1$ $0<1<\infty$

since $\int_1^\infty\frac{1}{x^2}dx$ is p-series $a>0$, $p>1$ $\to$ converges

$\boxed{\text{so by LCT, } \int_1^\infty\frac{1}{x^2-3x+5}dx \text{ converges}}$

5. Use one of the comparison tests to determine whether $\int_1^\infty \frac{1}{\sqrt{x}-3}dx$ converges or diverges.

$\frac{1}{\sqrt{x}-3}\geq\frac{1}{\sqrt{x}}$ for $x\geq1$

$\int_1^\infty\frac{1}{\sqrt{x}}dx$ is p-series, $p\leq1$ so diverges

since $\frac{1}{\sqrt{x}-3}\geq\frac{1}{\sqrt{x}}$ for $x\geq1$

$\boxed{\text{by DCT } \int_1^\infty\frac{1}{\sqrt{x}-3} \text{ also diverges}}$

6. Evaluate the following integrals.

a) $\int\tan^{-1}x\,dx$ $u=\tan^{-1}x$ $dv=dx$

$du=\frac{1}{1+x^2}dx$ $v=x$

$x\tan^{-1}x-\int\frac{x}{1+x^2}dx$ $u=1+x^2$

$du=2x\,dx$

$\frac{du}{2}=x\,dx$

$-\frac{1}{2}\int\frac{1}{u}du \to \boxed{x\tan^{-1}x-\frac{1}{2}\ln|1+x^2|+C}$

b) $\int\frac{\cos(\sqrt{x})}{\sqrt{x}}dx$

$u=\sqrt{x}$

$du=\frac{1}{2\sqrt{x}}dx$

$2du=\frac{1}{\sqrt{x}}dx$

$2\int\cos u\,du$

$2\sin u+C$

$\boxed{2\sin\sqrt{x}+C}$

c) $\int x^3\sqrt[4]{x}\,dx$

$\int x^3\cdot x^{1/4}dx$

$\to\int x^{13/4}dx$

$\boxed{\to\frac{4}{17}x^{17/4}+C}$

d) $\int\sin x\,e^{\cos x}dx$

$u=\cos x$

$-du=\sin x\,dx$

$-\int e^u du\to -e^u+C$

$\boxed{\to -e^{\cos x}+C}$