Polar Area

Name

1. Which of the following integrals represents the area enclosed by the smaller loop of the graph of $r = 1 + 2\sin\theta$?

(A) $\frac{1}{2}\int_{7\pi/6}^{11\pi/6} (1+2\sin\theta)^2\,d\theta$

(B) $\frac{1}{2}\int_{7\pi/6}^{11\pi/6} (1+2\sin\theta)\,d\theta$

(C) $\frac{1}{2}\int_{-\pi/6}^{7\pi/6} (1+2\sin\theta)^2\,d\theta$

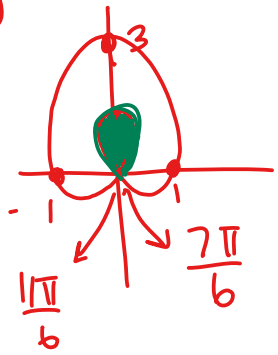
(D) $\int_{-\pi/6}^{7\pi/6} (1+2\sin\theta)^2\,d\theta$

(E) $\int_{-\pi/6}^{7\pi/6} (1+2\sin\theta)\,d\theta$

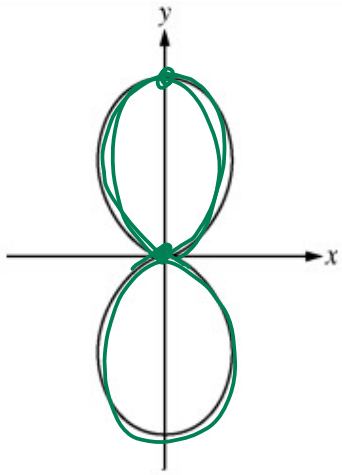
$r = 0 \quad \sin\theta = -\frac{1}{2} \quad \theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

$-\frac{\pi}{6}, -\frac{5\pi}{6}$

$\frac{1}{2}\int r^2\,d\theta$



2.



$\theta = 0 \quad r = 0$

$\theta = \frac{\pi}{2} \quad r = 1$

$\theta = \pi \quad r = 0$

$\theta = \frac{3\pi}{2} \quad r = 1$

$\frac{1}{2}\int_0^{2\pi} r^2\,d\theta$

$\frac{1}{2}\int_0^{2\pi} \sin^4\theta\,d\theta$

Which of the following expressions gives the total area enclosed by the polar curve $r = \sin^2\theta$ shown in the figure above?

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(A) $\frac{1}{2}\int_0^{\pi} \sin^2\theta\, d\theta$

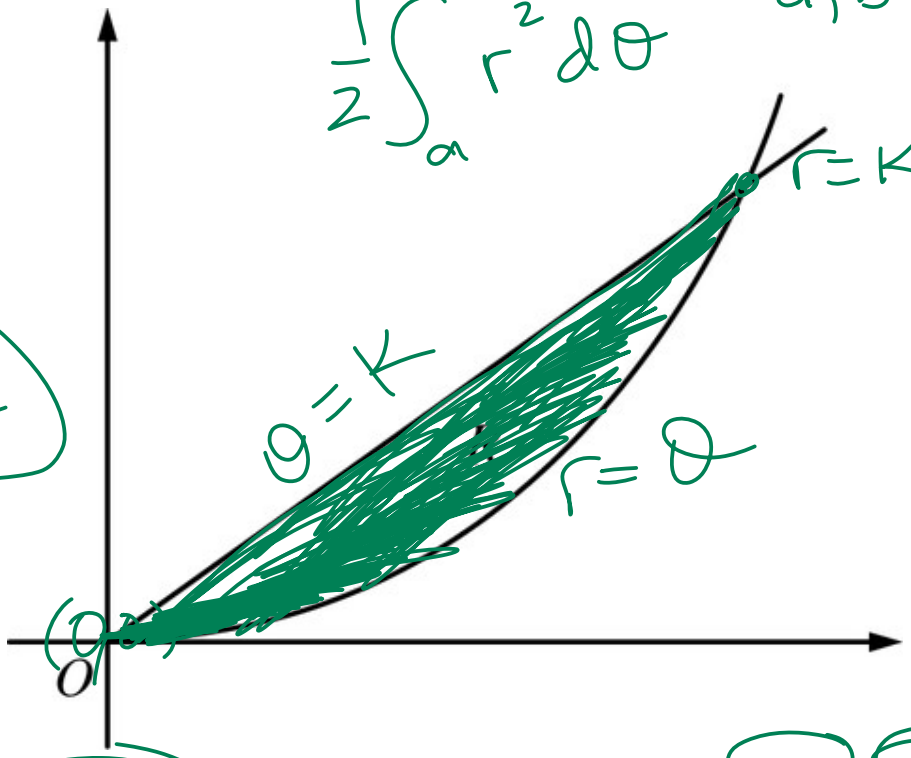
(B) $\int_0^{\pi} \sin^2\theta\, d\theta$

(C) $\frac{1}{2}\int_0^{\pi} \sin^4\theta\, d\theta$

(D) $\int_0^{\pi} \sin^4\theta\, d\theta$

(E) $2\int_0^{\pi} \sin^4\theta\, d\theta$

3.



Let R be the region in the first quadrant that is bounded by the polar curves $r = \theta$ and $\theta = k$, where k is a constant, $0 < k < \frac{\pi}{2}$, as shown in the figure above. What is the area of R in terms of k?

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(A) $\frac{k^3}{6}$

(B) $\frac{k^3}{3}$

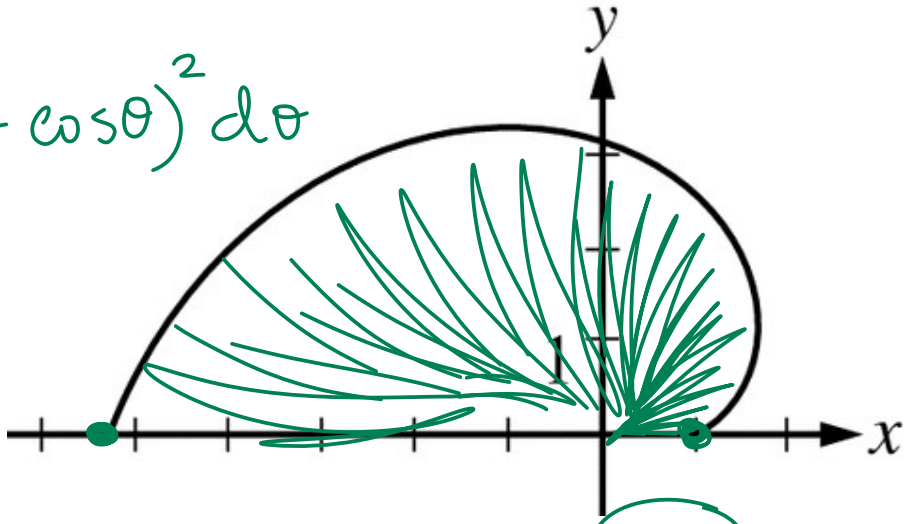
(C) $\frac{k^3}{2}$

(D) $\frac{k^2}{4}$

(E) $\frac{k^2}{2}$

4.

$$\frac{1}{2}\int_0^{\pi}(2\theta+\cos\theta)^2\,d\theta$$



The graph above shows the polar curve $r = 2\theta + \cos\theta$ for $0 \le \theta \le \pi$. What is the area of the region bounded by the curve and the *x*-axis?

$r =$ at 0

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(A) 3.069

(B) 4.935

(C) 9.870

(D) 17.456

(E) 34.912

5. Which of the following gives the total area enclosed by the graph of the polar curve r = θ sin 2θ for $0 \leq \theta \leq 2\pi$?

(A) $\int_0^{2\pi} \frac{1}{2}\left|\theta \sin 2\theta\right| d\theta$

(B) $\int_0^{2\pi} \left|\theta \sin 2\theta\right| d\theta$

(C) $\int_0^{2\pi} \frac{1}{2}\left|\theta \sin 2\theta\right|^2 d\theta$

(D) $\int_0^{2\pi} \left|\theta \sin 2\theta\right|^2 d\theta$

(E) $\int_0^{2\pi} \frac{\pi}{2}\left|\theta \sin 2\theta\right|^2 d\theta$

$\frac{1}{2}\int_0^{2\pi} (\theta \sin 2\theta)^2 \, d\theta$

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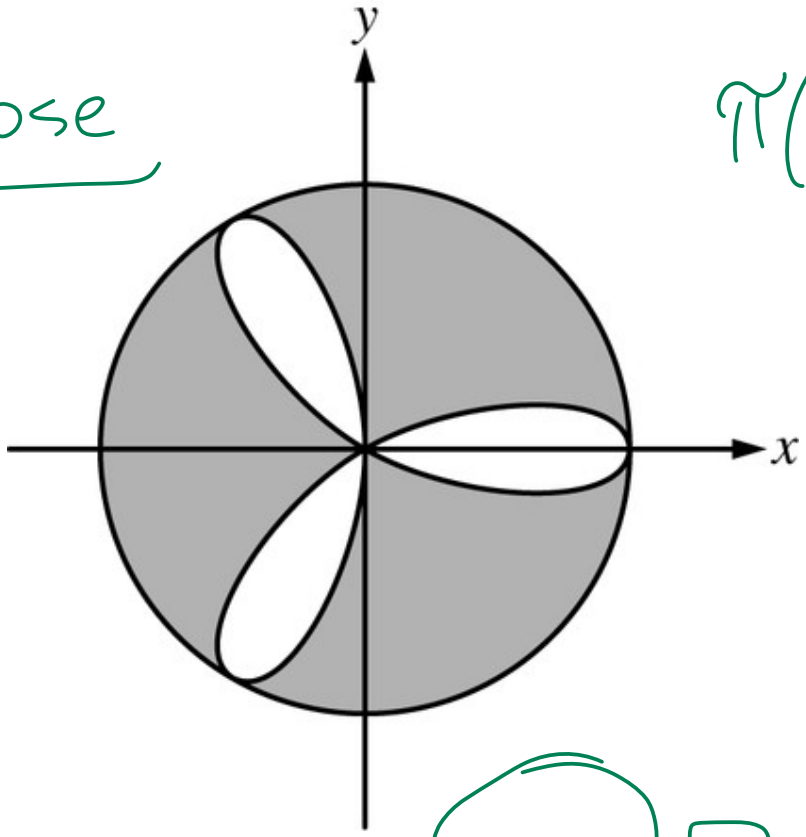
6.



The figure above shows the graphs of the polar curves $r = 2\cos(3\theta)$ and $r = 2$. What is the sum of the areas of the shaded regions?

(A) 6.858

(B) 3.142

(C) 8.566

(D) 9.425

(E) 15.708

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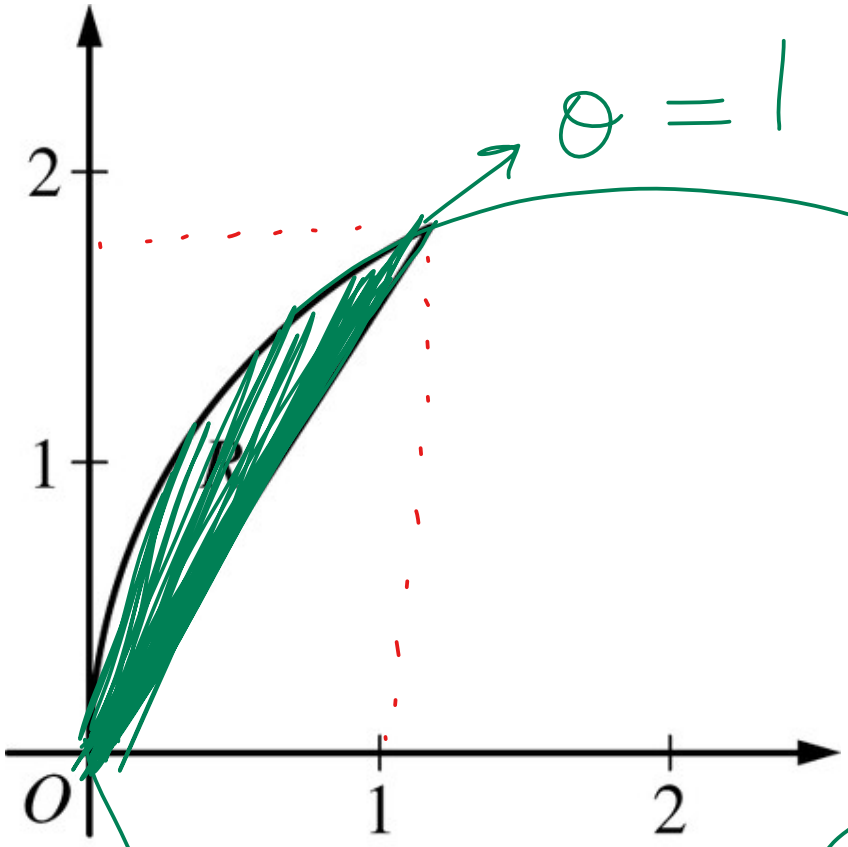
7.



Let R be the region in the first quadrant that is bounded above by the polar curve $r = 4\cos\theta$ and below by the line $\theta = 1$, as shown in the figure above. What is the area of R ?

(A) 0.317

(B) 0.465

(C) 0.929

(D) 2.618

(E) 5.819

$$\frac{1}{2}\int_{1}^{\pi/2} (4\cos\theta)^2 \, d\theta$$

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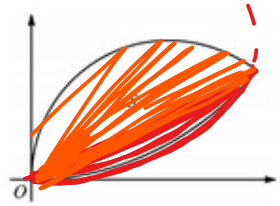
8.



Let S be the region in the first quadrant bounded above by the graph of the polar curve $r = \cos\theta$ and bounded below by the graph of the polar curve $r = 2\theta$, as shown in the figure above. The two curves intersect when $\theta = 0.450$. What is the area of S ?

$$\frac{1}{2}\int_0^{.450} (2\theta)^2\,d\theta + \frac{1}{2}\int_{.450}^{\pi/2} (\cos\theta)^2\,d\theta$$

(A) 0.232

(B) 0.243

(C) 0.271

(D) 0.384

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9. What is the total area between the polar curves r = 5 sin(3θ)and r = 8 sin(3θ)?

$$\frac{1}{2}\int_0^{\pi} \left((8\sin 3\theta)^2 - (5\sin 3\theta)^2\right) d\theta$$

(A) 2.000

(B) 7.069

(C) 30.631

(D) 61.261

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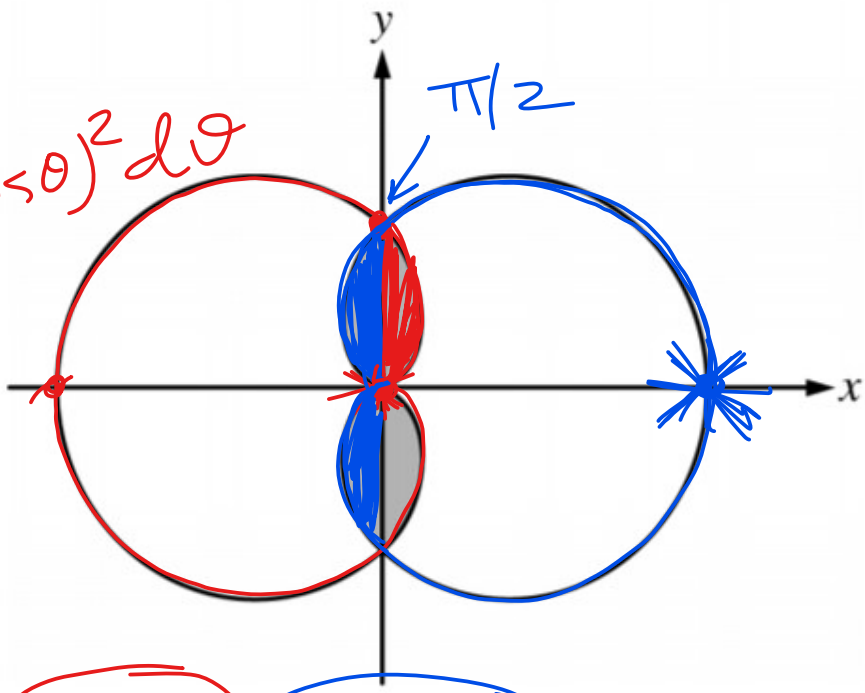
10.



The polar curves $r = 1 - \cos\theta$ and $r = 1 + \cos\theta$ are shown in the figure above. Which of the following expressions gives the total area of the shaded regions?

(A) $\int_0^{\pi} (1 + \cos\theta)^2 \, d\theta$

(B) $\int_{\pi/2}^{\pi} (1 + \cos\theta)^2 \, d\theta$

(C) $2\int_0^{\pi/2} (1 - \cos\theta)^2 \, d\theta$

(D) $\int_0^{\pi/2} \left((1 - \cos\theta)^2 + (1 + \cos\theta)^2 \right) d\theta$